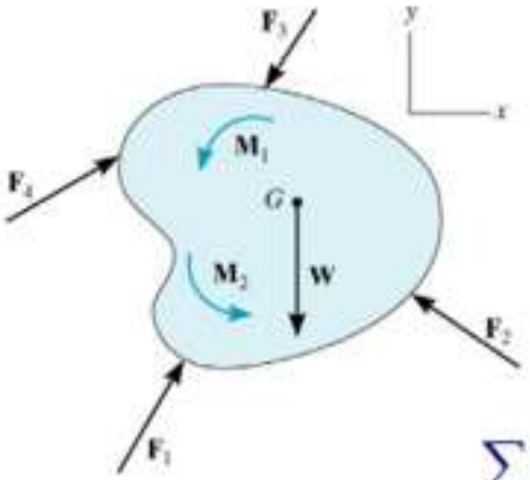
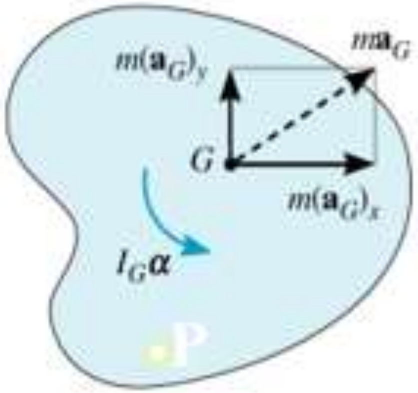
General plane motion



Sometimes, it may be convenient to write the moment equation about some point P other than G. Then the equations of motion are written as follows.



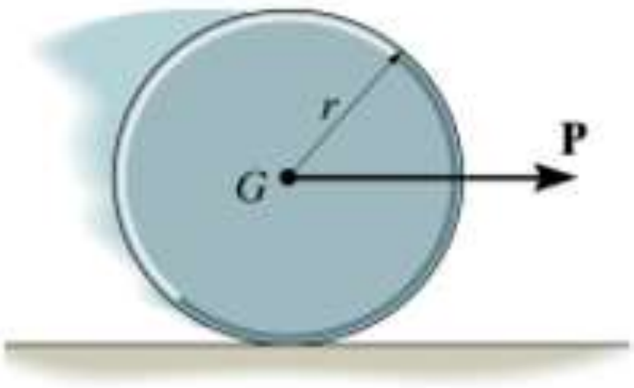
$$\sum F_x = m\,(a_G)_x$$

$$\sum F_y = m\,(a_G)_y$$

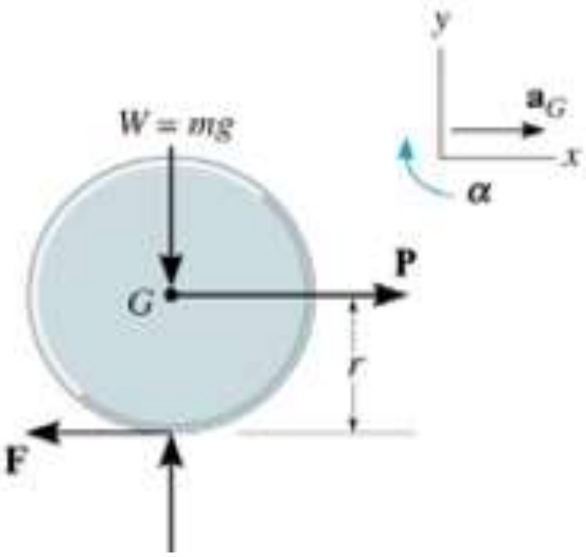
$$\sum M_P = \sum (M_k) = I_G\alpha + r_G\, m\,(a_G)_t = (I_G + m\,(r_G)^2)\,\alpha$$

In this case, $\sum (M_k)_P$ represents the sum of the moments of $I_G\boldsymbol{\alpha}$ and $m\boldsymbol{a}_G$ about point P.

Frictional Rolling



For example, consider a disk with mass m and radius r, subjected to a known force P.



The equations of motion will be

$\sum F_x = m(a_G)_x \quad => \quad P - F = m a_G$

$\sum F_y = m(a_G)_y \quad => \quad N - mg = 0$

$\sum M_G = I_G \alpha \quad => \quad F\, r = I_G \alpha$

There are 4 unknowns (F, N, α, and a_G) in these three equations.

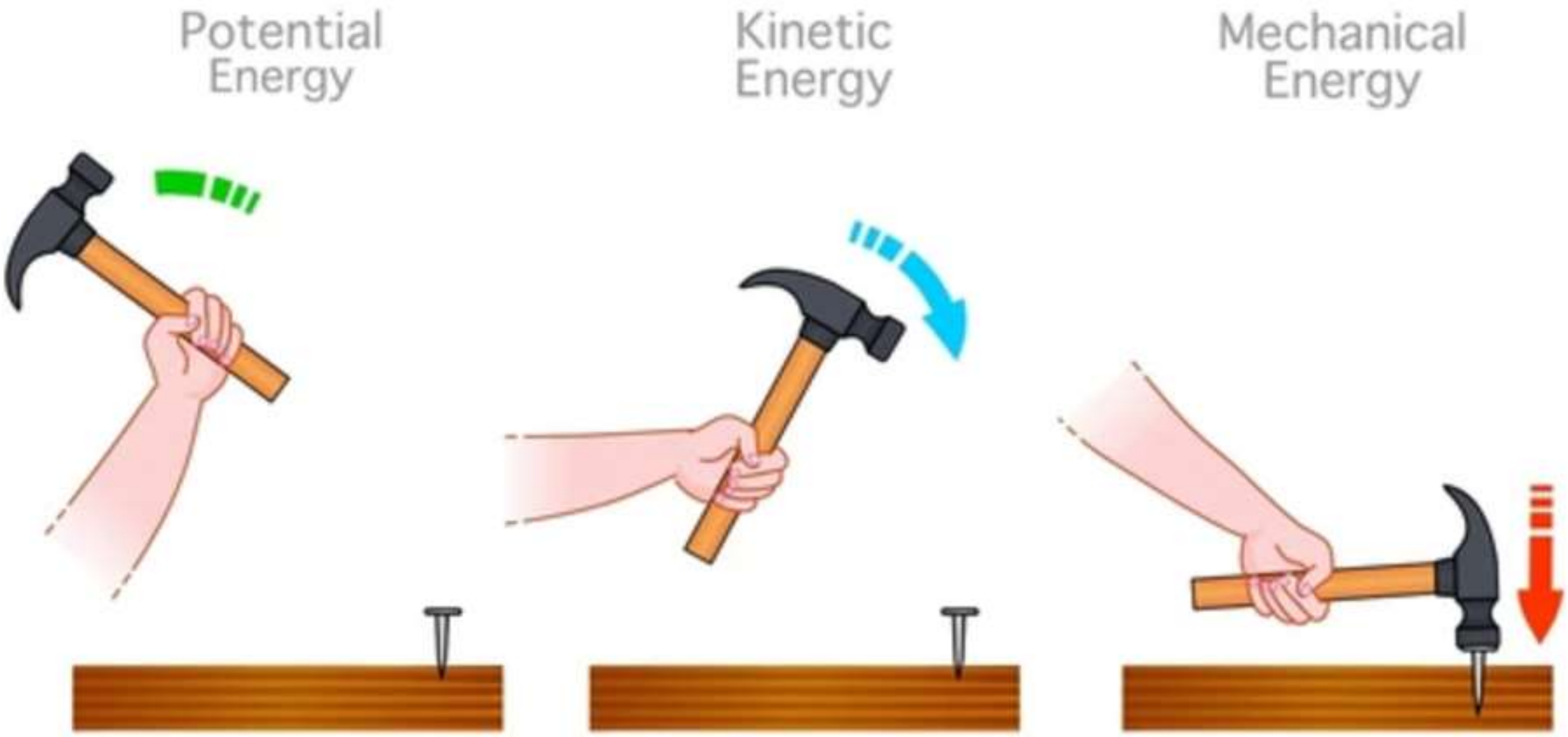
Planar kinetics of a rigid body: Work and Energy

Kinetic energy

Work of a force

Work of a couple

Principle of work and energy



Potential Energy
Kinetic Energy
Mechanical Energy

Kinetic energy

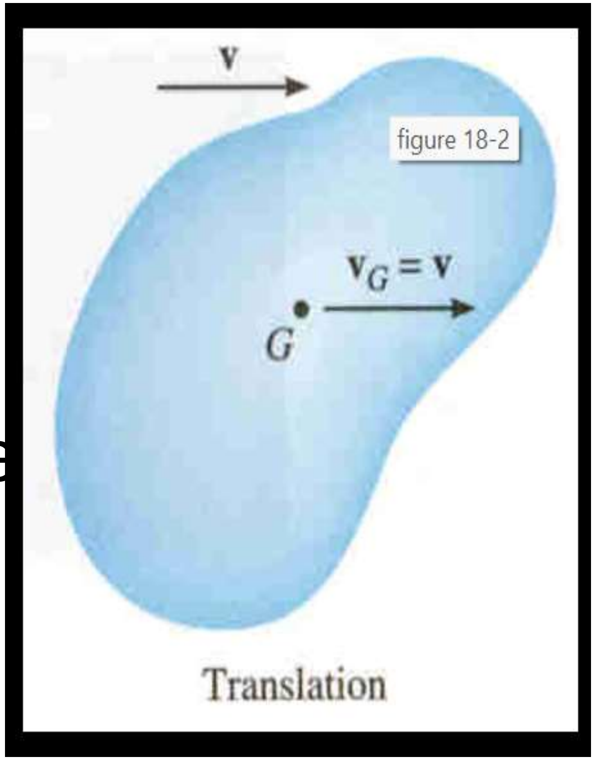
The kinetic energy of a rigid body can be expressed as the sum of its translational and rotational kinetic energies.

In equation form, a body in general plane motion has kinetic energy given by

$$T = 1/2\ m\ (v_G)^2 + 1/2\ I_G\ \omega^2$$

Sever… cur.

1. Pure Translation: When a rigid body is subjecte
   to only curvilinear or rectilinear translation,
   the rotational kinetic energy is zero



2. Pure Rotation: When a rigid body is rotating about a fixed axis passing through point O, the body has both translational and rotational kinetic energy. Thus,

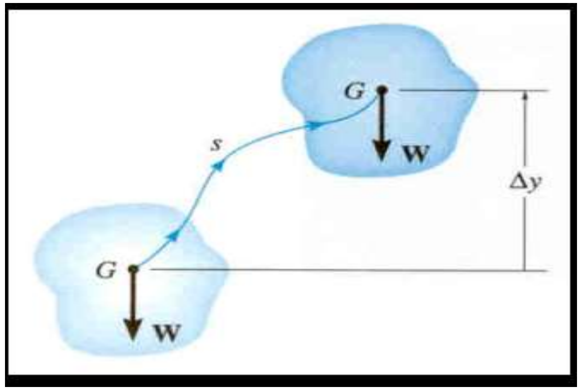
$$T = \frac{1}{2}\, m\, (v_G)^2 + \frac{1}{2}\, I_G\, (\omega)^2$$

Since $v_G = r_G\, \omega$, we can express the kinetic energy of the body as

$$T = \frac{1}{2}\, (I_G + m\, (r_G)^2 = \frac{1}{2}\, I_O\, (\omega)^2$$

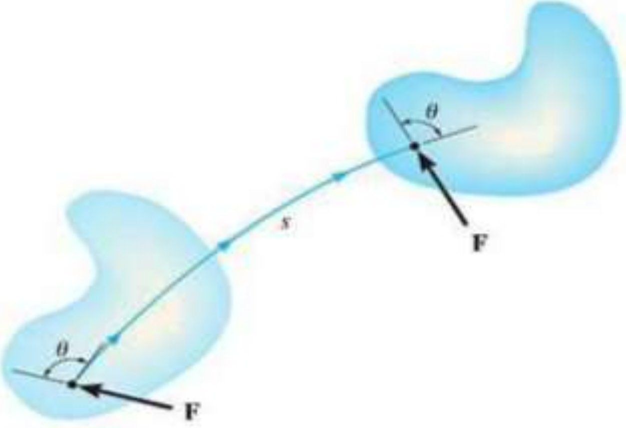
Work of a force

The force does work when it undergoes a displaceme
In the direction of the force $U_w = -W\Delta y$
Weight

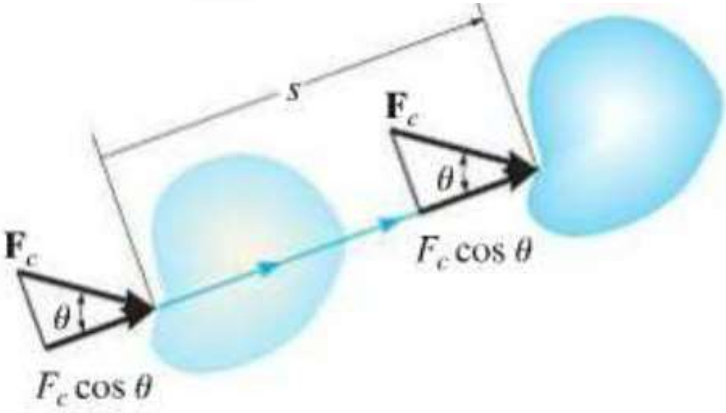


Recall that the work done by a force can be written a

$$U_F = \int \mathbf{F} \cdot d\mathbf{r} = \int (F \cos \theta)\, ds.$$

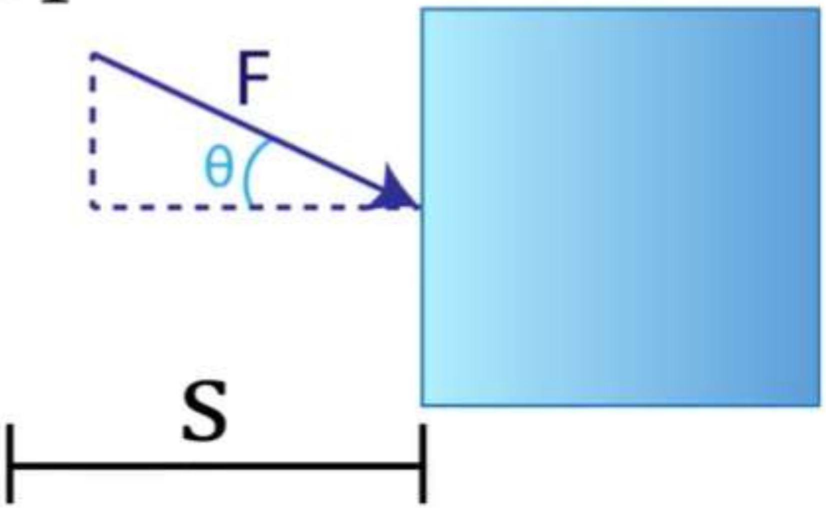


When the force is constant, this equation reduces to
$U_{Fc} = (F_c \cos\theta)\, s$ where $F_c \cos\theta$ represents the component of the force acting in the direction of displacement s.



$$U_F = (F \cos\theta)s$$

$$U_F = \int_{s_1}^{s_2} F \cos\theta \, ds$$



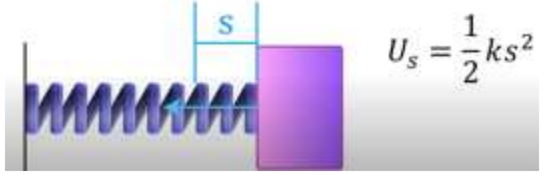
Work Done by a Spring

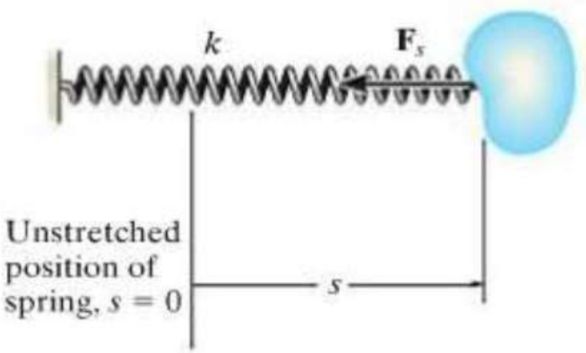
$$U_w = -\frac{1}{2}ks^2$$

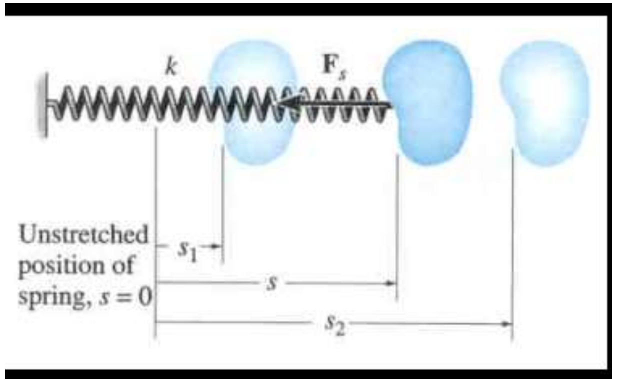
Remember, if the force and movement are in the same direction, the work is positive.

Work of a spring force: For a linear spring, the work is

$$Us = -\frac{1}{2}k[((s_2)^2 - (s_1)^2]$$

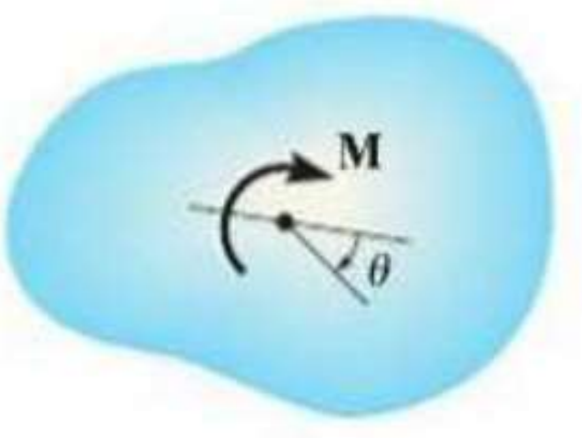


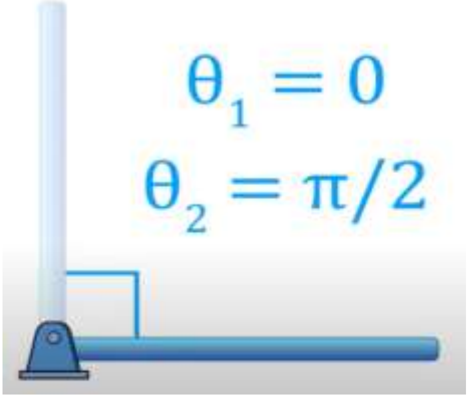




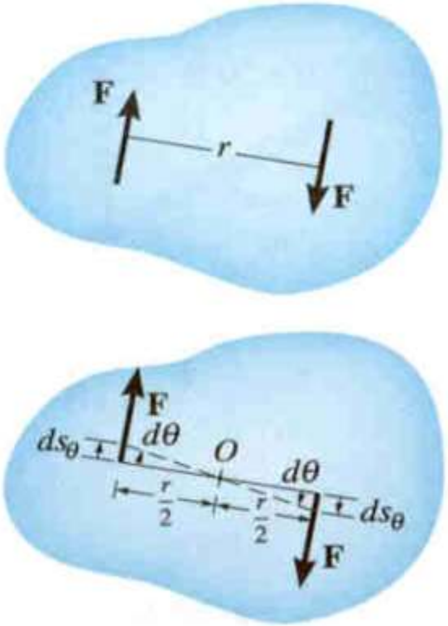
$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

$$U_M = M(\theta_2 - \theta_1)$$





The Work of a couple



Rotation

When a body subjected to a couple experiences general plane motion, the two couple forces do work only when the body undergoes rotation.

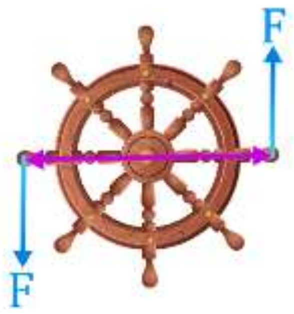
If the body rotates through an angular displacement dθ, the work of the couple moment, M, is

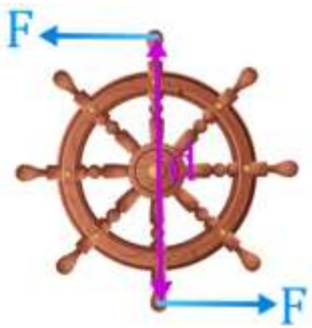
$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

If the couple moment, M, is constant, then

$$U_M = M(\theta_2 - \theta_1)$$

Here the work is positive, provided M and $(\theta_2 - \theta_1)$ are in the same direction.





$M = Fd$

Principle of Work and Energy

$$T_1 + \sum U_{1-2} = T_2$$

- T_1: Initial Kinetic Energy
- $\sum U_{1-2}$: Work
- T_2: Final Kinetic Energy

Principle of Work of and Energy

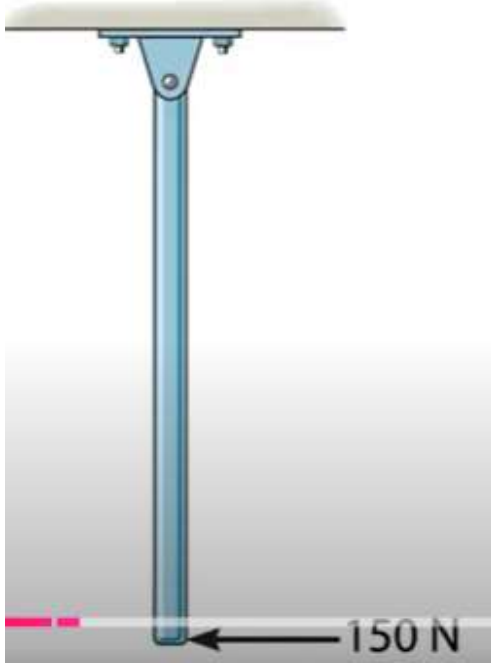
Problems that involve velocity, force and displacement can be solved using the principle of work and energy.

$$T_1 + \sum U_{1-2} = T_2$$

This equation states that the body s initial translational and rotational kinetic energy, plus the work done by all the external forces and couple moments acting on the body as the body moves from its initial to its final position, is equal the body s final translational and rotational kinetic energy.

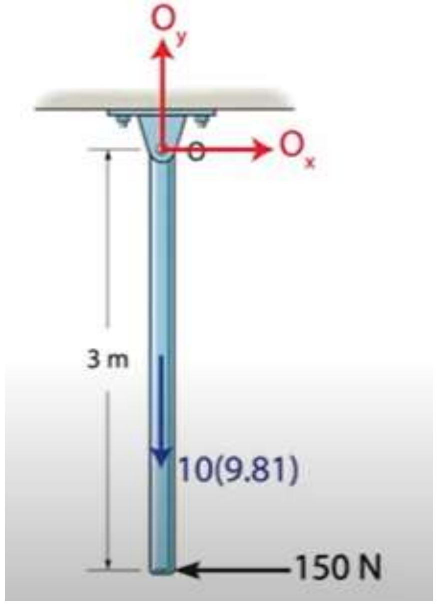
Example 6

The 10 kg uniform slender rod is suspended at the rest when the force of F= 150 N is applied to its end. Determine the angular velocity of the rod when it has rotated 90 clockwise from the position shown. The force always perpendicular to the rod.



Solution

$$T_1 + \sum U_{1-2} = T_2$$



$$I_O = \frac{1}{3}ML^2$$

$$I_O = \frac{1}{3}(10)(3)^2$$

$$I_O = 30 \text{ kg} \cdot \text{m}^2$$

$$T_2 = \frac{1}{2}I_O\omega^2$$

$$T_2 = \frac{1}{2}(30)\omega^2$$

$$T_1 = 0$$

$$T_2 = 15\omega^2$$

Work

$$90° = \frac{\pi}{2}$$

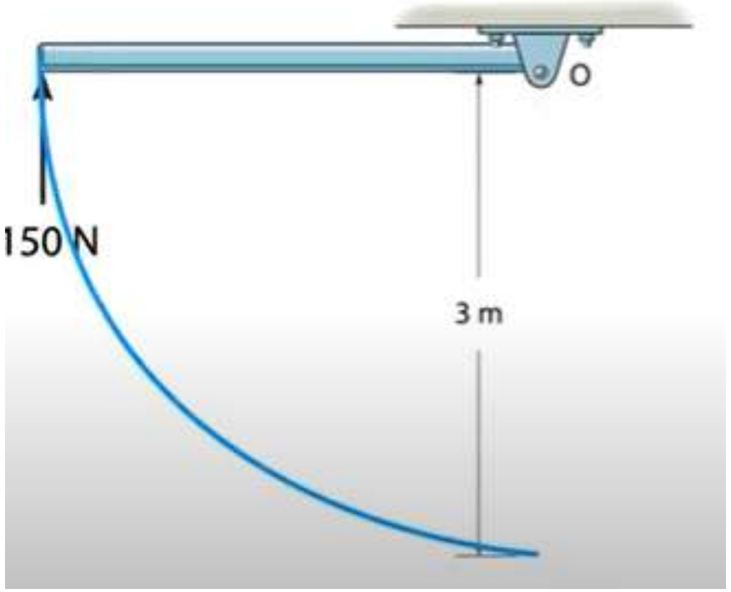
$$s_F = \theta r$$

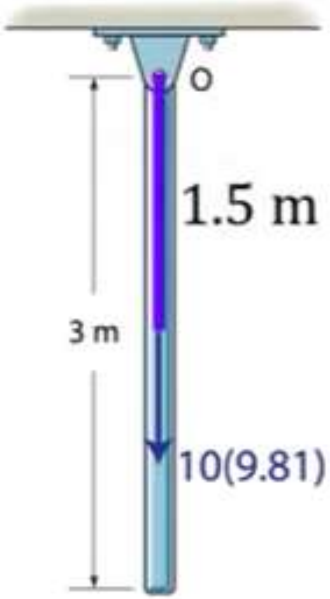
$$s_F = \frac{\pi}{2}(3)$$

$$s_F = \frac{3\pi}{2}$$

$$U_F = Fs$$

$$U_F = 150\left(\frac{3\pi}{2}\right)$$





$$U_W = -(10)(9.81)(1.5)$$

$$U_W = -147.15 \text{ J}$$

$$T_1 = 0$$

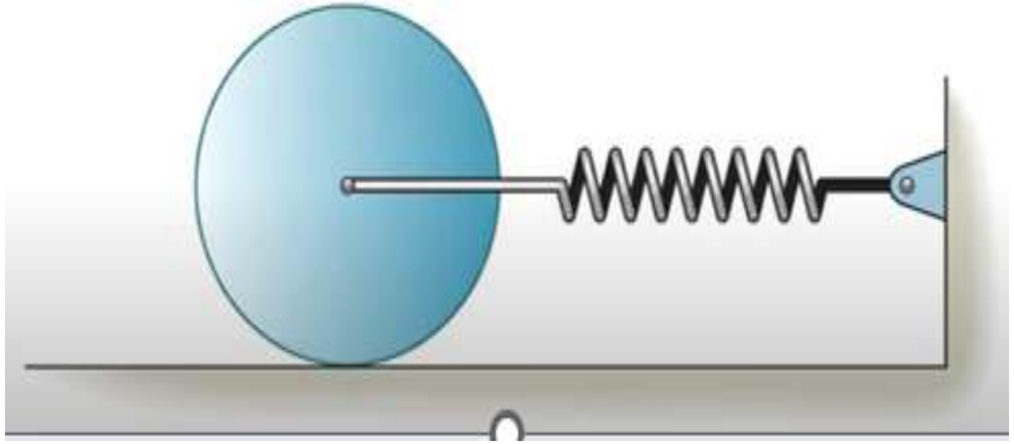
$$T_2 = 15\omega^2$$

$$U_W = -147.15 \text{ J}$$

$$U_F = 225\pi \text{ J}$$

Example 7

The 30 kg disk is originally at rest, the spring is unstretched, A couple moment M= 80 N.m is then applied to the disk as shown. Determine how far the center or mass of the disk travels along the plane before it momentarily stops. The disk rolls without slipping.



Solution

$$T_1 + \sum U_{1-2} = T_2$$

$T_1 = 0$

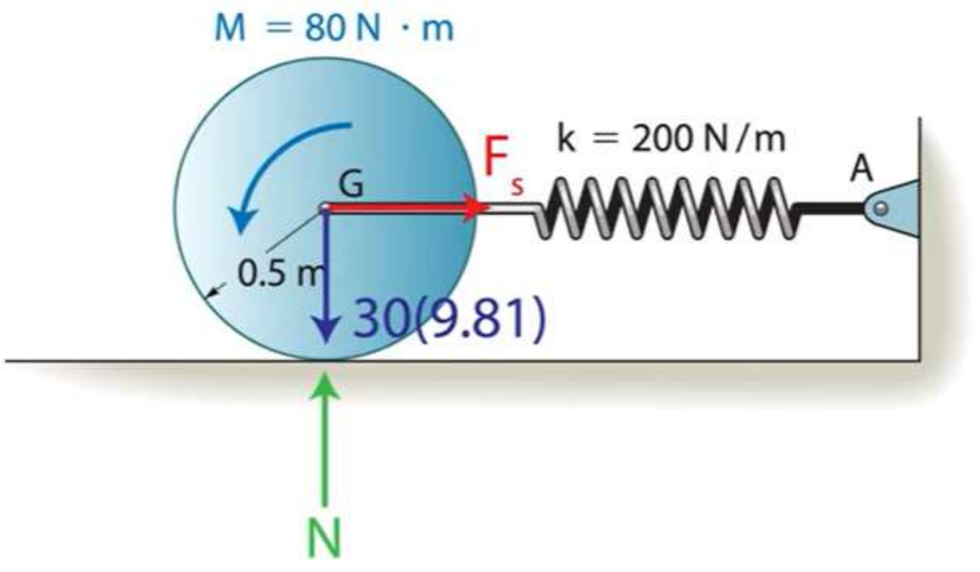
$T_2 = 0$

$U_M = M\theta$

$s_G = \theta r$

$U_M = 80(2s_G)$

$$\theta = \frac{s_G}{0.5} = 2s_G$$



$$T_1 + \sum U_{1-2} = T_2$$

$T_1 = 0$

$T_2 = 0$

$$U_f = -\frac{1}{2}ks^2$$

$U_M = 160s_G$

$$U_f = -\frac{1}{2}(200)(s_G^2)$$

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 160s_G - 100s_G^2 = 0$$

$T_1 = 0$

$T_2 = 0$

$$\boxed{s_G = 1.6\text{ m}}$$

$U_M = 160s_G$

$U_f = -100s_G^2$

Conservation of energy

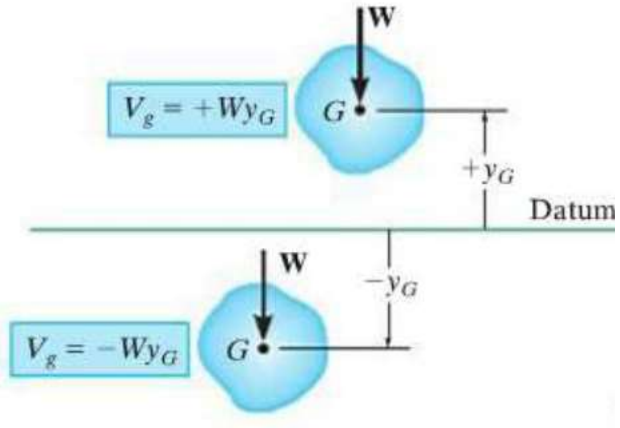
If a rigid body is subjected only to conservation forces , then the conservation of energy equation can be used to solve the problem. This equation requires that the sum of the potential and kinetic energies of the body remain the same at any two points along the path.

The potential energy is the sum of the body s gravitational and elastic potential energies.

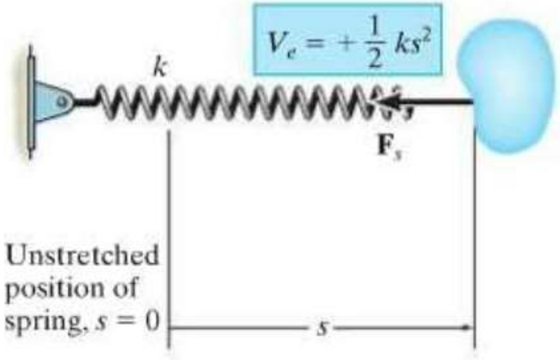
$$V_g = Wy_G \qquad V_e = +\tfrac{1}{2}ks^2$$

In general, if a body is subjected to both gravitational and elastic forces, the total potential energy can be expressed as the algebraic sum.

$$V = V_g + V_e$$



Gravitational potential energy



Elastic potential energy

Realizing that the work of conservative forces can be written as a difference in their potential energies. We can rewrite the principle of work and energy for a rigid body as.

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{noncons}} = T_2 + V_2$$

$$\sum_{1-2} (U)_{noncons}$$

$$(\Sigma U_{1-2})_{\text{cons}} = V_1 - V_2$$

Represents the work of the nonconservative forces such as friction.If this term is zero, then

Initial potential energy — Final potential energy

$$T_1 + V_1 = T_2 + V_2$$

Initial kinetic energy — final kinetic energy

Equation of Motion for a Rigid Body: Work and Energy

Equation of translational motion:

$$U_F = \int \mathbf{F} \cdot d\mathbf{r} = \int_s F \cos\theta \, ds$$

$$T_1 + \Sigma U_{1-2} = T_2$$

Equation of rotational motion:

$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

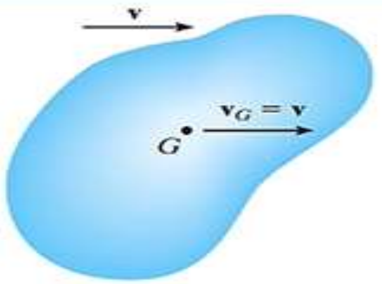
Equation of General Plane Motion

$$T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2$$

or

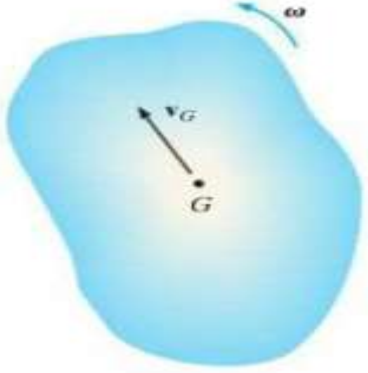
$$T = \tfrac{1}{2} I_{IC} \omega^2$$

$$T = \tfrac{1}{2} m v_G^2$$



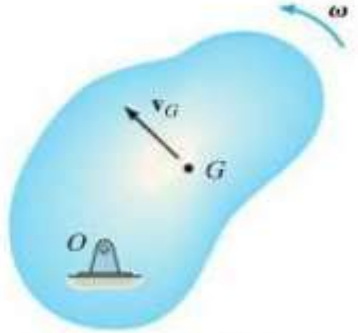
Translation

$$T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2$$



General Plane Motion

$$T = \tfrac{1}{2} I_O \omega^2$$



Rotation About a Fixed Axis