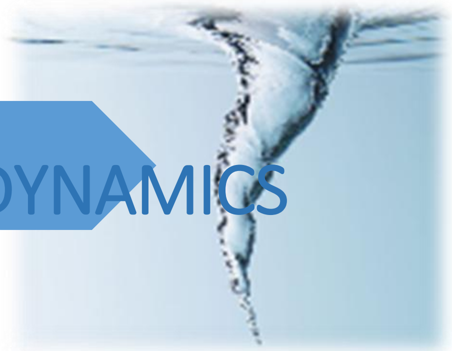
HYDRODYNAMICS



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Hydrodynamics is the branch of fluid mechanics that studies the motion of fluids (particularly liquids) and the forces acting on them. It encompasses the analysis of how liquids flow in various environments, the forces involved, and the resulting effects. Hydrodynamics has essential applications in engineering, physics, environmental science, and other fields where liquid flow plays a role. Hydrodynamics studies the movement of liquids while taking into account the forces that generate this motion. It involves examining the movement of fluid particles subjected to a system of forces, with compressibility forces being neglected. When viscosity forces do not come into play, there is no relative movement between liquid particles, leading to what is known as the hydrodynamics of perfect (ideal) fluids in motion. The presence of viscosity, however, induces energy loss, which is an irreversible transformation of mechanical energy into thermal energy, referred to as the hydrodynamics of real and incompressible fluids. Hydrodynamics generally divides into two parts: the hydrodynamics of perfect fluids and that of real fluids. Hydrostatics is a special case of hydrodynamics.

III.1. Hydrodynamics of Ideal (Perfect) Liquids.

The hydrodynamics of ideal, or perfect, liquids refers to the study of incompressible, non-viscous fluids that lack internal friction. These assumptions allow for simpler mathematical models, which provide insight into flow behavior without the complexities of real-world viscosity or turbulence. Although real fluids exhibit viscosity and can be compressible, the ideal liquid model provides a foundation for understanding fundamental fluid dynamics.

III.1.1. Key Characteristics of Ideal Liquids.

1. **Incompressibility**: Ideal liquids are assumed to have constant density, meaning they are incompressible. This simplifies the analysis since density doesn't change with pressure or temperature.
2. **Non-Viscosity**: Ideal liquids lack internal friction or resistance to flow, meaning they have zero viscosity. This assumption eliminates energy loss due to shear forces, making the flow smooth and free of internal friction.
3. **Steady Flow**: For an ideal liquid, flow is generally considered steady, meaning velocity at any point in the fluid does not change over time.

III.1.2. Fundamental Principles in Ideal Fluid Hydrodynamics.

1. **Bernoulli's Equation**: For ideal liquids, Bernoulli's equation is a crucial tool that describes the relationship between pressure, velocity, and height at different points along a streamline:

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Where:

- P= pressure,
- ρ = density of the fluid,
- v= velocity of the fluid,
- g= gravitationalacceleration,
- h = height above a reference level.

Bernoulli's principle implies that as a fluid's velocity increases, its pressure decreases and vice versa, as long as the flow is along a streamline and there is no energy loss.

2. **Continuity Equation**: For an incompressible fluid, the continuity equation states that the product of cross-sectional area and flow velocity remains constant along a streamline:

$$A_1 v_1 = A_2 v_2$$

where:

- A_1 and A_2 = cross-sectional areas at points 1 and 2,
- $v_1$and v_2 = velocities at points 1 and 2.

This relationship implies that if the area of flow decreases, velocity increases to maintain the same flow rate, which is essential for analyzing flow through pipes and channels.

3. **Euler's Equation**: Euler's equation for ideal fluids is derived from Newton's second law of motion, assuming no viscosity. It describes the balance between pressure forces, gravitational forces, and the rate of change of velocity in a fluid element:

$$\rho \frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} = -\nabla P + \rho \vec{g}$$

where $\vec{v}$ represents the velocity field of the fluid, P the pressure field, and $\rho\vec{g}$ the gravitational force per unit volume.

III.1.3. Applications of Ideal Fluid Hydrodynamics.

Although ideal fluids do not exist in nature, the assumptions of incompressibility and non-viscosity provide valuable approximations for many practical applications:

1. **Aerofoil and Lift**: Bernoulli's equation helps explain the generation of lift on aircraft wings. The pressure differential due to varying velocities on different parts of the wing creates an upward force.
2. **Pipe Flow Analysis**: While viscosity is a factor in real pipes, the continuity equation and Bernoulli's principle are often used as first approximations in calculating velocity and pressure distributions.

3. **Hydraulic Machines**: Ideal fluid models simplify the analysis of hydraulic machinery, such as turbines and pumps, by allowing engineers to predict flow rates and energy transfer without considering frictional losses.
4. **Ocean and Atmospheric Models**: Some large-scale fluid flows, like ocean currents or atmospheric patterns, can initially be modeled as ideal fluids, with corrections added for real-world effects later.

In summary, the hydrodynamics of ideal liquids allows for simplified analysis by removing the complexities of viscosity and compressibility. While these assumptions are theoretical, the resulting models provide essential insights and first-order approximations for understanding fluid flow across various fields.

Euler’s equation and Bernoulli’s theorem are fundamental principles in fluid dynamics, especially for the study of ideal (non-viscous and incompressible) fluids. They describe how pressure, velocity, and potential energy interact within a fluid flow.

III.2. Euler's Equation.

Euler's equation describes the motion of a fluid by balancing the pressure gradient, gravitational forces, and the acceleration of fluid particles. It is derived from Newton’s second law of motion for a fluid element in the flow and is particularly useful in ideal fluid dynamics where viscosity is neglected.

For a steady flow in the direction of a streamline, Euler’s equation is expressed as:

$$\frac{dP}{\rho} + gdz + vdv = 0$$

where:

- dP = differential change in pressure,
- ρ = density of the fluid,
- g = acceleration due to gravity,
- dz = differential change in height along the streamline,
- v= fluidvelocity,
- dv = differential change in velocity.

This form of Euler's equation highlights that changes in pressure, gravitational potential, and kinetic energy (due to velocity changes) must balance in an ideal fluid.

III.2.1. Expanded Form of Euler’s Equation (Vector Form).

In three dimensions, Euler's equation is commonly written in vector form as:

$$\rho \frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} = -\nabla P + \rho \vec{g}$$

where:

- $\vec{v}$ = fluid velocity vector,
- ∇P = pressure gradient,
- $\rho\vec{g}$= gravitational force per unit volume.

III.3. Bernoulli's Theorem.

Bernoulli's theorem is derived from Euler's equation for a steady, incompressible, and non-viscous flow. It states that the sum of pressure energy, kinetic energy, and potential energy per unit volume remains constant along a streamline.

The equation for Bernoulli's theorem is:

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

where :

- P = fluid pressure,
- $\frac{1}{2}\rho v^2$= kinetic energy per unit volume,
- ρgh = potential energy per unit volume (with h being the height above a reference level).

III.3.1. Physical Interpretation of Bernoulli's Equation.

Bernoulli's theorem implies that along a streamline:

- An increase in fluid velocity (kinetic energy) leads to a decrease in pressure.
- An increase in elevation (potential energy) also results in a decrease in pressure.

Thus, Bernoulli's theorem is widely used to analyze pressure differences in various points within the flow, especially in scenarios like fluid flow in pipes, airflow over wings, and river flow.

III.3.2. Relationship Between Euler's Equation and Bernoulli's Theorem.

- **Euler's equation** provides a differential form that describes the infinitesimal changes in pressure, velocity, and height along the flow direction.
- **Bernoulli's theorem** is an integrated form derived from Euler's equation under specific assumptions (steady, incompressible, and frictionless flow), describing the energy conservation along a streamline.

In summary:

- **Euler's Equation** applies generally and can handle complex flows (including varying pressure and gravitational effects).

- **Bernoulli's Theorem** is a specific application of Euler's equation, assuming ideal conditions, and is primarily used for simplified analyses where the fluid is ideal.

$$\begin{cases} \dfrac{\partial P}{\partial x} = 0 \\ \dfrac{\partial P}{\partial y} = 0 \\ \dfrac{\partial P}{\partial t} = -\rho g h \end{cases}$$

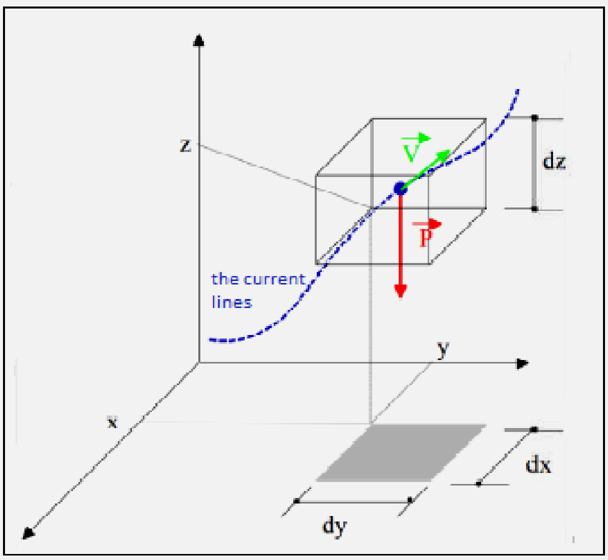


Figure III. 1:Elementary volume element

The fundamental relationship of dynamics being vectorial, we must work in projection on the 3 axes: According to the Ox axis:

$$-P_{(x,+dx,y,z)}dydz + \left(-P_{(x,y,z)}dydz\right) = \rho dydzdz\frac{dV_x}{dt}$$

After development we will have:$-\frac{\partial P}{\partial x} = \rho\frac{dv_x}{dt}$

According to Oy: :$-\frac{\partial P}{\partial x} = \rho\frac{dv_y}{dt}$

According to Oz: :$-\frac{\partial P}{\partial z} - \rho g = \rho\frac{dv_y}{dt}$

The differential of the pressure P is written: $dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz$

$\frac{\partial P}{\partial x}$: corresponds to the rate of variation of P according to x.

$\frac{\partial P}{\partial x}dx$: corresponds to the value of the pressure variation according to x when the variable "x" varies by a small quantity dx. Using the three previous equations, we obtain:

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz$$

$$dP = -\rho\frac{dV_x}{\partial t}dx - \rho\frac{dV_y}{dt}dy - \rho\frac{dV_z}{dt}dz + \rho g dz$$

$$dP = -\rho\frac{dx}{\partial t}dV_x - \rho\frac{dy}{dt}dV_y - \rho\frac{dz}{dt}dV_z + \rho g dz$$

$$dP = -\rho V_x DV_x - \rho V_y dV_y - \rho V_z dV_z + \rho g dz$$

where :

$$\vec{V}\begin{cases}V_x\\V_y\\V_z\end{cases} d\vec{V}\begin{cases}dV_x\\dV_y\\dV_z\end{cases} \vec{V}d\vec{V}=V_xdV_x+V_ydV_y+V_zdV_z$$

which gives us: $dP=-\rho\vec{V}d\vec{V}-\rho gdz$

deduce Euler's equation: $dP+\rho\vec{V}d\vec{V}-\rho gdz=0$

The Bernoulli equation is obtained by working between two points of the same vein of fluid (figure III.2.)

$$\int_1^2 dP+\rho\vec{V}d\vec{V}+\rho gdz=0$$

$$\int_1^2 dP+\int_1^2\rho\vec{V}d\vec{V}+\int_1^2\rho gdz=0$$

Hence the Bernoulli theorem for an incompressible fluid:

$$P_2-P_1+\frac{1}{2}\rho(v_2^2-v_1^2)+\rho g(z_2-z_1)$$

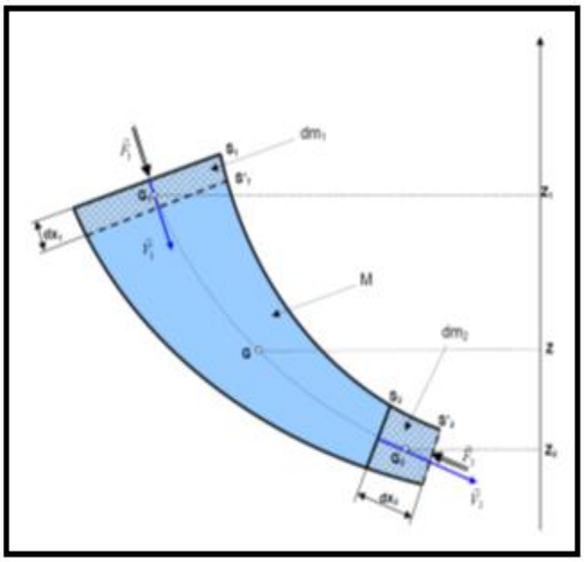


Figure III. 2. Diagram of the fluid vein

III.3.3. Interpretations of Bernoulli's Equation

Bernoulli's equation is a principle of conservation of energy in fluid dynamics, describing how the pressure, kinetic energy, and potential energy of a fluid behave as it moves along a streamline. This equation is crucial for understanding fluid motion and is used to analyze a wide range of scenarios in engineering and physics.

The equation for Bernoulli's theorem is:

$$P+\frac{1}{2}\rho v^2+\rho gh=constant$$

where :

- P = fluid pressure,
- $\frac{1}{2}\rho v^2$= kinetic energy per unit volume,
- ρgh = potential energy per unit volume (with h being the height above a reference level).

III.3.4. Interpretation of Energy Conservation.

Bernoulli's equation is essentially a statement of the conservation of mechanical energy for an ideal fluid (incompressible and non-viscous):

- **Pressure energy** (P) represents the force exerted by the fluid per unit area.
- **Kinetic energy** ($\frac{1}{2}\rho v^2$) accounts for the energy due to fluid motion.
- **Potential energy** (ρgh) represents the energy due to elevation or height in a gravitational field.

In a streamline, these three forms of energy balance out, meaning if one increases, at least one of the others must decrease to maintain a constant sum.

III.3.5. Static, Dynamic, and Hydrostatic Pressure.

Bernoulli's equation divides the total energy into:

- **Static Pressure** P: The pressure exerted by the fluid at rest or when moving parallel to the walls of the containing vessel.
- **Dynamic Pressure**$\frac{1}{2}\rho v^2$: The pressure due to the fluid's motion, proportional to the square of its velocity.
- **Hydrostatic Pressure**ρgh: The gravitational potential energy per unit volume, which varies with height.

Thus, Bernoulli's equation shows that an increase in one form of pressure results in a decrease in the others. For example, if fluid velocity (and hence dynamic pressure) increases, static pressure must decrease, assuming the height remains constant.

III.3.6. Applications in Fluid Flow.

Bernoulli's equation can help analyze various fluid flow scenarios:

- **Pipe Flow**: When fluid flows through pipes of varying diameter, velocity changes according to the continuity equation. If velocity increases in a narrow section, static pressure decreases, which explains why pipes with constrictions can lead to pressure drops.
- **Lift on an Airplane Wing**: Air moves faster over the top of the wing than the bottom, reducing pressure above the wing and creating lift due to a pressure difference.
- **Venturi Effect**: In a narrowing tube, the fluid velocity increases, leading to a drop in static pressure. This principle is utilized in Venturi meters to measure flow rates.
- **Hydraulic Systems**: Bernoulli's principle helps explain fluid behavior in hydraulic lifts and devices, where changes in height and pressure are critical to design.

III.3.7. Limitations of Bernoulli's Equation.

Bernoulli's equation applies strictly under specific conditions:

- The fluid must be incompressible and non-viscous (ideal).
- The flow must be steady (no changes over time).
- The equation applies along a single streamline, meaning it doesn't hold for flows with turbulence or circulation.

In real fluids, viscosity and turbulence create energy losses, causing deviations from ideal behavior, so Bernoulli's equation may only approximate actual conditions.

III.4. Summary.

Bernoulli's equation provides insight into the trade-offs between pressure, velocity, and height within an ideal fluid. It offers explanations for pressure changes in different flow scenarios, aids in the design of engineering applications, and is fundamental to understanding fluid behavior in pipes, around objects, and in open channels.

III.5. Applications of Bernoulli's Equation in the Case of Real Fluids.

In real fluids, viscosity and other dissipative forces cause energy losses that Bernoulli's equation, in its ideal form, does not account for. Therefore, when dealing with real (viscous) fluids, Bernoulli's equation is adjusted to include these energy losses, typically through the introduction of a "head loss" term.

This adjusted form is widely used in engineering to analyze real-world fluid systems, such as pipe flows, fluid transportation, and hydraulic machinery.

III.5.1. Modified Bernoulli's Equation for Real Fluids.

For real fluids, we add a term to account for energy loss due to viscosity and friction:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 + h_{loss}$$

where:

- P_1, P_2: Pressure at points 1 and 2,
- v_1, v_2: Fluid velocity at points 1 and 2,
- h_1, h_2: Heights at points 1 and 2,
- h_{loss}: Head loss term, which represents the energy dissipated due to friction and turbulence in the flow.

The head loss h_{lossh} is often estimated using empirical relations, such as Darcy-Weisbach or Hazen-Williams equations, depending on the type of flow and fluid properties.

III.5.2. Applications in Engineering and Real-World Scenarios

1. **Pipe Flow Analysis**

- In pipelines, real fluids experience pressure drops due to friction with the pipe walls, especially in long or narrow pipes.
- The Darcy-Weisbach equation is commonly used to calculate head loss in pipe flows:

$$h_{loss} = f \frac{L}{D} \frac{v^2}{2g}$$

where:

- f: Darcy friction factor (depends on pipe roughness and Reynolds number),
- L: Length of the pipe,
- D: Pipe diameter,
- v:Fluidvelocity.
- This modified approach is critical for designing efficient water distribution systems, HVAC systems, and fuel transportation pipelines.

2. **Venturi Meters and Flow Measurement**
- Venturi meters measure flow rates by narrowing a section of pipe to increase fluid velocity and decrease pressure, which is then measured.
- In real applications, a correction factor, known as the discharge coefficient, is applied to account for viscous effects and turbulence.

3. **Hydraulic Machinery (Turbines and Pumps)**
- Bernoulli's equation with head loss terms helps evaluate energy losses and efficiency in pumps, turbines, and hydraulic machines.
- For pumps, the head loss can be used to determine the power required to overcome friction and lift water to a certain height.
- Turbines use a modified Bernoulli's equation to account for energy extraction and efficiency losses, crucial for optimizing performance.

4. **Open Channel Flow (e.g., Rivers and Waterways)**
- In open channels, factors such as surface friction and turbulence affect the flow. Manning's equation is often used to estimate head loss in open channels:

$$h_{loss} = \frac{nL}{R^{4/3}} \frac{v^2}{2g}$$

where:

- n:Manning'sroughness coefficient,
- R: Hydraulic radius (cross-sectional area divided by wetted perimeter).

5. **Airflow Over Surfaces (e.g., Wings and Car Bodies)**
- Real fluids like air also lose energy due to frictional drag and boundary layer effects.
- Engineers apply a form of Bernoulli's equation corrected with drag coefficients to predict airflow around structures, important in aerodynamics, vehicle design, and architecture.

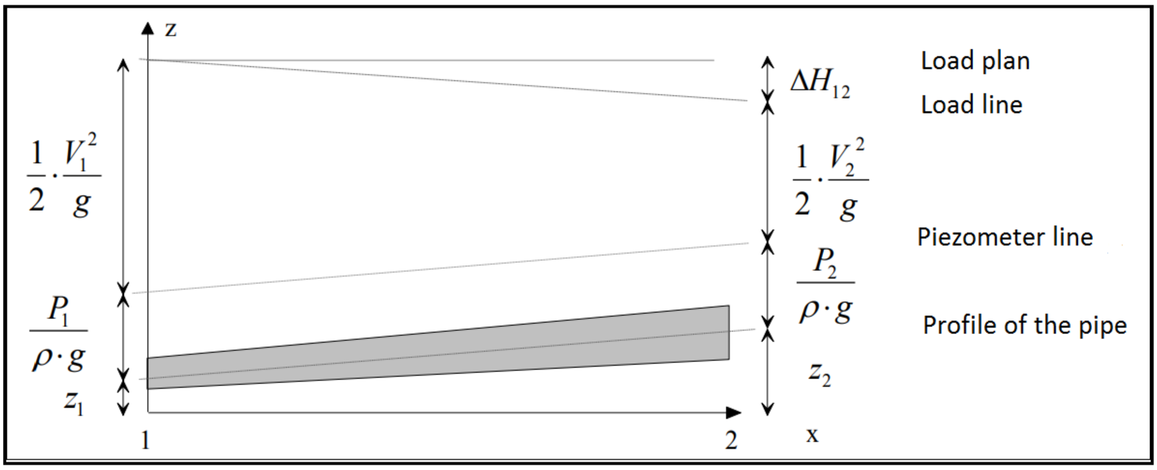


Figure III. 3. Graphical interpretation of the generalized Bernoulli equation.

III.5.3. Key Considerations.

- **Reynolds Number**: Determines whether flow is laminar or turbulent, significantly affecting the head loss and friction factors.
- **Frictional Effects**: More pronounced in high-viscosity fluids, narrow pipes, and rough surfaces.
- **Turbulence**: Increased turbulence leads to higher energy losses and requires empirical corrections.

III.6. Summary.

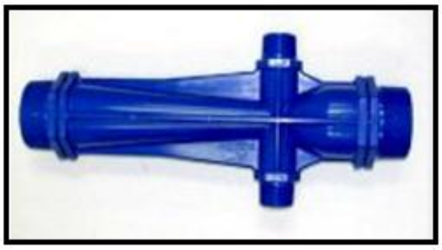
In real fluids, Bernoulli's equation serves as a foundation, modified to include head losses that represent energy dissipated through viscosity and turbulence. This approach is essential in engineering, enabling accurate predictions of pressure, flow rates, and energy requirements in systems where friction and turbulence cannot be ignored.

III.7. Applications of Bernoulli's Theorem.

Bernoulli's theorem is widely used in various fields to analyze and optimize systems where fluid flow is a key factor. Below are some of the main applications of Bernoulli's theorem:

III.7.1. *Venturi Meters and Flow Measurement.*

- **Description**: A Venturi meter is a device that narrows a section of pipe to create a pressure difference, which is then used to calculate fluid flow rate.



- **Application**: By measuring the pressure drop between the wide and narrow sections, Bernoulli's equation helps determine the velocity and flow rate of the fluid. This principle is used in water supply systems, chemical processes, and even in respiratory medical devices.

III.7.2. *Aerospace and Lift Generation on Wings.*

- **Description**: The shape of an airplane wing causes air to flow faster over the top surface and slower underneath.
- **Application**: According to Bernoulli's principle, the pressure on top of the wing decreases due to the higher speed, creating lift. This lift force allows airplanes to fly. This principle also applies to rotor blades in helicopters and propellers in aircraft and ships.

III.7.3.*Pipe Flow and Hydraulic Engineering.*

- **Description**: In fluid systems, pressure varies depending on pipe diameter, fluid speed, and elevation changes.
- **Application**: Bernoulli's theorem helps engineers design efficient piping systems by calculating pressure drops, selecting pipe sizes, and estimating pump requirements. This is essential in water distribution networks, sewage systems, and oil and gas pipelines.

III.7.4. *Automotive Design (Aerodynamics).*

- **Description**: The shape of a car influences airflow around it, affecting drag and stability.
- **Application**: Engineers use Bernoulli's principle to design vehicle bodies that reduce air pressure around specific areas to minimize drag, improve fuel efficiency, and increase stability. Spoilers on cars work by reducing lift on the rear of the vehicle, improving traction at high speeds.

III.7.5. *Blood Flow and Medical Devices.*

- **Description**: Blood flows through arteries and veins with varying diameters, creating pressure differences.
- **Application**: Bernoulli's principle is applied in cardiology to analyze blood flow velocities and detect abnormalities, such as aneurysms or arterial stenosis. Medical devices like Venturi masks, used in respiratory care, rely on this principle to control oxygen flow.

III.7.6. *Atomizers and Sprays.*

- **Description**: Atomizers create fine sprays by forcing liquid through a small nozzle, increasing its velocity and decreasing pressure.
- **Application**: Bernoulli's theorem explains how this pressure drop draws liquid from a reservoir, allowing it to mix with air and form a mist. Atomizers are commonly used in perfume bottles, paint sprayers, and carburetors in engines.

III.7.7. *Water Jets and Fountains.*

- **Description**: Water jets and fountains rely on pressurized water being forced through narrow openings.

- **Application**: Using Bernoulli’s equation, designers can calculate the required pressure and velocity to achieve specific fountain heights and patterns, making it a key principle in fountain design and hydraulics.

III.7.8. *Building and Architecture (Wind Effects).*

- **Description**: Wind speeds and pressure differences around buildings can affect structural stability.
- **Application**: Bernoulli’s theorem helps engineers assess wind forces on high-rise buildings, determining how the building shape and height influence wind speed and pressure differences. This is essential for ensuring structural safety and reducing wind-induced vibrations.

III.7.9. *Sailing.*

- **Description**: The curved surface of a sail causes differences in wind speed on either side of the sail.
- **Application**: Faster airflow on one side of the sail decreases pressure, while slower airflow on the other side increases pressure. This pressure difference generates a force that propels the sailboat forward, even upwind.

III.7.10. *Sports Applications.*

- **Description**: Bernoulli's principle is used to analyze the movement of balls in sports such as golf, soccer, and baseball.
- **Application**: The spin of a ball creates a pressure difference, influencing its trajectory (Magnus effect). This is essential for understanding and controlling the flight path of balls in various sports.

III.8. Summary.

Bernoulli’s theorem is foundational for systems involving fluid motion, from engineering applications in water management and aerospace to medical devices and sports dynamics. Its ability to relate pressure, velocity, and elevation changes allows engineers and designers to predict and control fluid behavior in diverse settings.

III.9. Exemple

III.9.1. Venturi

Student in Venturi Meters and Flow Measurement:

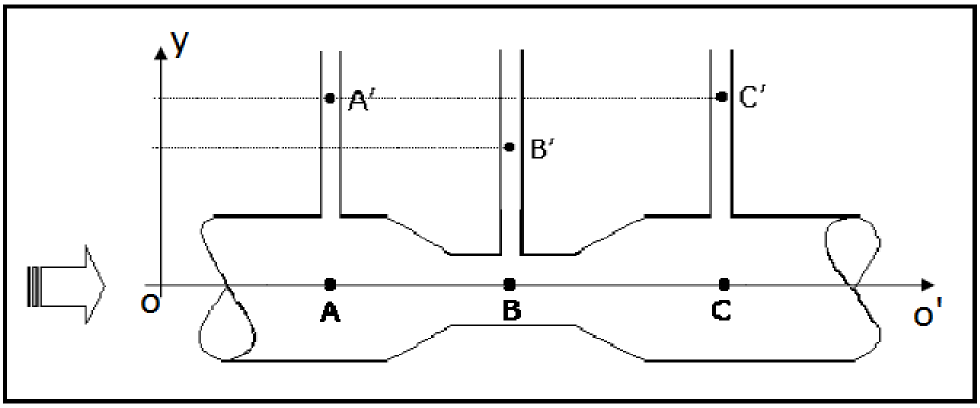


Figure III. 4. Venturi phenomenon.

We have the following pressures:

$$\begin{cases} P_A = P_{A'} + \rho g z_{A'} \\ P_B = P_{B'} + \rho g z_{B'} \\ P_C = P_{C'} + \rho g z_{C'} \end{cases} \qquad \text{or } P_{A'} = P_{B'} = P_{C'} = P_{atm}$$

Let us apply Bernoulli on the streamline passing through A, B and C

$$P_A + \rho g z_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g z_B + \frac{1}{2}\rho v_C^2 = P_C + \rho g z_C + \frac{1}{2}\rho v_C^2$$

$$P_{atm} + \rho g z_{A'} + \frac{1}{2}\rho v_A^2 = P_{atm} + \rho g z_{B'} + \frac{1}{2}\rho v_C^2 = P_{atm} + \rho g z_{C'} + \frac{1}{2}\rho v_C^2$$

$$z_A = z_B = z_C = 0$$

$$z_{A'} + \frac{1}{2}\frac{V_A^2}{g} = z_{B'} + \frac{1}{2}\frac{V_B^2}{g} = z_{C'} + \frac{1}{2}\frac{V_C^2}{g}$$

We also know that the volume flow rate is conserved

$q_V = S_A V_A = S_B V_B = S_C V_C$ (we assume that the speed is uniform over the same section).

If $S_A > S_B \Longrightarrow V_A < V_B \Longrightarrow z_{A'} > z_{B'}$ and $z_{A'} = z_{C'}$then $S_A = S_C \; and \; V_A = V_C$

The 3rd probe will only be used for a study of pressure losses.

$$z_{A'} + \frac{1}{2}\frac{V_A^2}{g} = z_{B'} + \frac{1}{2}\frac{V_B^2}{g} \Longrightarrow \Delta Z = z_{A'} - z_{B'} = \frac{1}{2g}(V_B^2 - V_A^2)$$

$$S_AV_A = S_BV_B \Longrightarrow V_B = V_A\frac{S_A}{S_B}$$

$$\Delta Z = \frac{1}{2g}V_A^2\left(\frac{S_A^2}{S_B^2} - 1\right) \Longrightarrow V_A = S_A\sqrt{\frac{2g\Delta Z}{S_A^2/(S_B^2 - 1)}}$$

The flow rate in the pipe is obtained by: $q_v = S_A\sqrt{\frac{2g\Delta Z}{S_A^2/(S_B^2-1)}}$

III.9.2. The Pitot tube

From a static pressure measurement and a total pressure measurement, the Pitot tube (figure III.5.) will allow us to determine the flow rate of a fluid in the pipe. We will write Bernoulli at point A and M and compare the expressions.

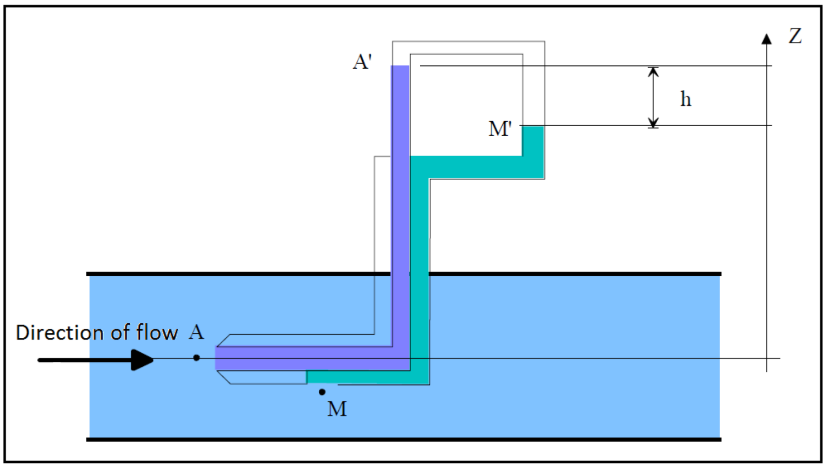


Figure III. 5. Pitot phenomenon.

At point A, we have a stopping point:

$P + \rho g z_A = constant$ The speed can be considered zero.

At point M, we have the expression:

$$\frac{P_M}{\rho g} + z_M + \frac{V_M^2}{2g} = \frac{P_A}{\rho g} \Longrightarrow P_A - P_M = \frac{1}{2}\rho V_M^2$$

We have the relations:

$$P_A - P_{A\prime} = \rho_{liquide}\, g(z_{A\prime} - z_A)$$

$$P_{A\prime} - P_{M\prime} = \rho_{Gaz}\, g(z_{M\prime} - z_{A\prime})$$

$$P_{M\prime} - P_M = \rho_{liquide}\, g(z_M - z_{M\prime})$$

By adding these three equalities member by member, we obtain:

$$P_A - P_M = \rho_{liquide}\, g(z_{A'} - z_A + z_M - z_{M'}) + \rho_{Gaz}\, g(z_{M\prime} - z_{A\prime})$$

Considering $z_A = z_M$ *and* $\rho_{Gaz} g(z_{M'} - z_{A'})$ negligible in front of the term related to water, we obtain:

$$\frac{1}{2}\rho_{liquide}V_M^2 = \rho_{liquide}gh$$

$$V_M = \sqrt{2gh}$$

III.9.3. Flow through an Orifice-Torricelli formula

Consider a large, thin-walled tank pierced with an orifice in its lower part. At the free surface of the fluid and at the orifice we are in the air and therefore at atmospheric pressure. The speed at point A is assumed to be zero (figure III.6)

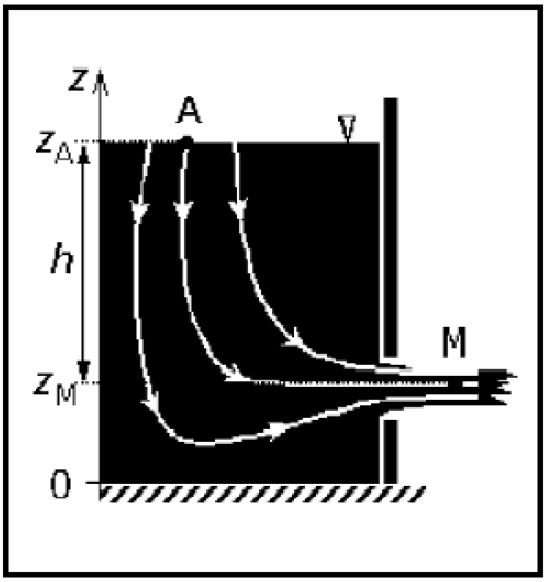


Figure III. 6. Emptying a tank.

Apply Bernoulli between the point A of the free surface and a point M in jet:

$$P_A + \rho g z_A + \frac{1}{2}\rho V_A^2 = P_M + \rho g z_M + \frac{1}{2}\rho V_M^2$$

Since there is no static pressure in the jet is equal to atmospheric pressure.

$$P_A = P_m = P_{atm}$$

$$P_{atm} + \rho g z_A + \frac{1}{2}\rho V_A^2 = P_{atm} + \rho g z_M + \frac{1}{2}\rho V_M^2$$

$\rho g z_A + \frac{1}{2}\rho V_A^2 = \rho g z_M + \frac{1}{2}\rho V_M^2$ With $V_A \gg V_M$

Therefore: $\rho g(z_A - z_M) = \frac{1}{2}\rho(V_M^2 - V_A^2) = \frac{1}{2}\rho V_M^2$

$V_M = \sqrt{2gh}$ Torricelli formula

In the case of real fluids, we always have an energy loss. The real velocity is therefore lower than that calculated by the Torricelli formula.To be able to calculate the real speed, we will determine coefficients of contraction, speed and flow.

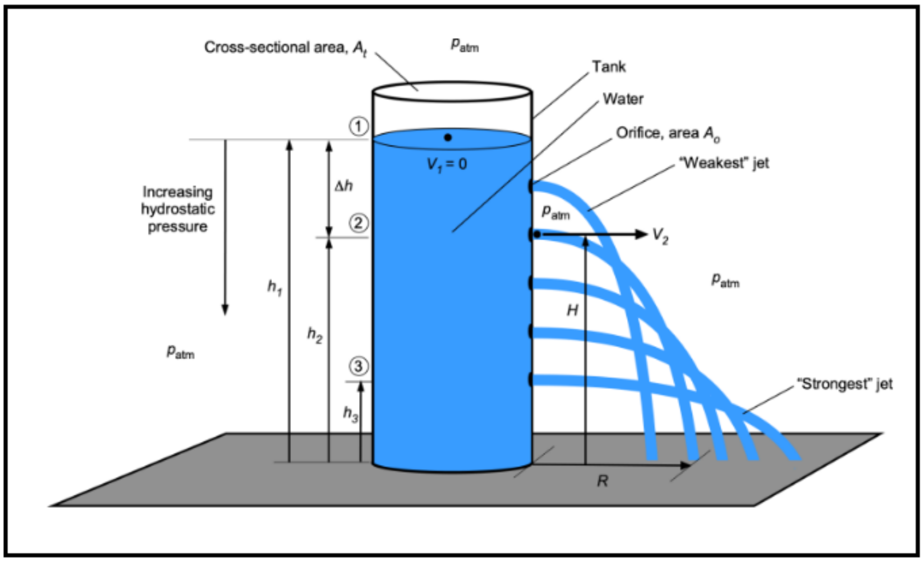


Figure III. 7. Torricelli's law relates the velocity of a fluid flowing out of an orifice to the height or hydrostatic head of the fluid above the orifice's level.

Relates the velocity of a liquid flowing out of an orifice to the height of the liquid above the level of the orifice, as shown in the figure below. The derivation of this law assumes ideal conditions where the liquid is quasi-steady, incompressible, and inviscid. While the problem is fundamentally unsteady in that the fluid properties at different points in the flow change with time as the liquid drains out of the tank because their time rates of change are small, then the flow can be assumed quasi-steady. Therefore, the applicability of the Bernoulli equation can be assumed as it describes the instantaneous balance of energy between different points in the flow.

$$q_V = \sigma V_M = \sigma\sqrt{2gh}$$

$$\sigma = C_C S$$

With: contraction coefficient depends on the geometry of the orifice.

III.9.4. Applications

Torricelli's formula is used in various fluid flow applications, including:

- Calculating flow rates in tanks and reservoirs.
- Estimating drainage speeds in open tanks.
- Designing outlets for tanks and pipes to control fluid discharge speed.

This formula illustrates how gravitational potential energy at a fluid's surface is converted into kinetic energy as it exits through an orifice.

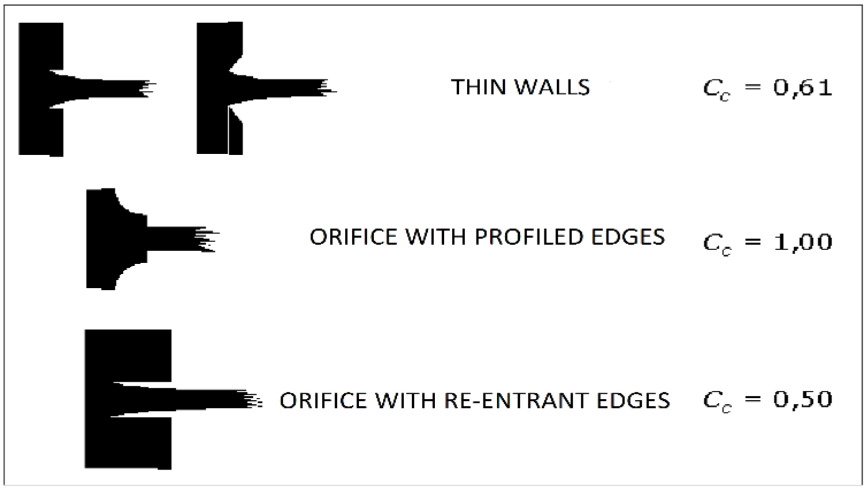


Figure III. 8. Contraction coefficient depends on the geometry of the orifice.

III.9.5. Applications of Hydrodynamics

- **Engineering**: Design and analysis of pipelines, water supply systems, and dams.
- **Environmental Science**: Modeling river currents, ocean flows, and groundwater movements.
- **Medical Applications**: Understanding blood flow in arteries and veins (hemodynamics).
- **Marine and Aerospace Engineering**: Studying drag and resistance for vessels, submarines, and aircraft moving through liquids.

In summary, hydrodynamics provides a framework for understanding and predicting the behavior of liquid flows, enabling the design of efficient systems and solving fluid-related challenges across various fields.