FREE-SURFACE FLOW AND HYDROLOGY

AbdelhafidBoussouf University Center - Mila
Science and Technology Institutes
Department of Civil Engineering and Hydrolic

INTRODUCTION

V.1. Flow in Open Channels

Open channel flow refers to the movement of a fluid (usually water) with a free surface exposed to the atmosphere, as opposed to flow confined within closed conduits like pipes. This type of flow is driven primarily by gravity and influenced by the channel's geometry, slope, and surface roughness.

V.1.1. Key Characteristics

1. **Free Surface**: The interface between the fluid and the air is exposed to atmospheric pressure.
2. **Gravity-Driven Flow**: The primary driving force is the gravitational pull acting on the fluid along the channel's slope.
3. **Types of Channels**:
   - **Natural channels**: Rivers, streams, and floodplains.
   - **Artificialchannels**: Irrigation canals, drainage ditches, and spillways.

V.1.2. Flow Classification

- **Steady vs. Unsteady**: Based on whether flow properties (velocity, depth) change with time.
- **Uniform vs. Non-Uniform**: Based on whether flow properties remain constant or vary along the channel length.
- **Laminar vs. Turbulent**: Determined by the Reynolds number, with most open-channel flows being turbulent.

V.1.3. Applications

- River engineering and flood management.
- Irrigation and drainage systems.
- Hydraulic structures like spillways and weirs.
- Environmental and ecological studies related to sediment transport and water quality.

Understanding open channel flow is fundamental in hydrology, civil engineering, and water resource management to ensure the efficient design and operation of water transport systems.

Flow in an open channel occurs when a liquid, flowing under the influence of gravity, is only partially confined by its solid boundaries. In open channel flow (Figure V.27), the flowing liquid has a free surface and is subject only to pressures created by its own weight and atmospheric pressure.

P: Wetted perimeter
h: Flow depth
S: Wetted area.

$D_h = \frac{S}{B}$ $[m]$: Hydraulic diameter

$$R_h = \frac{S}{P}[m] : \text{Hydraulic radius}$$

$I = \frac{h_f}{L}[\%]$:Slope (gradient)

$B = \frac{dS}{dS}[m]$:Top width.

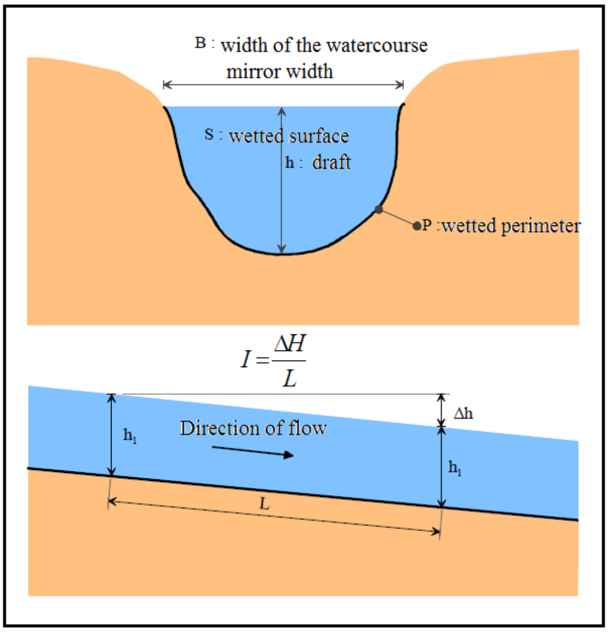

Figure V.1. Flow in an Open Channel.

Let us consider a channel with a constant cross-section, constant slope, a flow depth h, and a constant discharge Qv. We introduce a disturbance by rapidly opening and closing a gate (Figure V.28.).

At the free surface, two waves (gravity waves) are generated:

1. One wave always propagates downstream.
2. The other propagates upstream if the flow velocity in the channel is less than the gravity wave speed; otherwise, it propagates downstream.

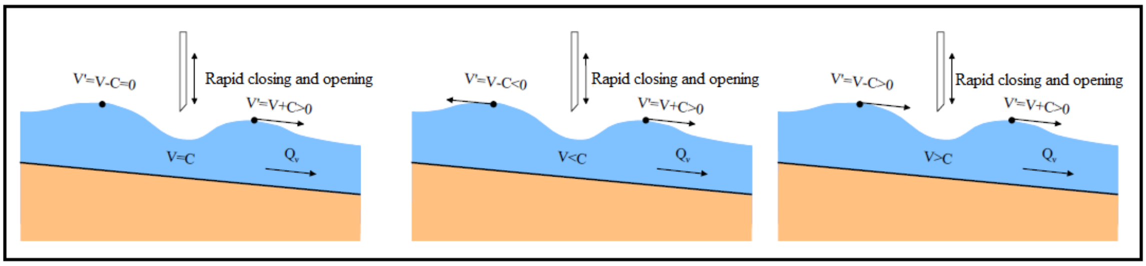

Figure.V.2. Creation of Disturbance in an Open Channel.

In the case where the fluid velocity is greater than the wave speed ccc, the upstream flow is not influenced by the downstream hydraulic conditions (torrential regime). However, in the opposite case, the wave propagates upstream, disturbing the upstream flow (fluvial regime). This phenomenon is called downstream influence.

The speed of the gravity wave is given by the relation:$C^2 = g.D_h$.The Froude number is defined to characterize the flow of a fluid and its behavior.

$$F_r = \frac{v}{\sqrt{gD_h}} \quad with\ v\ flow\ velocity$$

For a Froude number Fr less than 1, the flow is referred to as fluvial; otherwise, it is considered torrential. In the case of fluvial flow, the downstream conditions control the flow. In the case of torrential flow, the upstream conditions control the flow. A Froude number equal to 1 corresponds to a specific flow depth h_c, known as the critical depth.

V.2. Velocity Profiles and Limiting Velocities.

Velocity profiles and limiting velocities are fundamental concepts in fluid mechanics, particularly in the study of open channel flows and boundary layers. The velocity profile describes how the velocity of a fluid varies across a cross-section, typically from the channel bed (where velocity is zero due to the no-slip condition) to the free surface (where velocity is maximum).

Limiting velocities, on the other hand, are critical values beyond which certain flow behaviors or phenomena occur, such as turbulence onset or sediment entrainment. These parameters are essential for understanding flow dynamics, energy distribution, and the interaction between the flow and its boundaries.In open channels, velocity profiles depend on factors such as the channel's roughness, slope, and flow regime (laminar or turbulent). Studying these profiles helps in predicting flow behavior, optimizing hydraulic structures, and managing erosion or sediment transport.

V.2.1. Velocity Distribution in a Cross-Section

Due to viscosity, the velocity distribution in a straight section of a channel is not uniform. As in pipes, velocity is lower near the walls and increases as the distance from the walls increases, reaching a maximum. It then decreases near the surface because of friction at the water-air interface, causing the air to be dragged by the water.The figure on the side shows iso-velocity curves in a cross-section with a regular shape.

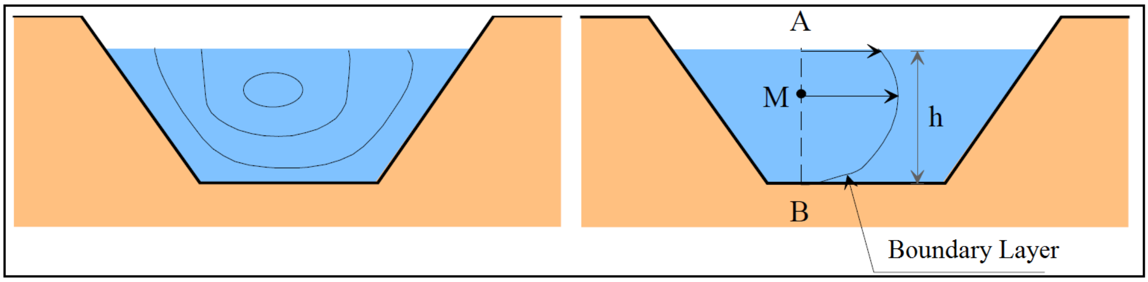

Figure.V.3.Velocity Distribution in a Straight Section of a Channel

V.3. Uniform and Steady Flow.

V.3.1. Head Losses Evaluated by the Manning-Strickler Formula.

The **Manning-Strickler formula** is widely used in open channel flow analysis to estimate head losses due to friction. The head loss, h_f, in a channel or pipe can be expressed as:

$$h_f = \frac{L}{R^{4/3}} \cdot \frac{v^2}{C^2}$$

V.3.2. Variables

1. **L**: Length of the channel or pipe (m)
2. **R**: Hydraulic radius (m) — R=$\frac{A}{P}$, where:
   - A: Cross-sectional area of flow (m²)
   - P:Wettedperimeter (m)
3. **v**: Flow velocity (m/s)
4. **C**: Strickler coefficient or Manning-Strickler roughness coefficient ($m^{1/3}/s$)

Alternatively, using the Manning coefficient n (which is the reciprocal of the Strickler coefficient), the formula becomes:

$$h_f = \frac{L.n^2.v^2}{R^{4/3}}$$

- The **Strickler coefficient** CCC depends on the roughness of the surface; smoother surfaces have higher CCC, while rougher surfaces have lower values.
- The formula is valid for fully developed flows and assumes uniform flow conditions.

This formula is often used in hydraulic engineering for rivers, channels, and culverts to determine head losses due to friction and ensure proper flow design.

The **Chézy coefficient**C depends on the roughness of the channel surface, and it is inversely related to the Manning coefficient n. For a rough channel bed, C will be smaller, and for a smoother bed, C will be larger.

The Chézy equation gives us:

$$V_m = C.\sqrt{I.R_h}$$

Where:

- C: Chézy coefficient, dependent on the resistance.
- I: Slope of the energy line.
- R_h : Hydraulic radius.

The **Manning-Strickler formula** expresses the Chézy coefficient CCC as a function of the bed roughness and hydraulic characteristics. It is written as:

$$C = K_s R^{1/6}$$

Where:

5. C: Chézy coefficient ($m^{1/2}/s$),
6. n: Manning coefficient (s/$m^{1/3}$), which depends on the surface roughness,

- R: Hydraulic radius (m), defined as R=A/P , where:
  - A: Cross-sectional flow area (m²),

- P:Wettedperimeter (m).

V3.3. Application in the Chézy Equation:

By substituting C into the Chézy equation ($v = C.\sqrt{R.S}$), we get:

$$v = \frac{1}{n}R^{2/3}\sqrt{S}$$

Where:

- v: Flow velocity (m/s),
- S: Energy slope or channel slope (dimensionless).
- This relation is widely used in hydraulics to estimate velocity or discharge in open channels.
- The Manning coefficient n depends on the surface material: smooth surfaces like concrete have low n values (approximately 0.012 to 0.015), while rough surfaces like natural riverbeds have higher n values (up to 0.05 or more).

This equation is only valid for turbulent flows in a rough domain, corresponding to 40≤Ks ≤80. The average discharge through the section is then determined by the relation:

$$Q_v = K_s.s.R_h^{2/3}.\sqrt{I}$$

Where:

- Q_v: Average discharge (m^3/s),
- Ks:Roughness coefficient,
- S: Cross-sectional area of flow (m^2),
- R : Hydraulic radius (m),
- I : Energyslope (dimensionless).

Table V.3. Some values of given as examples.

Channel Characteristic	Ks	n= 1/K
Board with poorly treated joints, sandstone	80	0.0125
Concrete with many joints	75	0.0134
Ordinarymasonry	70	0.0142
Old and very rough concrete, earth	60	0.0167
River in a rocky bed	40 to 50	0.0225

From the Manning equation, we have:

$$Q_v = K_s.s.R_h^{2/3}.\sqrt{I}$$

Where:

- h_f: Head loss in [m],
- L: Length of the channel in [m],

- $I = \frac{h_l}{L}$: Energy slope.

$$\left(\frac{V_m}{K_s.R_h^{2/3}}\right)^2 l = h_l$$

Normal Depthh_nOnce the nature of the wall and the slope are set, in a steady and uniform regime, we have a relation linking the depth h to the discharge Q:

$$\frac{Q_v}{\sqrt{I}} = K_s.s.R_h^{2/3}$$

This equation allows us to relate the water depth h to a given steady and uniform regime, which we will call the normal depthh_n (FigureV.30.).In flared sections, the discharge always increases as the water depth rises. However, this is not the case for arched sections, as in the upper part of these sections, the wetted perimeter increases more rapidly than the area. This leads to a decrease in the hydraulic diameter and, consequently, a reduction in discharge.In a straight section of a channel, the velocity distribution is influenced by viscosity, turbulence, and boundary effects. The velocity is typically:

1. **Lowest near the channel walls and bed**: Due to the no-slip condition and friction, the velocity is minimal at the boundaries.
2. **Maximum at some distance from the walls and the free surface**: Away from the boundaries, the velocity increases and reaches a maximum.
3. **Affected by surface drag**: At the free surface, velocity slightly decreases due to friction with the air.

This distribution is not uniform, and the exact profile depends on factors such as the channel shape, flow regime (laminar or turbulent), and roughness of the channel bed and walls.Iso-velocity curves in regular-shaped sections (e.g., rectangular or trapezoidal channels) show this variation, highlighting regions of high and low velocities.

V.3.3.1. Weirs

A weir is an opening in the enclosure of a reservoir through which the upper layer of water flows out.Regardless of the type of weir, its discharge can be determined using the formula:

$$Q_v = m.l.h.\sqrt{2gh}$$

- m: Discharge coefficient, depending on the type of reservoir.
- l: Length of the weir [m].
- h: Water height above the crest [m].

V.3.3.2. Thin-Plate Weir

It is observed that when the water sheet flows freely (measurements should be taken in this condition), there is aeration of the water sheet just downstream of the weir (Figure.V.31.).

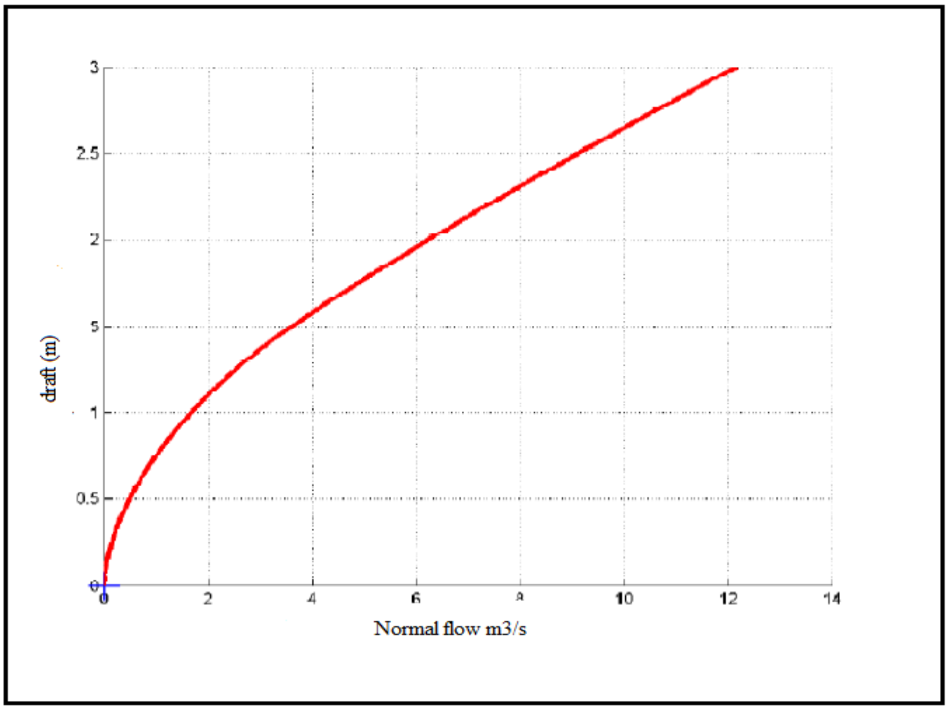

Figure V.4. Velocity Distribution in a Straight Section of a Channel.

In all cases of weirs, it is observed that the water level above the weir and slightly upstream is lower than the distant upstream level. In this situation, according to Bazin's experimentation, we have:

According to Bazin's experimentation, the discharge over a weir is expressed as:

$$Q = m.l.h^{3/2}$$

Where:

- Q:Discharge (m³/s),
- m: Discharge coefficient, determined experimentally,
- l: Effective length of the weir (m),
- h: Water height above the crest (m).

Bazin's experimentation highlights the influence of the water level upstream and just above the weir on the flow characteristics, accounting for energy losses and contraction effects.

$$m = \left(0.405 + \frac{0.003}{h}\right)\left[1 + 0.05\left(\frac{h}{p+h}\right)^2\right]$$

As a first approximation, we can take: 0.40<m<0.50.

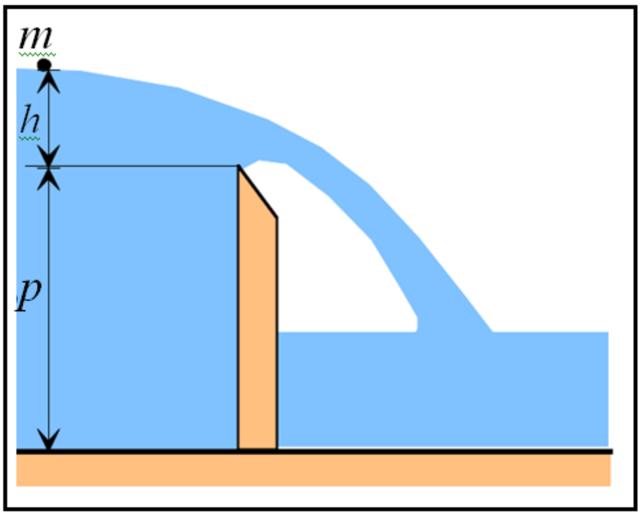

Figure.V.5. Thin-Plate Weir.

V.3.3.3.Thin-Plate Weir with Lateral Contraction

As shown in Figure V.31.32., the presence of lateral contraction reduces the flow passage section by more than just the value of the contraction itself.

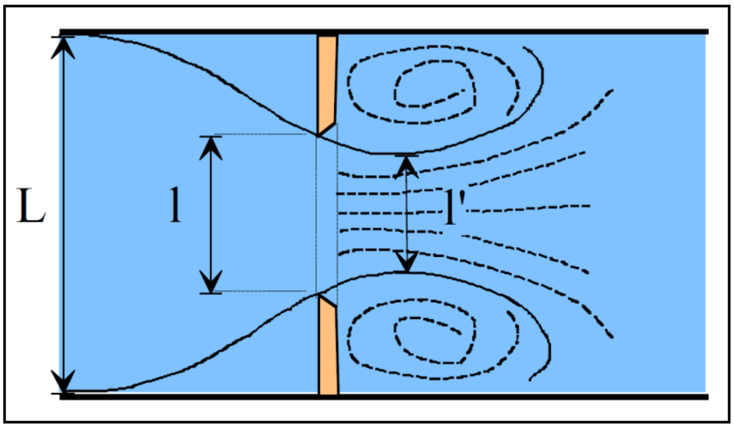

Figure V.6. Plan View of the Weir.

In this case, according to Hégly's experimentation, we have:

$$m = \left(0.405 - 0.006\frac{L-1}{L} + \frac{0.0027}{h}\right)\left[1 + 0.05\left(\frac{l}{L}\right)^2\left(\frac{h}{p+h}\right)^2\right]$$

Where:

- L: Total length of the weir (m),
- l: Effective flow width (m),
- h: Water height above the crest (m),
- p: Height of the weir crest above the channel bed (m).

As a first approximation, we can take m≈0.4.

V.3.3.4. Triangular Thin-Plate Weir

A triangular thin-plate weir, also known as a V-notch weir, is commonly used for measuring low flows with high accuracy. The discharge Q for such a weir is given by the formula:

$$Q = k.h^{5/2}$$

Where:

- Q:Discharge (m^3/s),
- k: Discharge coefficient (depends on the notch angle),
- h: Water height above the vertex of the notch (m).

For a triangular weir with a notch angle θ\thetaθ, the discharge coefficient kkk can be calculated as:

$$k = \frac{8}{15} C_d . \tan\frac{\theta}{2}$$

Where:

- C_d: Coefficient of discharge (dimensionless, typically close to 0.6–0.7 for most applications),
- θ: Angle of the notch (degrees).

V.3.4. Key Characteristics

- The triangular shape ensures higher sensitivity for small flow rates.

- Commonly used in laboratory experiments or small streams where precise measurements are needed.
- in degrees.

V.3.5. Derivation of the Formula

1. **Flow Velocity**: Based on the Bernoulli principle, the velocity vvv of water flowing through the notch is proportional to$\sqrt{2gh}$, where g is the gravitational acceleration.
2. **Flow Area**: The area of flow for a triangular notch at a height y is proportional to $2y\ \tan\frac{\theta}{2}$.
3. **Integration**: Integrating over the height hhh, we derive the total flow, resulting in $Q \infty h^{5/2}$.

V.3.5.1. Example Values for k

- For θ=90°, k≈1.38·C_d.
- For θ=60°, k≈0.64·C_d.

V.3.5.2. Applications

1. Measuring low flow rates in streams, canals, or laboratory setups.
2. Ideal for precise flow measurements where the flow is steady and free of debris.

V.3.5.3. Assumptions:

- The flow over the weir is free (not submerged).
- The water surface is stable and steady.
- The V-notch is sharp and clean for accurate results.

Would you like help with a specific calculation example?

$$v = \sqrt{2gz}$$

According to Thales, we have the relationship:

$$\frac{x}{l} = \frac{h-z}{h} \rightarrow x = l.\frac{h-z}{h}$$

Let μ be the discharge coefficient:

$$dQ_v = \mu.x.dz.\sqrt{2gz} \Rightarrow dQ_v = \mu.l.\frac{h-z}{h}dz\sqrt{2gz}$$

$$Q_v = \int_0^h dQ_v = \int_0^h \mu.l.\frac{h-z}{h}dz\sqrt{2gz} = \frac{\mu.l}{h}\sqrt{2g}\int_0^h (h-z)dz\sqrt{z}$$

$$q_v = \frac{\mu.l}{h}\sqrt{2g}\int_0^h \left(h\sqrt{z} - z^{3/2}\right)dz = \frac{\mu.l}{h}\sqrt{2g}\,.\left[\left(\frac{2}{3}hz^{3/2} - \frac{2}{5}z^{5/2}\right)\right]_0^h$$
$$= \frac{\mu.l}{h}\sqrt{2g}.\left(\frac{2}{3}h^{5/2} - \frac{2}{5}h^{5/2}\right)$$

In the case of a triangle, it is more practical to work with the angle α rather than the length l, as it varies with the water level. In the right triangle, we have:

$$\frac{\alpha}{2}, \frac{l}{2}, h$$

Using the relation:

$$\tan\frac{\alpha}{2} = \frac{l}{2h}\ orl = 2h\ \tan\frac{\alpha}{2}$$

The discharge is given by:

$$Q_v = \frac{8}{15}\mu h^2 \tan\left(\frac{\alpha}{2}\right)\sqrt{2gh}$$

In this scenario, according to Heyndrickx's experimentation:

$$\mu = (0.5775 + 0.214h^{-1.25})\left[1 + \frac{h^2}{l^2(h+z)^2}\right]$$

Where:

- h, l, and z are measured in centimeters (cm).

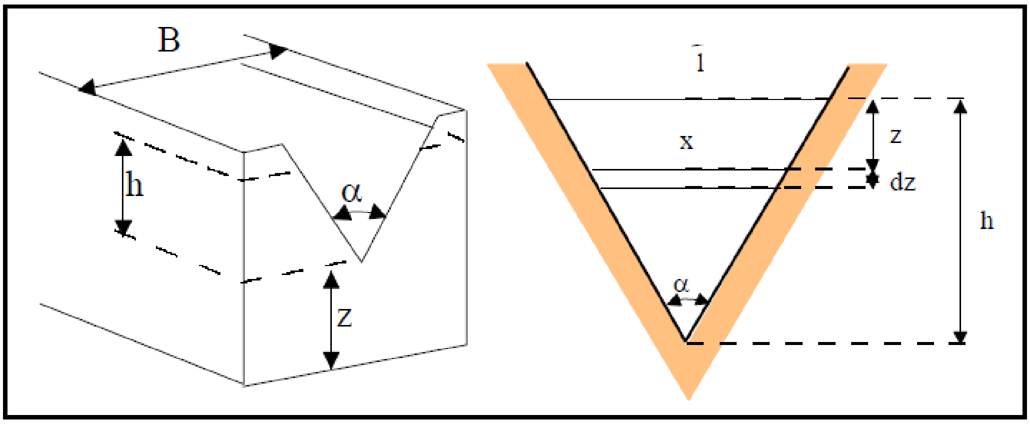

Figure.V.7. Triangular Thin-Plate Weir.

V.3.6. Characteristics

1. **Sensitivity**: Highly effective for small flow rates, as the discharge depends on$h^{5/2}$.
2. **Ease of Use**: The triangular shape simplifies measurement and calibration.
3. **Geometry**: The V-notch ensures smooth and uniform flow distribution over the weir.

Would you like to explore this further with an example or application?

V.3.7.Thick-Crested Weir

A thick-crested weir, also known as a broad-crested weir, is a structure where the crest (top surface) is wide compared to the flow depth, allowing the water to stabilize as it flows over. These weirs are commonly used for measuring and controlling flows in open channels, irrigation systems, and dams.

V.3.7.1. Key Characteristics

1. **Stabilized Flow**: The flat, broad crest enables the flow to adapt its velocity and depth before falling downstream.
2. **Geometry Impact**: Unlike thin-plate weirs, the flow over a thick-crested weir is influenced by the width of the crest.
3. **Applications**: Often used in hydraulic engineering due to their durability and effectiveness for large flows.

V.3.7.2. Discharge Formula

For a thick-crested weir, the discharge Q is given by:

$$Q = C.B.\sqrt{2g}h^{3/2}$$

Where:

- Q: Flow rate (m^3/s),
- C:Discharge coefficient (dimensionless),
- B: Effective width of the weir (m),
- h: Height of the water above the crest (m),
- g: Gravitational acceleration (9.81 m/s^2).

V.3.7.3. Determination of C (Discharge Coefficient)

- The coefficient C depends on the ratio of the crest width b to the upstream water depth h.
- For very wide crests, C approaches 1.0.
- Experimental or empirical methods are often used to estimate C.

V.3.7.4. Special Case

When the flow is subcritical and the crest is not excessively wide, the discharge is less sensitive to variations in h, making thick-crested weirs reliable for steady flow control.

Would you like additional details or an example application?

Bernoulli Equation Applied from Point M to M'**:**

$$\frac{P_M}{\rho g} + \frac{V_M^2}{2g} + h = \frac{P_{M'}}{\rho g} + \frac{V_{M'}^2}{\rho g} + h'$$

Assumptions:

1. The velocity at M is negligible compared to the velocity at M′: $V_M \ll V_{M'}$.
2. The pressures $P_M$and $P_{M'}$ are equal: $P_M = P_{M'}$.

Resulting Velocity at M′:

$$V_{M'} = \sqrt{2g(h - h')}$$

DischargeEquation:

$$Q_v = l.h'.\sqrt{2g(h - h')}$$

Where:

- l: Width of the channel,
- h: Initial water height,
- h′: Water height at M′,
- g:Gravitationalacceleration.

Maximum Flow Analysis:

The discharge function Q_v equals zero at h′=0and h′=h. The flow achieves a maximum for a value of h′ between 0 and h.

Derivation for Maximum Flow:

$$u = (h - h')\ so\ u' - 1$$

Rewrite Q_v:

$$Q_v = l.h.\sqrt{2g}.\sqrt{(h-h')}$$

Differentiate Q_v with respect to h′:

$$\frac{dQ_v}{dh'} = l.h.\sqrt{2g}.\sqrt{(h-h')} - 1.h.\sqrt{2g}\frac{1}{2\sqrt{h-h'}} = 1.\sqrt{2g}.\left(\sqrt{(h-h')} - \frac{h'}{2\sqrt{h-h'}}\right)$$

$$\frac{dQ_v}{dh'} = 0\ or\ \sqrt{(h-h')} - \frac{1}{2\sqrt{h-h'}} = 0\ so\ 2(h-h') - h = 0$$

the maximum discharge is:

With $h' = \frac{2h}{3}$ $or\ Q_v = \frac{2}{3}l.h.\sqrt{2g}.\sqrt{(h - \frac{2}{3}h)}$

Final Expression for Maximum Discharge:

$$Q_v = \frac{2}{3\sqrt{3}}.h.l.\sqrt{2gh} \Longrightarrow Q_v = 0.385.l.h.\sqrt{2gh}$$

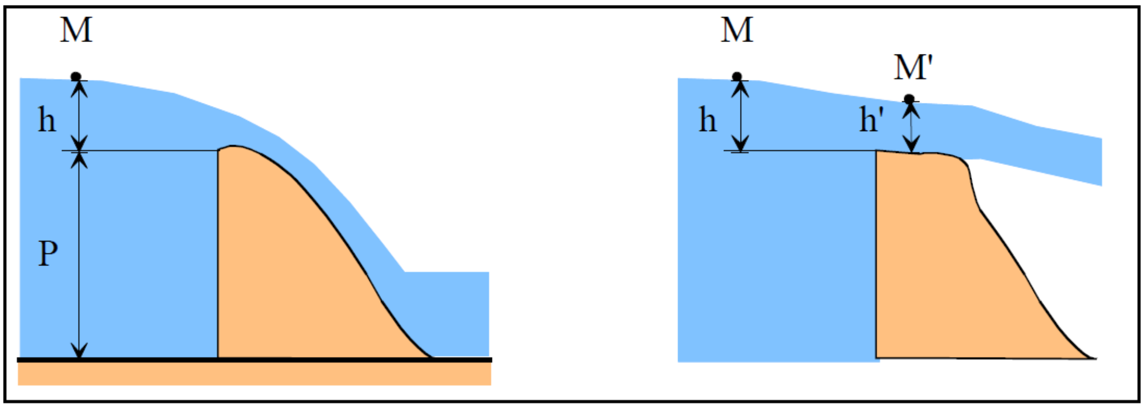

Figure V.8.déversoir à seuil épais.

V.4. Conclusion on Free-Surface Flows and Hydrology

Free-surface flows play a fundamental role in hydrology and environmental engineering, as they describe water movement in open channels, rivers, and weirs. The study of these flows helps in understanding natural water systems and designing hydraulic structures to manage water effectively.

V.4.1. Key points include:

1. **Energy and Momentum Principles**: The Bernoulli equation and conservation of momentum provide the theoretical basis for analyzing free-surface flows, accounting for energy losses, velocity profiles, and flow depths.
2. **Empirical Formulas**: Practical flow measurements often rely on empirical equations such as Manning-Strickler for channel flows and specialized equations for hydraulic structures like weirs. These formulas allow for estimating flow rates with reasonable accuracy based on field data.
3. **Hydraulic Structures**: Weirs (thin-plate, thick-crested, triangular, etc.) are essential tools in flow regulation and measurement. They illustrate the relationship between geometry, flow behavior, and discharge.
4. **Critical Flow Conditions**: Understanding flow transitions (e.g., from subcritical to supercritical states) and maximum discharge conditions is crucial for designing systems that optimize flow control and mitigate risks.

V.4.2. Hydrological Applications:

- **Flood Management**: Predicting flow rates and water depths in natural channels aids in flood control and risk mitigation.
- **Irrigation Design**: Free-surface flow principles guide the efficient design of canals, gates, and reservoirs.
- **Environmental Protection**: Accurate modeling of river systems supports ecological conservation and sediment transport studies.

In summary, free-surface flow analysis bridges theoretical fluid dynamics and practical hydrology, enabling sustainable management of water resources in natural and engineered