Foreword

This course

The document covers the majority of concepts in fluid mechanics and is structured into five chapters as follows:

- **Chapter I** deals with the properties of fluids, such as density, specific gravity, and viscosity, among others. These properties are applied in the following chapters.
- **Chapter II** deals with the study of fluids at rest. It examines the fundamental law of fluid statics and the forces exerted by fluids on solid objects. This section provides the essential foundations for the study of static fluids.
- **Chapter III** deals with the flow of ideal fluids. The equations governing this type of flow, such as the continuity equation and the Bernoulli equation, are derived. These equations are fundamental to various hydraulic applications, including the design of water supply and drainage networks, as well as in many instruments used to measure pressure and flow in hydraulic systems.
- **Chapter IV** deals with the study of real fluid flows. Concepts such as flow regimes and the calculation of frictional head losses are explained. These concepts are crucial for the design of various industrial installations.
- **Chapter V** examines open channel flows.

At the end of each chapter, exercises and practical work are provided to help students test their knowledge and prepare for exams.

In preparing this booklet, I have referred to many documents listed in the bibliography. I hope that this booklet will encourage readers to delve deeper into these works.

DEFERENCE REMINDERS ON FLUID MECHANICS

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A fluid can be considered a substance made up of a large number of material particles, very small and free to move relative to one another. It is thus a continuous, deformable material medium with very low rigidity, so that the fluid is a shapeless body that takes the form of the container holding it.

It is important to emphasize that a fluid is assumed to be a continuous medium: even if a very small volume element is selected, it will still be much larger than the size of the molecules that constitute it. Among fluids, a distinction is often made between liquids and gases. Fluids can also be classified into two families based on their viscosity. Viscosity is one of their physicochemical characteristics, which will be defined later in the course, and it describes the internal friction of fluids. Fluids canbecategorizedintotwo main families :

- The "Newtonian" family, with a constant viscosity or one that can only vary with temperature.
- The second family consists of "non-Newtonian" fluids, characterized by viscosity that changes depending on the speed and stresses they experience as they flow.

This course is limited only to Newtonian fluids, which will be classified as follows.

I.1. Ideal Fluid.

A perfect fluid is an idealized fluid with several key characteristics:

1. **Incompressibility**: A perfect fluid does not change its density, regardless of pressure variations. This means that its volume remains constant even when subjected to different pressures.
2. **No Viscosity**: Perfect fluids have zero viscosity, meaning they experience no internal friction or resistance to flow. This lack of viscosity allows perfect fluids to flow without dissipating energy through friction.
3. **No Thermal Conductivity**: A perfect fluid does not conduct heat. There is no transfer of heat within the fluid or between the fluid and its surroundings.
4. **No Surface Tension**: Perfect fluids do not exhibit surface tension, which simplifies calculations involving fluid behavior at interfaces, such as the fluid-air surface.
5. **Ideal Flow**: The flow of a perfect fluid is generally considered to be steady and inviscid (without viscosity), following an idealized model that makes theoretical analysis simpler.

In reality, no fluid perfectly meets all these criteria. However, some fluids (such as water or air under certain conditions) can be approximated as perfect fluids to simplify analysis in fluid dynamics, especially in ideal theoretical scenarios like the study of potential flow.

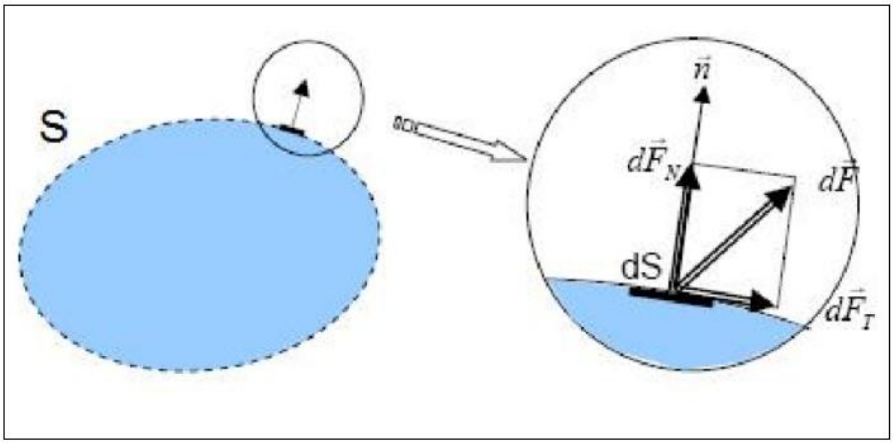

Figure. I.1. Surface élément dS

I.2. Real Fluid.

A real fluid is any fluid that exhibits properties deviating from those of an idealized or "perfect" fluid. Real fluids have characteristics that account for friction, viscosity, compressibility, and heat transfer, making them much more complex to analyze than perfect fluids. Key properties of real fluidsinclude:

1. **Viscosity**: Real fluids have a non-zero viscosity, meaning they experience internal friction as they flow. This viscosity causes resistance to flow, resulting in energy dissipation as heat and affecting the velocity profile within the fluid.
2. **Compressibility**: Real fluids can be compressible, meaning their density can change with pressure. While some fluids (like water) are nearly incompressible under standard conditions, gases are highly compressible.
3. **Thermal Conductivity**: Real fluids conduct heat, meaning that there is heat transfer within the fluid and between the fluid and its surroundings. This property is essential for understanding convection and energy transfer in fluid systems.
4. **Surface Tension**: Real fluids exhibit surface tension, which affects the behavior of fluids at interfaces, such as liquid droplets or bubbles, and plays a significant role in phenomena like capillarity.
5. **Flow Characteristics**: The flow of real fluids is often complex and may involve turbulence, where fluid particles move in chaotic and irregular paths, especially at high velocities or in large-scale systems.

Because real fluids have viscosity and other interactive forces, they obey the **Navier-Stokes equations**, which are fundamental in fluid dynamics for describing the motion of viscous fluids. Real fluid properties are crucial for practical applications in engineering, meteorology, and biology, as they allow for a more accurate representation of fluid behavior in natural and man-made systems.

I.2. physical characteristics.

The physical characteristics of fluids are essential properties that define their behavior in various conditions. These characteristics include:

1. **Density (ρ)**: The mass per unit volume of a fluid, typically measured in kg/m³. Density influences buoyancy, pressure, and flow behavior.
2. **Viscosity (μ)**: A measure of a fluid's resistance to deformation and flow, often described as internal friction. It determines how easily fluid particles slide past each other and is divided into:
   - **Dynamic viscosity**: Measured in Pa·s, representing internal resistance.
   - **Kinematic viscosity**: The ratio of dynamic viscosity to density, measured in m²/s.
3. **Surface Tension (σ)**: The force per unit length at the surface of a fluid, which makes it resist external forces. Surface tension causes droplets to form spherical shapes and influences capillary action.
4. **Compressibility**: The degree to which a fluid can change its volume in response to pressure changes. Gases are highly compressible, whereas liquids are almost incompressible.
5. **Thermal Conductivity (k)**: The ability of a fluid to conduct heat. This property is crucial for heat transfer applications, such as in cooling systems and heat exchangers.
6. **Specific Heat Capacity (c)**: The amount of heat required to raise the temperature of a unit mass of fluid by one degree Celsius. It influences how much a fluid's temperature changes with energy input.
7. **Pressure (P)**: The force exerted by the fluid per unit area, typically measured in Pascals (Pa). Pressure plays a significant role in fluid behavior, especially in compressible fluids like gases.

These physical characteristics vary depending on factors like temperature, pressure, and fluid composition, and they are fundamental for studying and analyzing fluid flow, stability, and heat transfer in both natural and engineered systems.

I.2.1. Mass, Specific Weight and Density.

Here’s a breakdown of the concepts of **mass**, **specific weight**, and **density** in the context of fluids:

1. **Mass (m)**: Mass is the amount of matter in a fluid, measured in kilograms (kg). In fluid mechanics, the mass of a fluid sample can be calculated as:

   $$m = \rho \times V$$

   where:

   - ρ = density of the fluid (kg/m³)
   - V= volume of the fluid (m³).
2. **Specific Weight (γ)**: The specific weight of a fluid, also known as **weight density**, is the weight per unit volume. It is represented by γ\gammaγ and depends on both the density of the fluid and the gravitational acceleration g. The formula for specific weight is:

$$\gamma = \rho \times g$$

where:

- γ= specific weight (N/m³).
- ρ= density of the fluid (kg/m³).
- g= gravitational acceleration (approximately 9.81m/s^2 on Earth).

Specific weight tells us how much the fluid "weighs" per unit of volume, distinguishing it from mass, which is independent of gravity.

3. **Density (ρ)**: Density is the mass of the fluid per unit volume, and it is a fundamental property that influences buoyancy, flow behavior, and other fluid dynamics phenomena. It isgivenby:

$$\rho = \frac{m}{V}$$

where:

- ρ = density (kg/m³)
- m= mass (kg)
- V = volume (m³)

Density is a critical property in fluid mechanics because it helps determine how fluids interact with each other and with solids, especially in phenomena involving buoyancy and pressure.

I.2.2. Key Points about Specific Weight.

1. **Units**: Specific weight is measured in newtons per cubic meter (N/m³) because it represents the weight (force) per unit volume.

2. **Relation to Density**: Specific weight depends directly on the density of the fluid. Higher density means a higher specific weight, assuming gravity remains constant.
3. **Dependence on Gravity**: Since γ includes the gravitational acceleration g, specific weight will vary with changes in gravity. For example, on the Moon, a fluid would have a lower specific weight than on Earth due to the lower gravitational pull, even if its density remains the same.
4. **Applications**: Specific weight is used in engineering and fluid mechanics to analyze forces in fluids, calculate hydrostatic pressure, and evaluate the buoyancy of objects submerged in fluids.

In summary, specific weight is a measure of how much weight a fluid has per unit volume and is directly related to both the fluid's density and the gravitational field.

I.2.3. Key Points about Density.

1. **Units**: The SI unit of density is kilograms per cubic meter (kg/m³). However, other units, such as grams per cubic centimeter (g/cm³), are also commonly used in certain fields.
2. **Influence on Buoyancy**: Density plays a major role in buoyancy. Objects with a density lower than the fluid they are in will float, while objects with a higher density will sink.
3. **Dependence on Temperature and Pressure**: Density can vary with temperature and pressure, especially in gases. For example:
    - **Temperature**: Generally, increasing temperature decreases the density of fluids, as they expand when heated.
    - **Pressure**: Increasing pressure usually increases density, particularly in gases, as the particles are forced closer together.
4. **Relative Density (Specific Gravity)**: Density is often compared with the density of a reference substance, usually water at 4°C for liquids and solids. This comparison is called **specific gravity** or **relative density** and is dimensionless:

$$\text{Specific Gravity} = \frac{\rho\text{substance}}{\rho\text{water}}$$

$$d = \frac{\rho_{substance}}{\rho_{water}}$$

In summary, density is a primary characteristic that affects a fluid's weight, pressure, and flow behavior. It is central to understanding fluid dynamics, buoyancy, and material properties in various scientific and engineering applications.

I.2.4. Compressibility.

is a property of fluids (and solids) that describes how much they can be compressed under the application of pressure? Compressibility essentially measures the change in volume of a substance in response to a change in pressure. The compressibility factor β\betaβ is defined as:

$$\beta = -\frac{1}{V}\frac{\Delta V}{\Delta P}$$

where:

- β= compressibility (often given in 1/Pa),
- V = initial volume,
- ΔV = change in volume,
- ΔP= change in pressure.

I.2.5. Key Points about Compressibility.

1. **Units**: Compressibility is measured in terms of inverse pressure, typically 1/Pa in SI units.

2. **Relation to Fluid Type**:
    - **Liquids** are generally **incompressible**; they exhibit very low compressibility, so their volumes change very little even under high pressure.
    - **Gases** are **highly compressible**; they can significantly change in volume with pressure changes, which is why gas density and volume vary with pressure.
3. **Bulk Modulus (K)**: The **bulk modulus** is the inverse of compressibility and represents a substance's resistance to compression. It isgivenby :

$$K = \frac{\Delta P}{-\frac{\Delta V}{V}}$$

    A higher bulk modulus means lower compressibility and vice versa.

4. **Applications in Fluid Dynamics**: Compressibility is essential in studying sound waves, gas flow, and high-speed fluid dynamics (like in aerodynamics and high-speed flows). When flows reach speeds near or above the speed of sound, they become **compressible flows**, where density variations need to be accounted for.
5. **Effect on Engineering Systems**: In practical applications, the compressibility of a fluid affects system design and performance, especially in hydraulic systems, gas pipelines, and even in the design of aircraft.

I.2.6. Practical Example of Compressibility in Action.

- **Gases**: The volume of a gas will decrease substantially if the pressure increases in a confined space (following Boyle's law for ideal gases, where PV=constant).
- **Liquids**: While usually considered incompressible, liquids do exhibit slight compressibility, which can be observed in systems operating under very high pressures, like deep-sea environments or hydraulic systems.

In summary, compressibility is crucial for understanding how fluids behave under pressure, particularly for gases, and has important implications for both fluid dynamics and engineering applications.

I.2.7. Viscosity.

Is a measure of a fluid's resistance to flow? It represents the internal friction within the fluid as layers move past one another. High-viscosity fluids (like honey) resist flow more than low-viscosity fluids (like water), making viscosity a key property in fluid mechanics.

I.2.8. Types of Viscosity.

1. **Dynamic (or Absolute) Viscosity (μ)**: Dynamic viscosity quantifies a fluid’s internal resistance to flow. It is defined as the force required to move one layer of fluid in relation to another. The unit of dynamic viscosity is **pascal-second (Pa·s)** or **N·s/m²**.

2. **Kinematic Viscosity (ν)**: Kinematic viscosity measures the fluid's resistance to flow under gravitational force and is given by:

$$\upsilon = \frac{\mu}{\rho}$$

   where:

   - ν= kinematic viscosity (m^2/s),
   - μ = dynamic viscosity (Pa·s),
   - ρ= density of the fluid (kg/m^3).

   Kinematic viscosity is commonly measured in **Stokes (St)**, where $1St=10^{-4}m^2/s$.

I.2.9. Key Points about Viscosity.

1. **Flow Behavior**: Viscosity plays a significant role in determining whether a fluid flow is laminar (smooth) or turbulent (chaotic). Higher viscosity tends to promote laminar flow, while lower viscosity allows easier transition to turbulence.
2. **TemperatureDependence**:
   - **Liquids**: Viscosity decreases with increasing temperature because the molecules move faster, reducing internal resistance.
   - **Gases**: Viscosity increases with temperature as faster-moving gas molecules create more collisions, increasing internal friction.
3. **Newtonian vs. Non-Newtonian Fluids**:
   - **Newtonian Fluids**: These fluids have a constant viscosity regardless of the shear rate (e.g., water, air).
   - **Non-Newtonian Fluids**: Their viscosity changes depending on the shear rate or stress applied. Examples include shear-thinning fluids (like ketchup) and shear-thickening fluids (like cornstarch in water).
4. **Importance in Engineering and Science**:
   - **Pipelines**: Knowing a fluid's viscosity is essential for designing pipeline systems, as it affects the energy needed for pumping.
   - **Lubrication**: High-viscosity fluids like oils are used in engines to reduce friction and wear between moving parts.
   - **Biological Fluids**: Viscosity measurements in fluids like blood can help diagnose health conditions, as increased viscosity might indicate disease.

I.2.10. Formula for Viscosity in a Flow.

In fluid flow between two parallel plates, the force F required to move one plate at a velocity v relative to the other, separated by a distance d, is:

$$F = \mu\, A\, \frac{v}{d}$$

where:

- μ= dynamicviscosity,
- A = area of the plate,
- v= velocity of the moving plate,
- d = distance between plates.

This relationship underscores how viscosity directly affects the force needed to sustain movement in fluids.

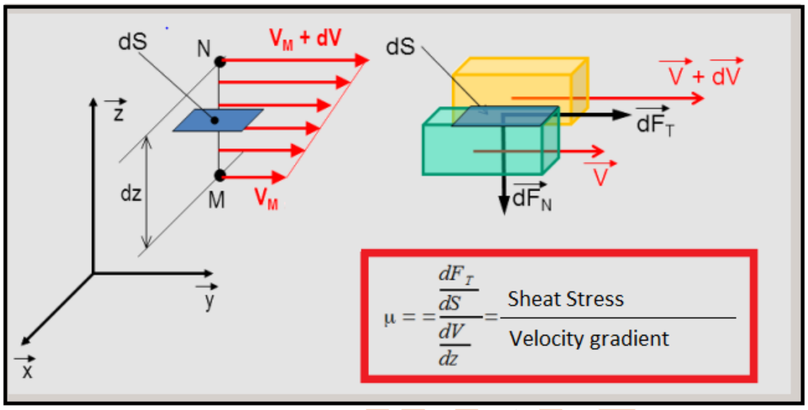

Figure.I.2. Viscosity of a fluid

I.2.11. Kinematic viscosity.

Kinematic viscosity is a property of fluids that describes the fluid’s resistance to flow under gravity. It combines the fluid's dynamic viscosity (its internal friction) with its density, allowing us to assess how easily a fluid flows under its own weight without the influence of external forces.

I.2.12. Definition and Formula.

The kinematic viscosity, denoted by the Greek letter ν\nuν, is defined as the ratio of dynamic viscosity η\etaη to the density ρ of the fluid:

$$\upsilon = \frac{\eta}{\rho}$$

where:

- ν(nu) is the kinematic viscosity, measured in m^2/s,
- η is the dynamic viscosity, measured in Pa·s or Ns/m^2,
- ρ is the density of the fluid, measured in kg/m^3.

I.2.13. Interpretation.

- **Units**: The units of kinematic viscosity are m^2/s, which makes it comparable to a diffusion coefficient. In fact, kinematic viscosity can be thought of as the rate at which momentum diffuses through a fluid.

- **Physical Meaning**: While dynamic viscosity measures the internal friction within a fluid, kinematic viscosity gives insight into how that frictional resistance interacts with the fluid’s density. A fluid with a high kinematic viscosity flows more slowly, as the internal friction has a significant effect on its movement.

I.2.14. PracticalExample.

1. **Water vs. Oil**: Water has a lower dynamic viscosity and density compared to oil, which gives it a relatively lower kinematic viscosity than oil. This iswhy water flows more easilythanoil.
2. **Air**: Air has a low density but also a very low dynamic viscosity, resulting in a kinematic viscosity value that is comparable to liquids under certain conditions.

I.2.15. Applications.

- **Fluid Flow Calculations**: Kinematic viscosity is used in calculations involving fluid flow, such as in the Reynolds number, which predicts flow regimes (laminar vs. turbulent flow).
- **Engineering and Environmental Studies**: Kinematic viscosity helps in understanding fluid behaviors in pipes, rivers, and oceans, aiding in the design of systems for fluid transport and environmental impact studies.

Understanding kinematic viscosity is essential in fluid dynamics and engineering fields where the flow characteristics of fluids play a critical role in design, prediction, and control of fluid behavior.

I.2.16. Measuring Viscosity.

Viscosity, which describes a fluid’s resistance to flow, can be measured using various methods and instruments, depending on the type of fluid and desired precision. Below are the main methods used to measure dynamic and kinematic viscosity:

a. Capillary Viscometer (Ostwald Viscometer)

This method is primarily used for measuring the viscosity of low-viscosity liquids, like water or dilute solutions.

- **Principle**: A capillary viscometer measures the time it takes for a fixed volume of fluid to flow through a thin capillary tube under the influence of gravity.
- **Procedure**: The fluid is placed in a U-shaped tube, and the time it takes to flow from one marked level to another is measured.
- **Formula**: The viscosity can be calculated using: $\eta = K \times \rho \times t$

  where:

  - η is the dynamic viscosity,
  - K is a calibration constant for the device,
  - ρ is the density of the fluid,
  - t is the time it takes for the fluid to pass between the two marks.

b. Falling Sphere Viscometer

This method is suitable for transparent fluids and provides an indirect measure of dynamic viscosity.

- **Principle**: A sphere falls through a fluid under the influence of gravity, and the time taken to fall a set distance is used to calculate viscosity.
- **Procedure**: A sphere of known density and size is dropped in the fluid, and the time it takes to reach a certain point is recorded.
- **Formula**: The viscosity is determined using Stokes' law:

$$\eta = \frac{2r^2\left(\rho_s - \rho_f\right)g}{9\upsilon}$$

where:

  - r is the radius of the sphere,
  - ρ_s and ρ_f are the densities of the sphere and fluid, respectively,
  - g is the acceleration due to gravity,
  - v is the terminal velocity of the sphere.

c. Rotational Viscometer

This method is widely used for measuring the viscosity of both Newtonian and non-Newtonian fluids, especially when they are highly viscous or have complex flow behaviors.

- **Principle**: A spindle or disk rotates in the fluid, and the resistance to this rotation, caused by the fluid's viscosity, is measured.
- **Procedure**: The device measures the torque required to rotate the spindle at a constant speed.
- **Formula**: The viscosity is calculated by relating the torque to the speed of rotation, using a calibration constant specific to the device.

d. Vibrational Viscometer

This method is often used for real-time viscosity measurements in industrial applications.

- **Principle**: A vibrating element (such as a tuning fork) is immersed in the fluid, and the damping effect of the fluid is measured, which correlates with viscosity.
- **Procedure:** The damping frequency or amplitude of vibration changes as a function of the fluid's viscosity.

f. Bubble Viscometer

This is a simple method for measuring relative viscosity and is useful for quality control.

- **Principle**: The time it takes for an air bubble to rise through a column of fluid is related to the viscosity of the fluid.

- **Procedure**: The bubble’s ascent time is recorded and compared to a reference fluid.

g. Ultrasonic Viscometer

This non-intrusive technique is suitable for monitoring viscosity in real-time without directly contacting the fluid.

- **Principle**: Ultrasound waves are passed through the fluid, and changes in wave propagation (attenuation and velocity) are used to determine viscosity.

I.2.17. Summary of Methods and Their Applications.

Each method has advantages depending on the fluid’s properties, application, and measurement precision needed, helping in industries from petrochemicals to pharmaceuticals where precise viscosity values are crucial.

Table I.1. Methods and Their Applications.

Method	Best For	Type of Viscosity
CapillaryViscometer	Low-viscosity, Newtonianfluids	Kinematic
FallingSphereViscometer	Transparent fluids	Dynamic
RotationalViscometer	Viscous and non-Newtonian fluids	Dynamic
VibrationalViscometer	Real-time, industrialfluids	Dynamic
BubbleViscometer	Quality control, comparative tests	Relative
UltrasonicViscometer	Non-intrusive, continuous monitoring	Dynamic

The rotational viscometer consists of two cylinders, as shown in Figure I.4. One cylinder, C_1, is fixed, has a radius R_1, and is connected to a torsion wire with a torsion constant C. The other cylinder, C_2, contains the liquid to be studied and is driven into rotation by a motor at a frequency of n [tr/s]. This cylinder has a radius R_2. We denote the height of cylinder C_1as h. During its rotation, cylinder C_2drives the fluid to be studied.

I.3. Description of a Moving Fluid (Review of Kinematics)

Fluid kinematics focuses on describing the motion of fluid particles without considering the forces or energies that produce this motion. Understanding the kinematics of fluids is essential for analyzing fluid behavior in applications ranging from pipe flow to atmospheric patterns.

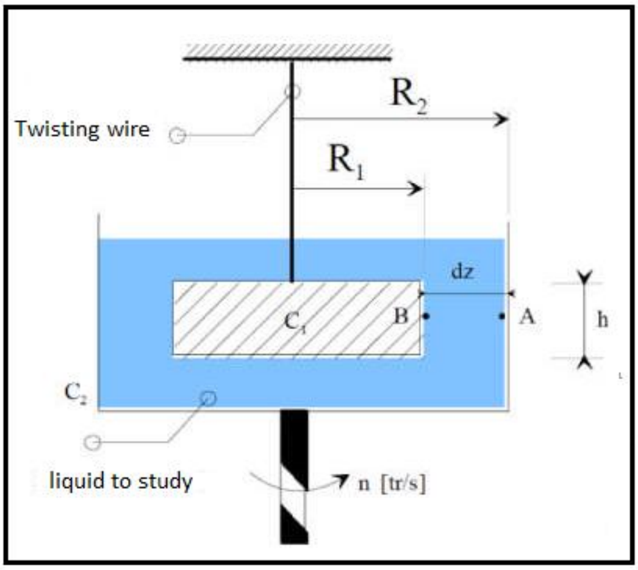

Figure I.3. Rotational viscometer.

I.3.1. Lagrangian and Eulerian Descriptions of Fluid Motion

- **Lagrangian Description**: Follows individual fluid particles as they move, tracking their position, velocity, and other properties over time. This approach describes the history of each particle but becomes complex with a large number of particles.
- **Eulerian Description**: Observes fluid motion at fixed points in space and analyzes how fluid properties (like velocity, pressure, and density) change over time at these points. This is the more common approach in fluid mechanics as it simplifies calculations by focusing on fields (e.g., velocity and pressure fields).

I.3.2. Flow Field and Streamlines

The flow field describes the fluid velocity at every point in space and time. The flow is defined by three primary elements:

- **Velocity Field** $\vec{V} = u(x,y,z,t)\vec{\imath} + v(x,y,z,t)\vec{\jmath} + w(x,y,z,t)\vec{k}$: Specifies the velocity of fluid particles at any location in the fluid. The components u, v, and w represent the velocity in the x-, y-, and z-directions, respectively.
- **Streamlines**: Lines that are tangent to the velocity vector at each point. They represent the path a particle would follow in steady flow, showing the fluid's instantaneous direction.
- **Pathlines and Streaklines**:
  - **Pathlines**: The actual path traced by a particle over time, useful in unsteady flows where the direction of flow changes.
  - **Streaklines**: Lines formed by particles that pass through a specific point, commonly visualized with dye or smoke in experiments.

I.3.3. Types of Flow

- **Steady vs. UnsteadyFlow** :

  - **Steady Flow**: Properties at any given point do not change with time $\left(\frac{\partial}{\partial t} = 0\right)$.
  - **Unsteady Flow**: Properties vary over time.
  -
- **Uniform vs. Non-Uniform Flow**:
  - **Uniform Flow**: Velocity is constant at each cross-section of the flow field.
  - **Non-Uniform Flow**: Velocity varies within the flow field.
- **Laminar vs. Turbulent Flow** :
  - **Laminar Flow**: Smooth, orderly flow where fluid particles move in parallel layers.
  - **Turbulent Flow**: Chaotic, disordered flow with significant mixing and fluctuations.

I.3.4. Acceleration in Fluid Flow

The acceleration of a fluid particle has both local and convective components:

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\nabla\vec{V}$$

where:

- $\partial\frac{\partial \vec{V}}{\partial t}$ is the local acceleration (change in velocity with time at a fixed point),
- $\vec{V}\nabla\vec{V}$is the convective acceleration (change in velocity due to movement within the velocity field).

I.3.5. Flow Rate and Continuity Equation

- **Flow Rate (Q)**: The volume of fluid flowing per unit time, given by:

$$Q = A.V$$

Where A is the cross-sectional area and V is the velocity.

- **Continuity Equation**: Expresses the conservation of mass in a fluid flow:

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho\vec{V}\right) = 0$$

For incompressible flow (ρ=constant):

$$\nabla.\vec{V} = 0$$

Indicating that the rate of mass entering a volume equals the rate leaving it.

I.3.6. Summary

The kinematic analysis of fluid motion provides foundational descriptions and mathematical tools for understanding fluid flows, enabling engineers and scientists to model and predict fluid behavior in diverse applications.

In fluid mechanics, the **Lagrangian** and **Eulerian** descriptions provide two different approaches to analyzing fluid motion. They differ in how they observe and track the properties of fluid particles.

I.3.7. Lagrangian Description

The **Lagrangian** approach follows individual fluid particles as they move through space and time. This method is akin to tracking the journey of each particle in a fluid flow, noting changes in properties like position, velocity, and pressure for each particle.

- **Concept**: Each particle's trajectory is mapped by identifying its position over time.
- **Advantages**: Provides detailed information about each particle's motion, making it useful in analyzing how specific particles behave or interact with others.
- **Applications**: Commonly used in particle-tracking methods, simulations involving small fluid elements, and in situations where tracing the history of each particle is critical (e.g., pollutant dispersion in fluids).
- **Mathematical Representation**: If a fluid particle is identified at an initial position (x_0,y_0,z_0) at time t=0 its trajectory over time can be expressed as:
- $x = x(x_0, y_0, z_0, t)$,$y = y(x_0, y_0, z_0, t)$, $z = z(x_0, y_0, z_0, t)$

I.3.8. Eulerian Description

In contrast, the **Eulerian** approach observes fluid properties at fixed points in space over time, focusing on how properties change at specific locations rather than tracking individual particles.

- **Concept**: The fluid's behavior is analyzed by observing what happens at each point in the field rather than following particles.
- **Advantages**: Simplifies the analysis, especially in large flows, since it focuses on fields (e.g., velocity field, pressure field) rather than tracking every particle. This is more suitable for many practical applications, like analyzing flow in pipes or open channels.
- **Applications**: Commonly used in fluid dynamics equations and computational fluid dynamics (CFD) simulations, where a fixed grid in space observes changes in velocity, pressure, or other properties.
- **Mathematical Representation**: For velocity at a fixed point, the Eulerian description specifies that velocity$\vec{V} = (u, v, w)$ varies as a function of space and time:$\vec{V} = (x, y, z, t)$

I.4.Summary of Différences

Table I.2. Descriptions are foundational in fluid.

Aspect	Lagrangian Description	Eulerian Description
Focus	Individual fluid particles	Fixed points in the flow field
Approach	Followsparticletrajectories	Observes changes in properties at specific locations
Information Obtained	Particle paths, history of each particle	Flow properties (e.g., velocity, pressure) at fixed points
Typical Applications	Particletracking, pollutant dispersion	Fluid dynamics equations, CFD, large-scale flow analysis

Both descriptions are foundational in fluid mechanics and can often be used complementarily to analyze complex fluid behaviors.

I.4.1. Lagrangian Variable (Louis-Joseph Lagrange, 1736-1813)

The **Lagrangian variable**, named after Louis-Joseph Lagrange (1736–1813), is a concept in fluid mechanics and continuum mechanics that focuses on describing the motion and properties of individual fluid particles over time. In the **Lagrangian framework**, each particle in a fluid is tracked as it moves through space, allowing for a detailed view of the particle's trajectory and changes in its properties. This approach contrasts with the Eulerian framework, which examines properties at fixed locations in space rather than following individual particles.

I.4.2. Key Aspects of the Lagrangian Variable Approach

1. **Particle Tracking**: In the Lagrangian perspective, each fluid particle is given a unique label based on its initial position. As time progresses, the motion and properties of this particle (like position, velocity, and acceleration) are tracked.
2. **Reference to Initial Position**: The behavior of each particle is described relative to its initial position(X_0, Y_0, Z_0) at t=0 . The current position (x,y,z)(x, y, z)(x,y,z) of the particle at any later time t can then be expressed as:

$$x = x(x_0, y_0, z_0, t), y = y(x_0, y_0, z_0, t), \qquad z = z(x_0, y_0, z_0, t)$$

3. **Function of Time**: The Lagrangian approach focuses on how properties change as a function of time for each particle, providing insight into the temporal evolution of its state.
4. **Advantages in Analysis**: By following individual particles, the Lagrangian approach can give detailed information on the history and interaction of particles, which is particularly useful in analyzing complex flow phenomena such as turbulence, mixing, or particle-laden flows.

I.4.3. Louis-Joseph Lagrange and Fluid Mechanics

Louis-Joseph Lagrange was a highly influential mathematician and physicist, contributing significantly to mechanics and fluid dynamics. His work laid the groundwork for the mathematical formulation of the Lagrangian framework, which became essential in both theoretical and applied mechanics. Lagrange's methods allowed scientists and engineers to model dynamic systems by focusing on individual elements, a concept that remains integral in modern physics, fluid dynamics, and computational modeling.

Let A be the position of a fluid particle at time $t_0$in the reference frame O,x,y,z(Figure I.4.).

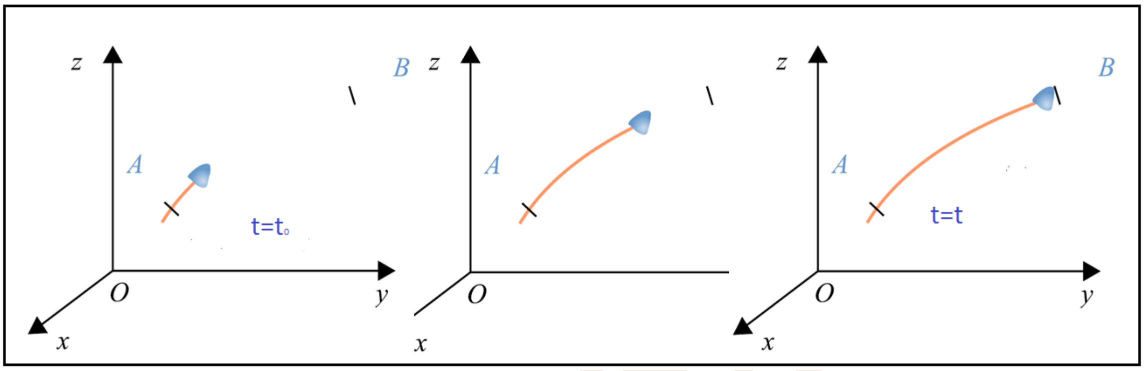

Figure I.4. Movement of a fluid particle from point A to point B.

We consider the movement of a fluid particle that, at time t_0, was at point A with coordinates (a_1, a_2, a_3). For$t > t_0$, this particle is located at point B with coordinates (x_1, x_2, x_3). The movement of the fluid particle is defined by the parametric equations:

$$x_1 = f_1(a_1, a_2, a_3, t, t_0)$$

$$x_2 = f_2(a_1, a_2, a_3, t, t_0)$$

$$x_3 = f_3(a_1, a_2, a_3, t, t_0)$$

The Lagrangian variables are the time t and the coordinates (a_1, a_2, a_3), and the functions to be determined are (x_1, x_2, x_3). In this description of fluid motion, each particle is tracked individually as it moves (Figure I.4.).

The **Eulerian variable**, named after Leonhard Euler (1703–1783), is a fundamental concept in fluid mechanics and continuum mechanics for describing the motion and properties of fluids. In the **Eulerian approach**, the focus is on fixed points in space through which the fluid flows, rather than tracking individual particles. This allows for the analysis of fluid properties (such as velocity, pressure, and density) at specific locations as the fluid moves through those points.

I.4.4. Key Aspects of the Eulerian Approach

1. **Fixed Spatial Observation**: In the Eulerian framework, properties of the fluid are described as functions of both space and time at particular positions in a flow field. This approach does not follow the particles themselves but rather observes how properties change at a given point in space over time.
2. **Spatial Representation**: The state of the fluid is expressed as:

$$V(x,t), \qquad p(x,t), \qquad \rho(x,t)$$

where V is the velocity vector, p is the pressure, ρ is the density, and x represents the spatial coordinates.

3. **Velocity Field**: One of the primary applications of the Eulerian description is the velocity field. The velocity at a specific point x and time t indicates how fast and in what direction the fluid is moving at that location.
4. **Simplified Analysis for Flow Fields**: The Eulerian perspective is particularly effective for analyzing steady and unsteady flows, where the goal is to understand how fluid properties evolve at specific locations rather than the movement of individual particles.
5. **Mathematical Application**: The Eulerian variable framework leads directly to important equations of fluid motion, such as the **continuity equation**, the **Navier-Stokes equations**, and the **Bernoulli equation** in fluid dynamics. These equations describe how the velocity field and other properties change over space and time.

I.4.5. Léonard Euler's Contribution to Fluid Mechanics

Leonhard Euler was a prominent mathematician whose contributions to fluid dynamics and mechanics laid the foundation for modern hydrodynamics. His development of the Eulerian description allowed for a comprehensive mathematical analysis of fluid motion.

The **Euler equations** for inviscid flow (without viscosity) are a direct result of his work, forming the basis for more complex models involving real-world applications such as air and water flow analysis. Euler's approach revolutionized the study of fluid dynamics by allowing scientists and engineers to model and predict fluid behavior efficiently in various engineering and natural systems.

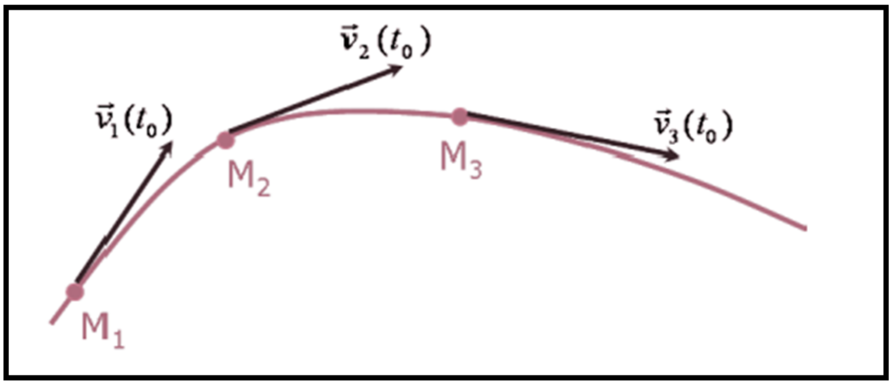

Figure I.5. Instantaneous photo of the flow.

Eulerian variables are the time t and the coordinatesx_i of the fluid particle at time t. The functions to be determined are the velocitiesU_i:

$$U_1 = g_1(a_1, a_2, a_3, t)$$

$$U_2 = g_2(a_1, a_2, a_3, t)$$

$$U_3 = g_3(a_1, a_2, a_3, t)$$

The Eulerian description consists of defining, at a given time t, a vector field $U(M,t)$that represents the velocity vector of the particles in a fluid domain, called the flow field.

The Eulerian perspective is more convenient for the experimenter because it involves examining a point M in the fluid and studying the variations of physical quantities (e.g., velocity) at different times (Figure I.5.). The Eulerian perspective is more practical in kinematics because:

1. For steady flows, the projection of velocities in the reference frame does not depend on time.
2. The velocity vectors of the flow form a vector field to which vector field properties can be applied.

A **ligne de courant** (streamline) is a fundamental concept in fluid dynamics that represents the path followed by fluid particles at a given instant. It is a visual representation of the flow of a fluid, showing the direction in which the fluid moves at every point in a flow field. Here are some key aspects of streamlines:

I.4.6. Key Characteristics of Streamlines

1. **Definition**: A streamline is a line that is tangent to the velocity vector of the flow at any given point. This means that at every point along a streamline, the direction of the line corresponds to the instantaneous direction of the fluid velocity.
2. **No Flow Across**: By definition, fluid cannot cross a streamline; fluid motion occurs only along the streamline. Therefore, streamlines indicate the paths that individual fluid particles will follow if the flow is steady.
3. **MathematicalRepresentation**:
   - For a velocity field represented by v (x,y,z), the streamline can be defined by the differential equations:

   $$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

   where u,v,w are the velocity components in the x, y and z directions, respectively.

4. **Flow Visualization**: Streamlines provide a snapshot of the flow pattern at a specific moment in time. In steady flow, the pattern of streamlines does not change over time, whereas in unsteady flow, the streamlines may shift and vary with time.
5. **Relation to Flow Properties**:
   - In a two-dimensional flow, streamlines can be visualized easily using diagrams.
   - In three-dimensional flows, computer simulations or experimental techniques such as particle image velocimetry (PIV) are often used to visualize streamlines.

I.4.7. Applications of Streamlines

- **Design and Analysis**: Engineers use streamlines to analyze flow patterns around objects, such as airfoils, car bodies, or pipes, to optimize performance and reduce drag.
- **Flow Prediction**: Streamlines help predict how pollutants or particles will move through a fluid.
- **Aerodynamics and Hydrodynamics**: Understanding streamline behavior is essential for designing efficient aircraft, ships, and other vehicles that interact with fluid environments.

I.4.8. Differences from Streaklines and Pathlines

- **Streaklines**: These show the path of particles that have passed through a specific point over time.
- **Pathlines**: These trace the trajectory that a single particle follows as it moves through the flow.
- **Streamlines**: These represent the direction of the velocity field at a given instant.

In steady flow, streamlines, streaklines, and pathlines are identical. However, in unsteady flow, they can differ significantly.

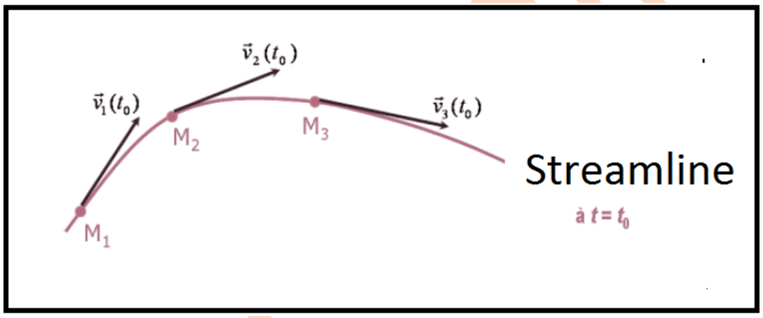

Figure I.6. Streamlines.

The trajectory of a fluid particle is the path described by the position of this particle over time as it moves within a fluid field. It is a physical representation that traces the exact route followed by an individual particle when it is carried by the fluid.

I.4.9. Characteristics of the trajectory

- **Dynamic description:** Unlike streamlines, which provide an instantaneous representation of the flow direction, the trajectory follows the actual movement of a fluid particle over time.
- **Time-dependent position:** The trajectory is a function of position r(t)\mathbf{r}(t)r(t), where ttt represents time. Each fluid particle has its unique trajectory that can be traced if its initial position and how it evolves over time are known.
- **Movement in the field:** In steady (or permanent) flow, the trajectory coincides with the streamline. However, in unsteady (or non-permanent) flow, the trajectory may differ from streamlines because the velocity field changes over time.

I.4.10. Applications of trajectories

- **Particle tracking:** Trajectories are used to study the transport of particles, such as contaminants in the air or nutrients in the water.
- **Flow analysis:** They help visualize how a particle moves in complex flows, such as in turbines, rivers, or the atmosphere.
- **Modeling and simulation:** In computational fluid dynamics (CFD) simulations, trajectories can be simulated to predict the behavior of fluid particles.

I.4.11. Differences from streamlines and instantaneous streamlines

- **Streamlines:** Tangent to the velocity vector at every point at a given instant, showing the direction but not the movement over time.
- **Trajectories:** Represent the actual and continuous movement of a particle over a period of time.

Example: Imagine a particle in a river. Its trajectory corresponds to the path it would follow if its position were tracked second by second. If the river flows uniformly, its trajectory might resemble a streamline. However, if the river has eddies or fluctuations, the trajectory would show how the particle is drawn into these variations.

In summary, the trajectory is essential for understanding the behavior of a fluid particle in motion, especially in unsteady or turbulent flows where the movement can be unpredictable and complex.

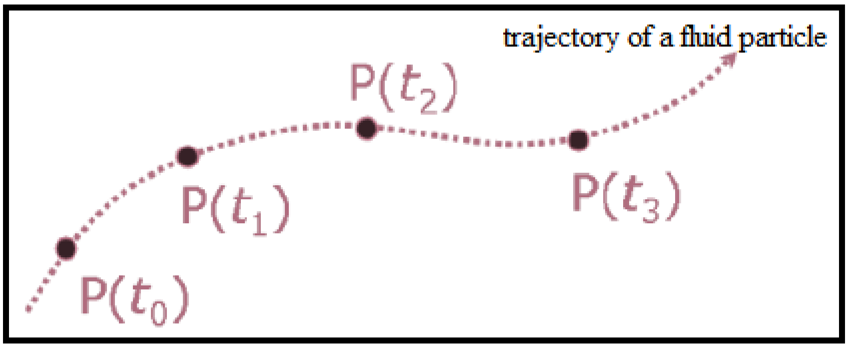

Figure I.7. The trajectory of a fluid particle.

I.5. Particle Derivative

The particle derivative, also known as the material or substantial derivative, is used in fluid dynamics to describe the rate of change of a physical quantity (such as velocity or temperature) as it moves with a fluid particle. It helps link the Eulerian (fixed point) perspective and the Lagrangian (moving particle) perspective of fluid motion.

I.5.1. Definition

The particle derivative $\frac{D}{Dt}$ of a field quantity v(x,y,z,t) (e.g., velocity or temperature) is defined as:

$$\frac{D\emptyset}{Dt} = \frac{\partial\emptyset}{\partial t} + u.\nabla\emptyset$$

where:

- $\frac{\partial\emptyset}{\partial t}$ is the local (Eulerian) rate of change of ϕ at a fixed point.
- $u\,.\nabla\emptyset$ represents the convective rate of change, showing how the movement of the particle through the spatial gradient ∇ϕaffects ϕ.

I.5.2. Explanation of Components

- **Local derivative**$\frac{\partial\emptyset}{\partial t}$: This term captures how ϕ\phiϕ changes at a given location over time.
- **Convective term u·∇ϕ**: This term accounts for changes due to the movement of the particle within the flow field, where u is the velocity vector of the fluid.

I.5.3. Physical Interpretation

The particle derivative represents the total rate of change experienced by a fluid particle as it moves through the flow field. It is a crucial concept for understanding how properties like temperature, pressure, or velocity evolve as they are carried along by the flow.

I.5.4. Application

In the case of a velocity field u, the particle derivative can describe the acceleration of a particle in the flow:

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + (u.\nabla)u$$

This equation is fundamental in deriving the Navier-Stokes equations, which describe the motion of viscous fluid substances.

I.5.5. Example

For a temperature field T(x,y,z,t), the particle derivative $\frac{DT}{Dt}$describes how the temperature changes as an observer follows a moving fluid particle:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u.\nabla T$$

This tells us how both the temporal change and the motion within the temperature gradient contribute to the overall change a particle experiences.

In summary, the particle derivative is vital for analyzing and predicting the behavior of fluid particles as they interact with the surrounding field, providing insight into fluid motion and transport phenomena.