Application

Déterminer la flèche maximale et les rotations aux appuis de la Poutre représentée sur la Fig.2.

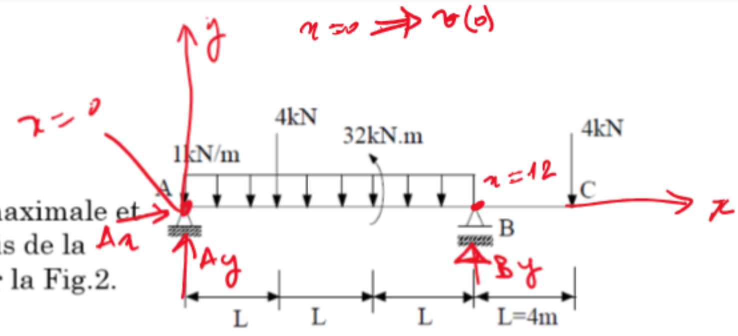

Figure 2

1/ Calculer les réactions d'appuis:

$$\sum f_x = 0 \Rightarrow \boxed{A_x = 0}$$

$$\sum f_y = 0 \Rightarrow A_y + B_y = 4 + 4 + 1(12) = 20\,kN$$

$$(+)\ \sum M_{/A} = 0 \Rightarrow -1(12).(6) - 4(4) - 4(16) + 32 + B_y(12) = 0$$

$$\Rightarrow -72 - 16 - 64 + 32 + 12B_y = 0$$

$$\Rightarrow 12B_y = 152 - 32 = 120$$

$$\Rightarrow \boxed{B_y = 10\,kN}\ ,\ \boxed{A_y = 10\,kN}$$

2/ Déterminer l'expression de la flèche et la rotation

$$EI\theta(x) = EI\theta_0 + \sum M\frac{(x-a)}{1!} + \sum P\frac{(x-b)^2}{2!} + \sum q_c\frac{(x-c)^3}{3!} - \sum q_d\frac{(x-d)^3}{3!} + \sum q_c'\frac{(x-c)^4}{4!} - \sum q_d'\frac{(x-d)^4}{4!} + \ldots$$

$$EI\theta(x) = EI\theta_0 + \frac{32(x-8)}{1!} - \frac{10(x-0)^2}{2!} + \frac{4(x-4)^2}{2!} - \frac{10(x-12)^2}{2!} + \frac{4(x-x)^2}{2!} + 1\frac{(x-0)^3}{3!} - \frac{1(x-12)^3}{3!}.$$

$$EI\theta(x) = EI\theta_0 + 32(x-8) - 5x^2 + 2(x-4)^2 - 5(x-12)^2 + \frac{1}{6}x^3 - \frac{(x-12)^3}{6}$$

$$EIv(x) = EIv_0 + EI\theta_0\frac{x}{1!} + \sum\frac{M(x-a)^2}{2!} + \sum\frac{P(x-b)^3}{3!} + \sum\frac{q_c(x-c)^4}{4!} - \sum\frac{q_d(x-d)^4}{4!} + \sum\frac{q_c'(x-c)^5}{5!} - \sum\frac{q_d'(x-d)^5}{5!} + \ldots,$$

$$EIv(x) = EIv_0 + EI\theta_0.x + \frac{32(x-8)^2}{2!} - \frac{10(x-0)^3}{3!} + \frac{4(x-4)^3}{3!} - \frac{10(x-12)^3}{3!} + \frac{4(x-x)^3}{3!} + \frac{1(x-0)^4}{4!} - \frac{1(x-12)^4}{4!}.$$

$$EIv(x) = EIv_0 + EI\theta_0.x + 16(x-8)^2 - \frac{5}{3}x^3 + \frac{2}{3}(x-4)^3 - \frac{5}{3}(x-12)^3 + \frac{x^4}{24} - \frac{(x-12)^4}{24}$$

3/ les conditions d'Appuis :

$v(0)=0\ ,\ v(12)=0$

$$EIv(0)=0 \Rightarrow EIv_0 + 16(-8)^2 + \frac{2}{3}(-4)^3 - \frac{5}{3}(-12)^3 - \frac{(-12)^4}{24} = 0$$

$$EIv_0 + 1024 - \frac{128}{3} + 2880 - 864 = 0$$

$$\boxed{v_0 = -\frac{2848}{3EI}} \boxed{= \frac{-949{,}33}{EI}}$$

$$EIv(12)=0 \Rightarrow \frac{-949{,}33}{3} + EI\theta_0(12) + 16(4)^2 - \frac{5}{3}(12)^3 + \frac{2}{3}(8)^3 + \frac{12^4}{24} = 0$$

$$\Rightarrow -\frac{949{,}33}{3} + 12\,EI\theta_0 + 256 - 2880 + \frac{341{,}33}{3} + 864 = 0$$

$$\Rightarrow 12\,EI\theta_0 = \frac{608}{3} + 1760 \Rightarrow \boxed{\theta_0 = \frac{163{,}55}{EI}}$$

$$EI\theta(x) = EI\theta_0 - \frac{10}{2}x^2 + \frac{x^3}{6} - \frac{(x-12)^3}{6} + 2(x-4)^2 + 32(x-8) - 5(x-12)^2$$

$$EIv(x) = EIv_0 + EI\theta_0 x - \frac{5}{3}x^3 + \frac{x^4}{24} - \frac{(x-12)^4}{24} + \frac{2(x-4)^3}{3} + 16(x-8)^2 - \frac{5}{3}(x-12)^3$$

$$EIv(0) = 0 \Rightarrow EIv_0 \Rightarrow v_0 = 0$$

$$EIv(12) = 0 \Rightarrow \theta_0 = \frac{118.22}{EI}$$

4) calculer la flèche max et les rotations

Aux Appuis :

Rotation aux Appuis :

$$\text{Appuis A : } EI\theta(x=0) = 163{,}55 + 32(0-8) - 5(0)^2 + 2(0-4)^2 - 5(0-12)^2 + \frac{1}{6}(0)^3 + \frac{1}{6}(0-12)^3$$

$$= 163{,}55 - 256 - 0 + 32 - 720 + 0 - 288$$

$$\boxed{\theta(0) = \frac{-1068{,}45}{EI}}$$

$$\text{Appuis B : } EI\theta(x=12) = 163{,}55 + 32(12-8) - 5(12)^2 + 2(12-4)^2 - 5\underbrace{(12-12)^2}_{0} + \frac{1}{6}(12)^3 + \frac{1}{6}\underbrace{(12-12)^3}_{0}$$

$$= 163{,}55 + 128 - 720 + 128 + 288 = -12{,}45$$

$$\Rightarrow \boxed{\theta(12) = \frac{-12{,}45}{EI}}$$