

## Series of exercises N 3

## Exercise 1

Determine the domain of definition of each of the following functions

❶  $f(x, y) = \sqrt{1 - x^2 - y^2}$

❷  $f(x, y) = \frac{x^2 + y^2}{x + y}$

❸  $f(x, y) = x^2 + y + \ln(x^2 + y^2)$

## Exercise 2

We consider the real function of two variables  $f$  defined by

$$f(x, y) = \frac{x^2}{y - 2x^2}$$

- ❶ Determine the domain of definition of  $f$
- ❷ Calculate the gradient of  $f$  at the point  $(1, 1)$ .

## Exercise 3

Calculate the partial derivatives of order 2 of the following functions

❶  $f(x, y) = xe^{xy}$

❷  $f(x, y) = x^3 + y^3 + 3x^2y$

❸  $f(x, y) = x^4 + y^3 + 2y \cos(x) + 5y$ .

#### Exercise 4

Calculate the following double and triple integrals

$$\textcircled{1} I_1 = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x + y) dx dy$$

$$\textcircled{2} I_2 = \int_1^2 \int_{-1}^1 \frac{x^2}{y} dx dy$$

$$\textcircled{3} I_3 = \iint_D \frac{1}{(x + y)^3} dx dy \quad \text{with } D = \{(x, y) \in \mathbb{R}^2 \mid x \geq 1, y \geq 1, x + y \leq 3\}.$$

$$\textcircled{4} I_1 = \iiint_P \frac{x^2 y}{z} dx dy dz \quad \text{with } P = [0, 1] \times [0, 1] \times [1, 2]$$