People's Democratic Republic of Algeria

Ministry of Higher Education and scientific Research

University Center abdelhafid boussouf (Alger)

Institute of mathematics and computer sciences

Department of mathematics

Serie of exercise
Introduction To Dynamics Systems .

Master 1 (first year) fundamental and applied mathematics

The first semester

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CHAPTER 1

NONLINEAR SYSTEMS: LOCAL THEORY

exercise 48

Linearize the nonlinear system

$$\dot{x} = e^{-x-3y} - 1$$
$$\dot{y} = -x\left(1 - y^2\right)$$

about the fixed point $(0, 0)$ and then classify the fixed point.

exercise 49

Draw the phase portraits of $\ddot{x} + x = ax^2$ for the three values $a = 0, -1, 1$.

exercise 50

Show that the equilibrium point is a stable focus for the damped linear pendulum $\ddot{x} + \dot{x} + x = 0$. Draw the phase diagram of the system.

exercise 51

Obtain all critical points of the system $\dot{x} = \sin y, \dot{y} = \cos x$. Linearize the system about the critical point $\left(\frac{\pi}{2}, 0\right)$. Find the equation of the phase path.

exercise 52

Classify the equilibrium points of the system

$$\dot{x} = x - y$$
$$\dot{y} = x^2 - 1$$

exercise 53

Consider a nonlinear system

$$\dot{x} = 1 - (a+1)x + bx^2 y$$
$$\dot{y} = ax - bx^2 y$$

where a and b are positive parameters.

(i) Show that $(1, a/b)$ is the only critical point of the system,

(ii) linearize the system about this critical point.

exercise 54

Find all fixed points of the system

$$\dot{x} = x(y - 1)$$
$$\dot{y} = 3x - 2y + x^2 - 2y^2$$

Linearize the system about the fixed point $(0, 0)$. Comment about the stability around this fixed point.

exercise 55

Show that the solution of the autonomous system $\dot{x} = y, \dot{y} = -x$ with $x(0) = 0, y(0) = 0$ is stable in the sense of Lyapunov.

exercise 56

Prove that each solution of the equation $\dot{x} + x = 0$ is asymptotically stable.

exercise 57

Prove that all solutions of the system $\dot{x} = \sin^2 x$ are bounded on $(-\infty, +\infty)$ but the solution $x(t) = 0$ is unstable as $t \to \infty$.

exercise 58

Using suitable Lyapunov functions, examine the stabilities for the following systems:
(i) $\ddot{x} + x = 0$,
(ii) $\dot{x} = x, \quad \dot{y} = -y$ at the origin.

exercise 59

Examine different stability criteria satisfied by the linear harmonic oscillator $\ddot{x} + x = 0$.

exercise 60

Show that the system $\dot{x} = -y\left(x^2 + y^2\right)^{1/2}, \dot{y} = x\left(x^2 + y^2\right)^{1/2}$ is orbitally stable but not Lyapunov stable.

exercise 61

Investigate the stability of the system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -(x - 2y)\left(1 - x^2 - 3y^2\right)$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = -(y + x)\left(1 - x^2 - 3y^2\right)$$

at the fixed point origin.

exercise 62

Using a suitable Lyapunov function shows that the origin is an asymptotically stable equilibrium point of the system

$$\begin{aligned}\dot{x} &= -2y + yz - x^3 \\ \dot{y} &= x - xz - y^3 \\ \dot{z} &= xy - z^3\end{aligned}$$

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