**Exercice 2.** Résoudre avec la méthode du simplexe :

Max z =2x1+x2

$$\left\{\begin{array}{c}x1-2x2+x3=2\\-2x1+x2+x4=2\\x1,x2, x3, x4\geq 0\end{array}\right.$$

On constate que le PL est sous la forme standard alors on peut directement lancer l’algorithme du simplexe.

1) Tableau initial :

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1**$\downright $ | **x2** | **x3** | **x4** |
| $\leftarrow $**x3** | **2** | **1** | **-2** | **1** | **0** |
| **x4** | **2** | **-2** | **1** | **0** | **1** |
| **Z** | **0** | **2** | **1** | **0** | **0** |

2) Première itération

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1**$\downright $ | **x2** | **x3** | **x4** |
| $\leftarrow $**x3** | **2** | **1** | **-2** | **1** | **0** |
| **x4** | **6** | **0** | **-3** | **2** | **1** |
| **Z** | **-4** | **0** | **5** | **-2** | **0** |

Tous les coefficients dans la colonne qui correspond à la plus grande valeur dans la ligne de l’objectif sont négatifs ou nuls alors le problème admet des solutions non-bornées.

**Exercice 3.**

**1. La formulation mathématique du problème**

* 1. Les variables de décision

X1 : quantité de A, X2 : quantité de B, X3 : quantité de C.

* 1. Les contraintes

x1+x2≤100 contrainte sur X.

x2+x3 ≤200 contrainte sur Y.

x1+x2+ x3≤400 contrainte sur Z.

x2 ≤80. La demande sur B.

La fonction objective :

Max Z=3x1+4x2+2x3.

Nous obtenons donc Le programme linéaire suivant :

Max Z=3$x\_{1}$+$4x\_{2}$+$2x\_{3}$
$\left\{\begin{array}{c}x\_{1}+x\_{2}\leq 100\\x\_{2}+x\_{3}\leq 200\\x\_{1}+x\_{2}+x\_{3}\leq 400\\x\_{2}\leq 80\\x\_{1},x\_{2},x\_{3}\geq 0\end{array}\right.$

2. La résolution du programme linéaire :

* 1. La forme standard :

Max Z=3$x\_{1}$+$4x\_{2}$+$2x\_{3}$
$$\left\{\begin{array}{c}x\_{1}+x\_{2}+x\_{4}=100\\x\_{2}+x\_{3}+x\_{5}=200\\x\_{1}+x\_{2}+x\_{3}+x\_{6}=400\\x\_{2}+x\_{7}=80\\x\_{1},x\_{2},x\_{3}\geq 0\\x\_{4},x\_{5},x\_{6}\geq 0\end{array}\right.$$

 Le tableau initial :

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **sm** |
| **X4** | **1** | **1** | **0** | **1** | **0** | **0** | **0** | **100** |
| **X5** | **0** | **1** | **1** | **0** | **1** | **0** | **0** | **200** |
| **X6** | **1** | **1** | **1** | **0** | **0** | **1** | **0** | **400** |
| **X7** | **0** | **1** | **0** | **0** | **0** | **0** | **1** | **80** |
| **Z** | **3** | **4** | **2** | **0** | **0** | **0** | **0** | **0** |

Itération 1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **sm** |
| **X4** | **1** | **0** | **0** | **1** | **0** | **0** | **-1** | **20** |
| **X5** | **0** | **0** | **1** | **0** | **1** | **0** | **-1** | **120** |
| **X6** | **1** | **0** | **1** | **0** | **0** | **1** | **-1** | **320** |
| **X2** | **0** | **1** | **0** | **0** | **0** | **0** | **1** | **80** |
| **Z** | **3** | **0** | **2** | **0** | **0** | **0** | **-4** | **-320** |

Itération 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **sm** |
| **X1** | **1** | **0** | **0** | **1** | **0** | **0** | **-1** | **20** |
| **X5** | **0** | **0** | **1** | **0** | **1** | **0** | **-1** | **120** |
| **X6** | **0** | **0** | **1** | **-1** | **0** | **1** | **0** | **300** |
| **X2** | **0** | **1** | **0** | **0** | **0** | **0** | **1** | **80** |
| **Z** | **0** | **0** | **2** | **-3** | **0** | **0** | **-1** | **-380** |

Itération 3

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **sm** |
| **X1** | **1** | **0** | **0** | **1** | **0** | **0** | **-1** | **20** |
| **X3** | **0** | **0** | **1** | **0** | **1** | **0** | **-1** | **120** |
| **X6** | **0** | **0** | **0** | **-1** | **-1** | **1** | **1** | **180** |
| **X2** | **0** | **1** | **0** | **0** | **0** | **0** | **1** | **80** |
| **Z** | **0** | **0** | **0** | **-3** | **-2** | **0** | **1** | **-620** |

## *Itération 4*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **sm** |
| **X1** | **1** | **1** | **0** | **1** | **0** | **0** | **0** | **100** |
| **X3** | **0** | **1** | **1** | **0** | **1** | **0** | **0** | **200** |
| **X6** | **0** | **-1** | **0** | **-1** | **-1** | **1** | **0** | **100** |
| **X7** | **0** | **1** | **0** | **0** | **0** | **0** | **1** | **80** |
| **Z** | **0** | **-1** | **0** | **-3** | **-2** | **0** | **0** | **-700** |

## *Tous les coefficient*s dans la ligne de la fonction objective sont négatifs ou nulle donc la solution optimale est : x1\*=100, x2\*=0, x3\*=200,x4\*=x5\*=x6\*=0, x7\*=80, z\*=-z=700.

**Exercice 4** Résoudre avec la méthode du simplexe :

 Max Z= 3x1+2x2+4x3

$$\left\{\begin{array}{c}x1+x2+2x3 \leq 4\\2x1+3x3\leq 5\\2x1+x2+3x3\leq 7\\x\&1 , x2,x3\geq 0\end{array}\right.$$

**1- La forme standard du PL :**

Max Z= 3x1+2x2+4x3

$$\left\{\begin{array}{c}x1+x2+2x3+x3=4\\2x1+3x3+x5=5\\2x1+x2+3x3+x6=7\\x\&1 , x2,x3, x4,x5,x6\geq 0\end{array}\right.$$

**2- Tableau initial**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **x6** |
| **x4** | **4** | **1** | **1** | **2** | **1** | **0** | **0** |
| $\leftarrow $**x5** | **5** | **2** | **0** | **3** | **0** | **1** | **0** |
| **x6** | **7** | **2** | **1** | **3** | **0** | **0** | **1** |
| **Z** | **0** | **3** | **2** | **4** | **0** | **0** | **0** |

**3- Première itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1** | **x2**$\downright $ | **x3** | **x4** | **x5** | **x6** |
| $\leftarrow $**x4** | **2/3** | **-1/3** | **1** | **0** | **1** | **-2/3** | **0** |
| **x3** | **5/3** | **2/3** | **0** | **1** | **0** | **1/3** | **0** |
| **x6** | **2** | **0** | **1** | **0** | **0** | **-1** | **1** |
| **Z** | **-20/3** | **1/3** | **2** | **0** | **0** | **-4/3** | **0** |

**4- Deuxième itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1**$\downright $ | **x2** | **x3** | **x4** | **x5** | **x6** |
| **x2** | **2/3** | **-1/3** | **1** | **0** | **1** | **-2/3** | **0** |
| $\leftarrow $**x3** | **5/3** | **2/3** | **0** | **1** | **0** | **1/3** | **0** |
| **x6** | **4/3** | **1/3** | **0** | **0** | **-1** | **-1/3** | **1** |
| **Z** | **-8** | **1** | **0** | **0** | **-2** | **0** | **0** |

**5- Troisième itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1** | **x2** | **x3** | **x4** | **x5** | **x6** |
| **x2** | **3/2** | **0** | **1** | **1/2** | **1** | **-1/2** | **0** |
| **x1** | **5/2** | **1** | **0** | **3/2** | **0** | **1/2** | **0** |
| **x6** | **1/2** | **0** | **0** | **-1/2** | **-1** | **-1/2** | **1** |
| **Z** | **-21/2** | **0** | **0** | **-3/2** | **-2** | **-1/2** | **0** |

On constate que tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x1\*=x1=5/2, x2\*=x2=3/2, x6\*=x6=3/2, x3\*=x4\*=x5\*=0.

Z\*=-Z=21/2.

**Exercice 5.**

Max z =-5x1+5x2+13x3

$$\left\{\begin{array}{c}-x1+x2+3x3\leq 20\\12x1+4x2+10x3\leq 90\\x1,x2, x3\geq 0\end{array}\right.$$

Max z =-5x1+x2+13x3

$$\left\{\begin{array}{c}-x1+x2+3x3+x4=20\\12x1+4x2+10x3+x5=90\\x1,x2, x3,x4, x5\geq 0\end{array}\right.$$

1) Tableau initial :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **Bi** |
| $\leftarrow $**x4** | **-1** | **1** | **3** | **1** | **0** | **20** |
| **x5** | **12** | **4** | **10** | **0** | **1** | **90** |
| **Z** | **-5** | **5** | **13** | **0** | **0** | **0** |

2) Première itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **X2**$\downright $ | **x3** | **x4** | **x5** | **Bi** |
| $\leftarrow $**x3** | **-1/3** | **1/3** | **1** | **1/3** | **0** | **20/3** |
| **x5** | **46/3** | **2/3** | **0** | **-10/3** | **1** | **70/3** |
| **Z** | **-2/3** | **2/3** | **0** | **-13/3** | **0** | **-260/3** |

2) Deuxième itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **Bi** |
| $$x2$$ | **-1** | **1** | **3** | **1** | **0** | **20** |
| **x5** | **16** | **0** | **-2** | **-4** | **1** | **10** |
| **Z** | **0** | **0** | **-2** | **-5** | **0** | **-100** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x2\*=x2=20, x5\*=x5=10, x1\*=x3\*=x4\*=0.

Z\*=-Z=100

2- La nouvelle base en changeant le second membre de la contrainte n 1 :

B=$\left(\begin{array}{c}x2\\x5\end{array}\right)$ , AB=$\left(\begin{matrix}1&0\\4&1\end{matrix}\right)$ AB-1=$\left(\begin{matrix}1&0\\-4&1\end{matrix}\right)$

$\left(\begin{array}{c}x2\\x5\end{array}\right)$=$\left(\begin{matrix}1&0\\-4&1\end{matrix}\right)$ $\left(\begin{array}{c}30\\90\end{array}\right)$= $\left(\begin{array}{c}30\\-30\end{array}\right)$ x5<0 Solution non admissible.

2.1 La solution du problème par l algorithme du simplexe.

. 1) Tableau initial :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **Bi** |
| **x4** | **-1** | **1** | **3** | **1** | **0** | **30** |
| $\leftarrow $**x5** | **12** | **4** | **10** | **0** | **1** | **90** |
| **Z** | **-5** | **5** | **13** | **0** | **0** | **0** |

2) Première itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **X2**$\downright $ | **x3** | **x4** | **x5** | **Bi** |
| **x4** | **-4,6** | **-0,2** | **0** | **1** | **-0,3** | **3** |
| **x3** | **1,2** | **0,4** | **1** | **0** | **0,1** | **9** |
| **Z** | **-20,6** | **-0.2** | **0** | **0** | **-1.3** | **-117** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x3\*=x3=9, x4\*=x4=3, x1\*=x2\*=x5\*=0.

Z\*=-Z=117.

L intervalle de validité de la base B=$\left(\begin{array}{c}x2\\x5\end{array}\right) $ :

$\left(\begin{matrix}1&0\\-4&1\end{matrix}\right)$ $\left(\begin{array}{c}b\\90\end{array}\right)$=$\left(\begin{array}{c}b\\-4b+90\end{array}\right)\geq \left(\begin{array}{c}0\\0\end{array}\right)$ => b$\in [0, 22.5]$.