## Introduction

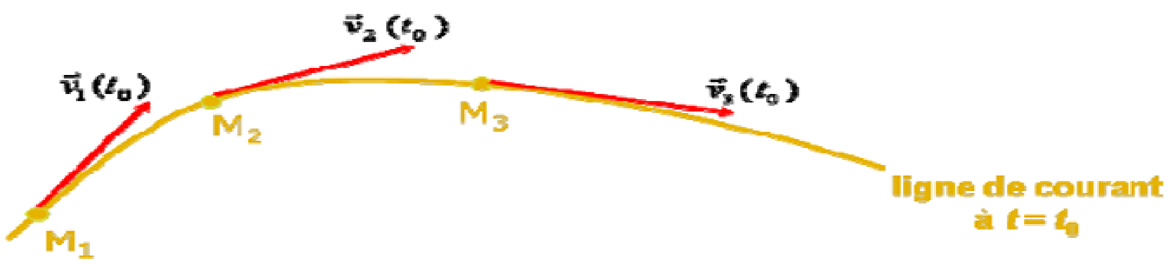
This chapter is devoted to the study of the movements of a fluid determining the causes and the laws which govern it (without taking into account the forces which give rise to it). We therefore only consider the relationships between the positions of the liquid particles and time.

In fluid kinematics, the study of the movement of fluid particles constitutes the study of the movement of the fluid itself..

## III.1 Définition

### Power lines

Une ligne de courant est une courbe de l'espace décrivant le mouvement d'un fluide correspond à une ligne du champ de vitesse de l'écoulement dans un volume de contrôle à un instant donné. C'est une ligne où le vecteur vitesse est tangent en tout point comme le montre la figure suivante.

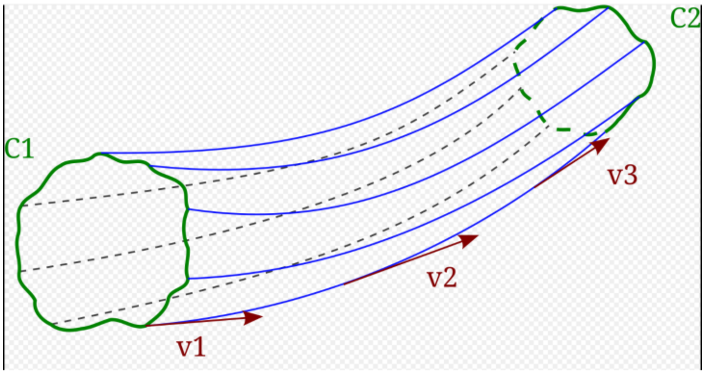



From a mathematical point of view, if we consider an elementary displacement dl= 〖 $\boldsymbol{dl} = \boldsymbol{dl_i e_i}$ on a current line, it will be defined by the following relation: $\vec{u} \wedge \overrightarrow{dl} = \vec{0}$ **(the two vectors are parallel).**

Which implies in the Cartesian coordinate system the following relationship:

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$ (**The mathematical definition of streamline**)

A set of current lines based on a closed contour placed inside the flow is called a current tube.




**Trajectory:** It is the curve described over time by a fluid particle, i.e. each fluid particle that we follow in its movement (Lagrangian description) describes a trajectory that we determine by solving the following equation:

We have: $\frac{dx}{dt} = u(t, x, y, z)$ ; $\frac{dy}{dt} = v(t, x, y, z)$ ; $\frac{dz}{dt} = w(t, x, y, z)$

So: $dt = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

That in ***steady state*** the trajectories of the fluid particles and the streamlines are confused. They constitute a network of curves constant over time. However, in ***unsteady regime***, the current lines evolve over time and do not coincide with each other nor with the trajectories of the particles; while the trajectories do not intrinsically depend on it.

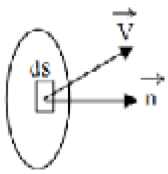
## III.2 Conservation of mass

### III.2.1 Mass flow and volume flow

**Mass flow**

The mass flow rate $Q_m$ is the amount of fluid mass flowed through a given section in unit time. This flow rate is given by the relation:

$$\boldsymbol{Q_m = \frac{dm}{dt} = \int \rho \vec{V}\, ds\, \vec{n}} \qquad Q_m \text{ is expressed in } (Kg/s)$$



**Volume flow**

We call flow rate the volume of fluid flowing in units of time through a given section. This flow rate $Q_V$ is given by the relation:

$$\boldsymbol{Q_V = \frac{dV}{dt} = \rho \frac{dm}{dt} = \int \vec{V}\, ds\, \vec{n}}$$

$Q_m$ *is expressed in* $(m^3/s)$

The density **ρ** is given by the following relation $\boldsymbol{\rho = \frac{dm}{dV}}$

Therefore$\boldsymbol{Q_m = \frac{dm}{dt} = \frac{\rho dV}{dt} = \rho\, Q_V}$ the two flow rates are linked by the following relationship:

$$\boldsymbol{Q_m = \rho\, Q_V}$$

**III.3 Continuity equation**

In a permanent and one-dimensional flow of a fluid, we consider an infinitely narrow current tube between two sections $S_A$ et $S_B$ .

During the infinitely small time interval dt, the mass $dm_A = \rho_A S_A dx_A$ of the fluid crosses the section $S_A$ and the mass $dm_B = \rho_B S_B dx_B$ crosses the section $S_B$.

The conservation of mass is as follows: $dm_A = dm_B \quad \Rightarrow \rho_A S_A dx_A = \rho_B S_B dx_B$

By dividing by dt :

$\rho_A S_A \frac{dx_A}{dt} = \rho_B S_B \frac{dx_B}{dt}$ We obtain:

$\boldsymbol{\rho_A S_A v_A = \rho_B S_B v_B}$ Either : $\boldsymbol{Q_{m(A)} = Q_{m(B)}}$

$$\boldsymbol{Mass\ flow\ in = Mass\ flow\ out}$$

- This relationship reflects **<u>the conservation of mass flow in a steady flow</u>**.

**The mass flow rate ($Q_m$):** is t<u>he mass of fluid</u> flowing through <u>a given section</u> in the unit of time.

$Q_m$: mass flow is expressed in (Kg/s)

S : area ($m^2$)

$\rho$: density of the fluid (Kg/$m^3$)

v : speed in (m/s)

- For an incompressible fluid $\rho = cste$

$\rho = cste$ so, we have $\rho_A = \rho_B = \rho \quad \Rightarrow \quad \rho S_A v_A = S_B v_B$ , we simplify by ρ We obtain:

$$\boldsymbol{S_A v_A = S_B v_B} \Rightarrow \boldsymbol{Q_{V(A)} = Q_{V(B)}}$$

$$\boldsymbol{Incoming\ volume\ flow = Outgoing\ volume\ flow}$$

This relationship reflects **the conservation of the volume flow in a steady flow**.

$Q_v$: volume flow is expressed ($m^3/s$)

We call **volume flow** the volume of fluid flowing in the unit of time through a given section.

### III.2.1 Permanent and non-permanent flow

Steady flow is the flow in which all the characteristic quantities of the flow and the properties of the fluid such as (pressure, temperature, density, speed etc.) at each point remain constant over time $\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{t}} = 0 \ , \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{t}} = 0 \ldots ..$

A flow is non-permanent if the characteristic quantities of the flow and the properties of the fluid such as (pressure, temperature, density, speed etc.) at each point vary as a function of space and time $\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{t}} \neq 0 \ , \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{t}} \neq 0 \ldots ..$