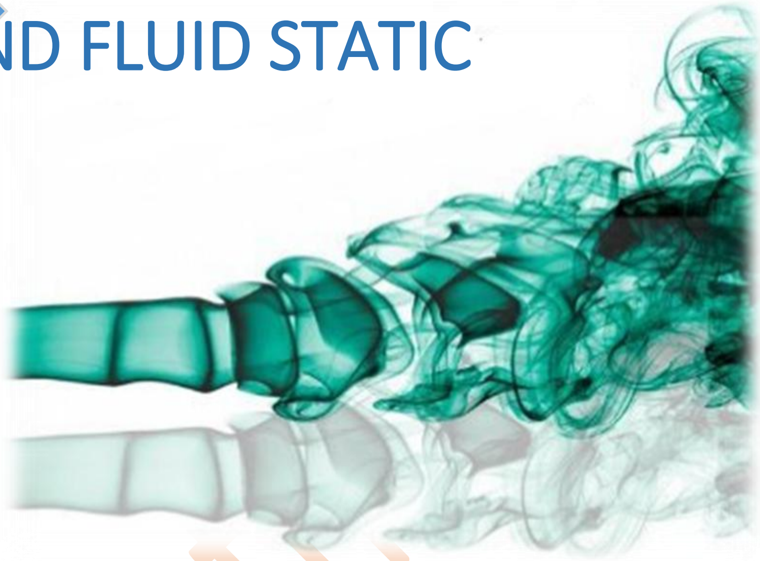
PRESSURE AND FLUID STATIC



Abdelhafid Boussouf University Center - Mila
Science and Technology Institutes
Department of Civil Engineering and Hydraulic

The term "pressure" refers to the force exerted per unit area on the surface of an object. It is a fundamental concept in physics, particularly in fluid mechanics, thermodynamics, and engineering. The pressure in a fluid (liquid or gas) arises due to the motion and collisions of molecules, and it can be caused by external forces like gravity or by the internal kinetic energy of the fluid's molecules.

II.1. Definition

Pressure is mathematically defined as the force applied per unit area:

$$P = \frac{F}{S} \tag{1}$$

where **P** is the pressure, **F** is the applied force, and **A** is the area over which the force is distributed.

II.1.1. SI Unit

The SI unit of pressure is the Pascal (Pa), where 1 Pa is equal to one Newton per square meter (N/m^2). Other common units include atmospheres (atm), bar, and pounds per square inch (psi).

II.1.2.Types of Pressure

- ***Absolute Pressure:*** Measured relative to a perfect vacuum (zero reference point).
- ***Gauge Pressure:*** Measured relative to atmospheric pressure. It is often used in practical applications like tire pressure.
- ***Differential Pressure:*** The difference in pressure between two points in a system.

II.1.3. Pressure in Fluids

In a static fluid, pressure at a point is isotropic (equal in all directions). This is described by Pascal's law, which states that any change in pressure applied to a fluid is transmitted uniformly in all directions.

The pressure at a depth in a fluid is given by:

$$P = P_0 + \rho g h \tag{2}$$

where P_0 is the surface pressure, ρ is the fluid density, g is gravitational acceleration, and h is the depth.

II.1.4. Applications of Pressure

- **Hydraulics:**

Pressure is the key principle behind hydraulic systems, where fluid pressure is used to generate force (e.g., car brakes, hydraulic lifts).

- **Atmospheric Pressure:**

Weather patterns are influenced by atmospheric pressure variations, leading to phenomena like high and low-pressure systems.

- **Engineering and Design:**

Pressure is critical in the design of pipelines, reactors, boilers, and various mechanical systems to ensure structural integrity.

- **Dynamic Pressure**:

In moving fluids, pressure is related to velocity, as described by Bernoulli's principle. The total pressure is the sum of static pressure, dynamic pressure (due to fluid motion), and gravitational effects. Understanding pressure is essential for analyzing systems involving fluid flows, gas laws, and structural stability, particularly in fields like aerospace, civil engineering, and thermodynamics.

For a static fluid, the only stress is the normal stress since by definition a fluid subjected to a shear stress must deform and undergo motion. Normal stresses are referred to as pressure **P**.

II.2. Definition of the Stress Tensor

The stress tensor is a second-order tensor that relates the forces within a material to the surface areas across which these forces act. It characterizes how forces are distributed inside the material. For the general case, the stress on a fluid element or at a point is a tensor:

τ_{ij}: stress tensor is

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \quad (3)$$

i: face, j: direction.

Tensor: Some mathematical object analogues to but more general than a vector, represented by an array of components that are functions of the coordinates of a space (Oxford)

The stress tensor is a mathematical construct used in physics and engineering to describe the internal forces (or stresses) that develop within a material when subjected to external forces.

It is essential for understanding how solids and fluids respond to loads, deformations, and other forces.

II.2.1. Components of the Stress Tensor

Each element τ_{ij} in the stress tensor represents the force per unit area in the i-direction acting on a surface with a normal vector in the j-direction: τ_{xx},τ_{yy},τ_{zz} are normal stresses: These represent stresses that act perpendicular to the surfaces. For example, τ_{xx} is the stress in the x-direction acting on a surface with a normal in the x-direction.$\tau_{xy}\tau_{xz}\tau_{yz}$ (and their symmetric counterparts) are shear stresses: These represent stresses acting tangentially to the surfaces. For example, τ_{xy} is the stress in the x-direction acting on a surface with a normal in the y-direction.

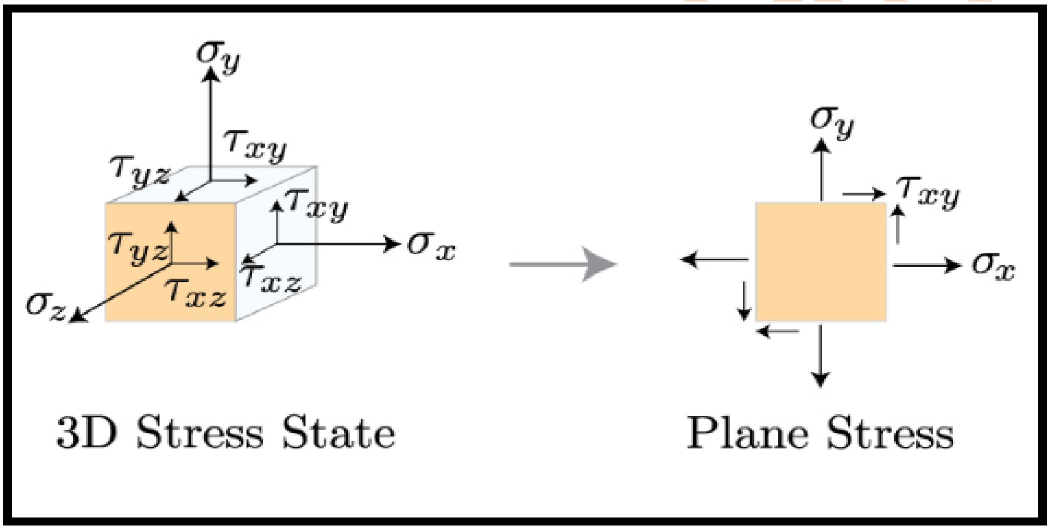


Figure 1: The 3D stress and plane stress.

II.2.2. Symmetry of the Stress Tensor

In most physical situations, the stress tensor is symmetric, meaning:

$$\tau_{ij} = \tau_{ji} \tag{4}$$

This symmetry arises from the equilibrium of rotational moments (i.e., no net torque). Thus, the stress tensor often reduces to six independent components instead of nine. This is important in fields like elasticity, where materials tend to have symmetric stress responses.

II.2.3. Interpretation of the Stress Tensor

The stress tensor can be thought of as describing the internal distribution of forces within a material at a given point.

- Given a surface with a normal vector **n**, the stress acting on this surface is given by:

$$T = \tau . n \qquad (5)$$

where T is the traction vector (the force per unit area on the surface), and **n** is the unit normal vector to the surface.

II.2.4. Principal Stresses and Eigenvalues

The principal stresses are the normal stresses that act on mutually perpendicular planes where the shear stress components are zero. These stresses correspond to the eigenvalues of the stress tensor, and the directions of these stresses correspond to the eigenvectors. These planes are known as the principal planes.

II.2.5. Applications of the Stress Tensor

Solid Mechanics: In analyzing how structures respond to forces, the stress tensor helps predict material failure, deformations, and strains.

- **Fluid Mechanics:** The stress tensor is critical in describing how fluids flow, particularly in relation to the concept of viscous stress (related to internal friction in fluids).
- **Continuum Mechanics:** The stress tensor, along with strain tensors, is essential in describing the behavior of materials under various loads.
- **Thermodynamics:** It plays a role in understanding the thermodynamic properties of materials, especially when dealing with forces and energy exchange.

II.2.6. Cauchy's Stress Principle

Cauchy's theorem states that at any point within a material, the stress vector on any plane can be determined using the stress tensor and the normal to the plane. This principle helps in describing the stress state of a material from different orientations.

II.3. Summary

The stress tensor is a fundamental concept for understanding internal forces within materials. It represents both normal and shear stresses and helps predict how materials respond to external loads. Its applications span across solid mechanics, fluid mechanics, and material science, making it a key tool for analyzing structural integrity and dynamic behaviors.

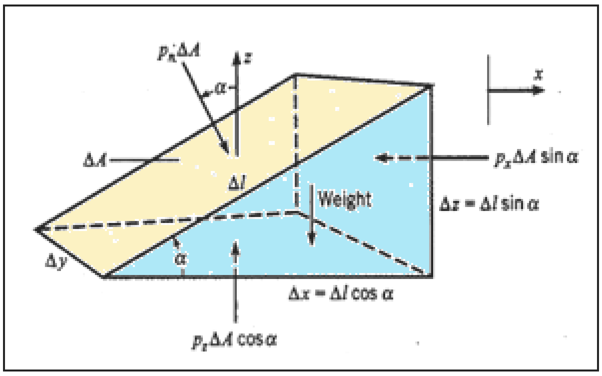
II.4. Pressure of Fluid

Also shows that p is isotropic, one value at a point which is independent of direction, a scalar.

- Shear stresses=0 $\tau_{ij} = 0 \quad i \neq j$
- Normal stresses =-P $\tau_{ii} = -P = \tau_{xx} = \tau_{yy} = \tau_{zz} \quad i = j$

$$P = \lim_{\partial A \to 0} \frac{\partial F}{\partial A} = \frac{dF}{dA} \frac{N}{m^2} = Pa(Pascal) \quad (6)$$

F = normal force acting over **A** as already noted, **P** is a scalar, which can be easily demonstrated by considering the equilibrium of forces on a wedge-shaped fluid element of geometry:



W: Weight

- $\Delta A = \Delta l \, \Delta y$
- $\Delta x = \Delta l \, \Delta \cos\alpha$
- $\Delta z = \Delta l \, \Delta \sin\alpha$

$\sum F_x = 0 \Longrightarrow P_n \Delta A \ \sin\alpha - P_x \Delta A \ \sin\alpha = 0 \Longrightarrow P_n = P_x$

$$\sum F_z = 0$$

$$-P_n \Delta A \ \cos\alpha + P_z \Delta A \ \cos\alpha - W = 0 \ \to W = mg = \rho V g = \frac{1}{2} \rho g \Delta x \Delta y \Delta z$$

$$-P_n \Delta A \ \cos\alpha + P_z \Delta A \ \cos\alpha - \frac{1}{2} \rho g \Delta x \Delta y \Delta z$$

$$\Longrightarrow -P_n \Delta l \ \Delta y \cos\alpha + P_z \Delta l \ \Delta y \cos\alpha - \frac{1}{2} \rho g \Delta l^{2 \cos\alpha \sin\alpha} \Delta y = 0$$

$P_n + P_z - \frac{1}{2} \rho g \Delta l \sin\alpha = 0$

$P_n = P_z$ For $\Delta l \to 0$ i.e., $P_n = P_x = P_y = P_z$

P is single valued at a point and independent of direction. A body/surface in contact with a static fluid experiences a force due to P.

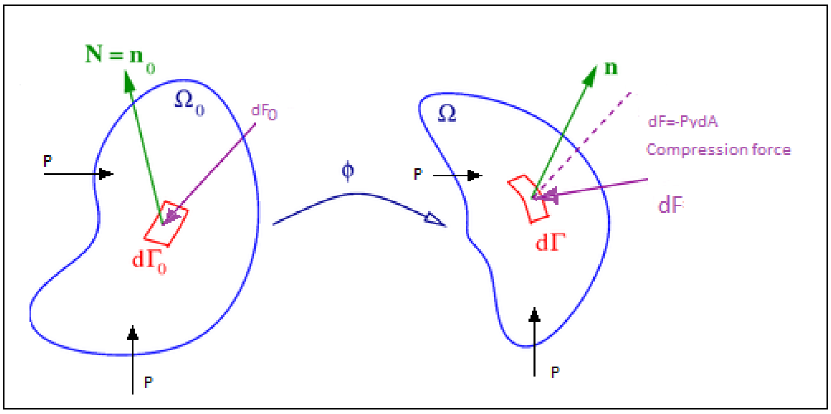


Figure 2:Quantités utilisées dans la définition des mesures de stress.

Note: if P=constant, $F_P = 0$ for a closed boby. Scalar from of Green's Theorem:

$$\int_S fnds = \int_A \nabla f dA \qquad f = constant \Rightarrow \nabla f = 0 \tag{7}$$

II.4.1. Pressure Transmission

Pascal's law: in a closed system, a pressure change produced at one point in the system is transmitted throughout the entire system.

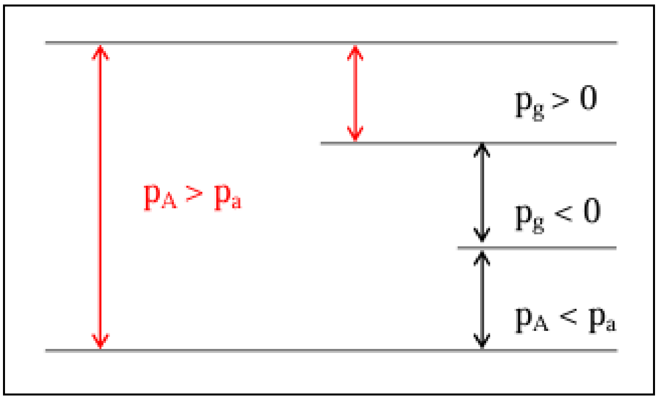
Generally, refers to the way pressure is propagated through a fluid within a system, based on principles of fluid mechanics. In any enclosed fluid system, pressure changes applied at one point are typically transmitted uniformly throughout the fluid, a principle often summarized by Pascal's Law. Here are some key aspects :

1. **Pascal's Law**: This foundational principle states that in a confined fluid, any change in pressure applied at one point is transmitted undiminished to all points within the fluid. This property is the basis for hydraulic systems.
2. **Applications in Hydraulic Systems**: Pressure transmission enables hydraulic systems to multiply force, allowing small inputs to create large outputs. This principle is applied in equipment like hydraulic jacks, brakes, and industrial machinery.
3. **FactorsAffecting Pressure Transmission** :
    - **Fluid Density**: Denser fluids can transmit pressure more effectively in some contexts, as they have lower compressibility.
    - **Viscosity**: Higher viscosity can dampen pressure transmission over distances due to resistance to flow.

- **Temperature**: Increased temperatures can change fluid properties, affecting both viscosity and pressure transmission.

4. **Design Considerations**: Systems are often designed to optimize pressure transmission by minimizing energy losses, which is essential in fields such as aeronautics, hydraulics, and fluid power systems.

Would you like more details on a particular type of pressure transmission, such as in porous media or turbulent flow systems?



P_a = atmospheric pressure = 101.325 kPa

$P_A = 0$ = absolute zero

Figure 3:Absolute Pressure, Gage Pressure, and Vacuum.

- For $P_A > P_a P_g = P_A - P_a$ = gage pressure
- For $P_A < P_a P_{vac} = -P_g = P_a - P_A$ = vacuum pressure