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Series N°2: Kinematics of material point

Exercise 1

1. Represent the points (ρ, θ): $\left\{(2, \frac{\pi}{4}), (1, \frac{\pi}{2}), (-2, \frac{\pi}{4}), (-1, -\frac{\pi}{2}) \text{ and } (2\sqrt{2}, \frac{3\pi}{4})\right\}$ in polar coordinates and find their equivalent in the Cartesian system.
2. Find the cylindrical coordinates of the points A(2, 3, 1) and B(3, 4, 5).

Exercise 2

A material point moves in space with an acceleration $\vec{a} = 2e^t\vec{\imath} + 5\cos(\theta)\,\vec{\jmath} - 3\sin(\theta)\vec{k}$. At the initial time t_0, the vectors of position and velocity are respectively ;

$\vec{r} = \vec{\imath} + 3\vec{\jmath} - 2\vec{k}$, and $\vec{v} = \overrightarrow{4\imath} - 3\vec{\jmath} + 2\vec{k}$. Find the time equations of the motion for this material point.

Exercise 3

A material point moves with the time equations $x = 3t + 1$, and $y = 4t + 1$. Find:

1. Trajectory (path) of this point.
2. Velocity and acceleration.
3. What is it the nature of the movement.

Exercise 4

Let us assume that, at t_o a projectile leaves the earth with a velocity $\vec{v}_0$ which makes an angle 30° with the horizontal. The acceleration of the particle is given by $\vec{a} = -g\vec{\jmath}$. Find:

- The time equation of the motion.
- The trajectory.
- Horizontal Range and Maximum Height of the Projectile.

Exercise 5

A particle moves with velocity $\vec{v} = \overrightarrow{4\imath} + (2t - 3)\vec{\jmath}$. At $t = 0$, the vector position $\vec{r} = 2\vec{\imath} - 3\vec{\jmath}$.

- Determine the trajectory of the particle.
- Determine t_1 where $\vec{v} \perp \vec{a}$, and deduce the coordinate at that time.
- Find the components of the acceleration (a_t and a_n).
- Find the radius of curvature R

Exercise 6

The time equations of a material point which moves on (x y) plane are given by $x = 2t$ and $y = 4t(t-1)$.

1- Plot the trajectory (path) on xy plane.
2- Find the vectors of velocity and acceleration and their magnitude.
1- Find the components of acceleration (tangent a_t and normal a_n).
3- Write the radius of curvature R as function of time.
4- Find the time where the velocity and acceleration are parallel.

Exercise 7

The polar coordinates of a particle are $\begin{cases} \rho = 2a\cos(\theta) \\ \theta = wt \end{cases}$, a and w are constants. Find:

2- The vectors of velocity and acceleration as a function of a and w. Deduce their magnitude.
3- Both components of acceleration (tangent a_t and normal a_n).
4- The radius of curvature R, and what do we conclude?
5- The curvilinear coordinate $s(t)$, we take $s(0) = 0$.
6- The trajectory equation in Cartesian system and represent the polar coordinates on the same plot.

Exercise 8

The polar coordinates of a material point are $\begin{cases} \rho = a(1+\cos(\theta)) \\ \theta = wt \end{cases}$, a and w are constants. Find:

- The vectors of velocity and acceleration as a function of a and w. Deduce their magnitude.
- Both components of acceleration (tangent a_t and normal a_n).
- The radius of curvature R, what do we conclude?
- The curvilinear coordinate s(t), we take s(0)=0.

Exercise 9

A particle m moves in space, and its vector position is written in the cylindrical coordinate a,

$\begin{cases} \vec{r} = a\vec{u}_\rho + bt\vec{k} \\ \theta = wt^2 \end{cases}$, where a, b and c are constants.

1. Determine the velocity and the acceleration.
2. The radius of curvature R after the particle makes a complete cycle around the z-axis.