Learning Objectives

- To define the external and internal forces.
- To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions.
- To define the moment of a force about a point.
- To provide a method for finding the moment of a force about a specified axis.
- To define the moment of a couple.
- To present methods for determining the resultants of nonconcurrent force systems.
- To indicate how to reduce a simple distributed loading to a resultant force having a specified location

External and Internal Effects

We can separate the action of a force on a body into two effects, ***external*** and ***internal.*** When external forces are applied to an object from outside, internal forces are induced in the object to counteract the externally applied forces and maintain equilibrium (Figure 11.(a)). Think of the forces you apply to stretch a rubber band. Your externally applied forces result in internal forces being set up in the rubber band. So when you let go of the band these internal forces pull the band back to its original unstretched state. Internal forces are produced in reaction to the application of external forces and so are often called the *reactive forces* or *reactions*.

Division in the system

-Internal force:

obtained by notionally cutting the body (Figure 11.(c)). This force acts between the parts of a body or system (normal force N, shear force Q and bending moment M).

-External force:

acts on a body from the outside (e.g. weight, wind pressure, snow load, adhesive force and support force).

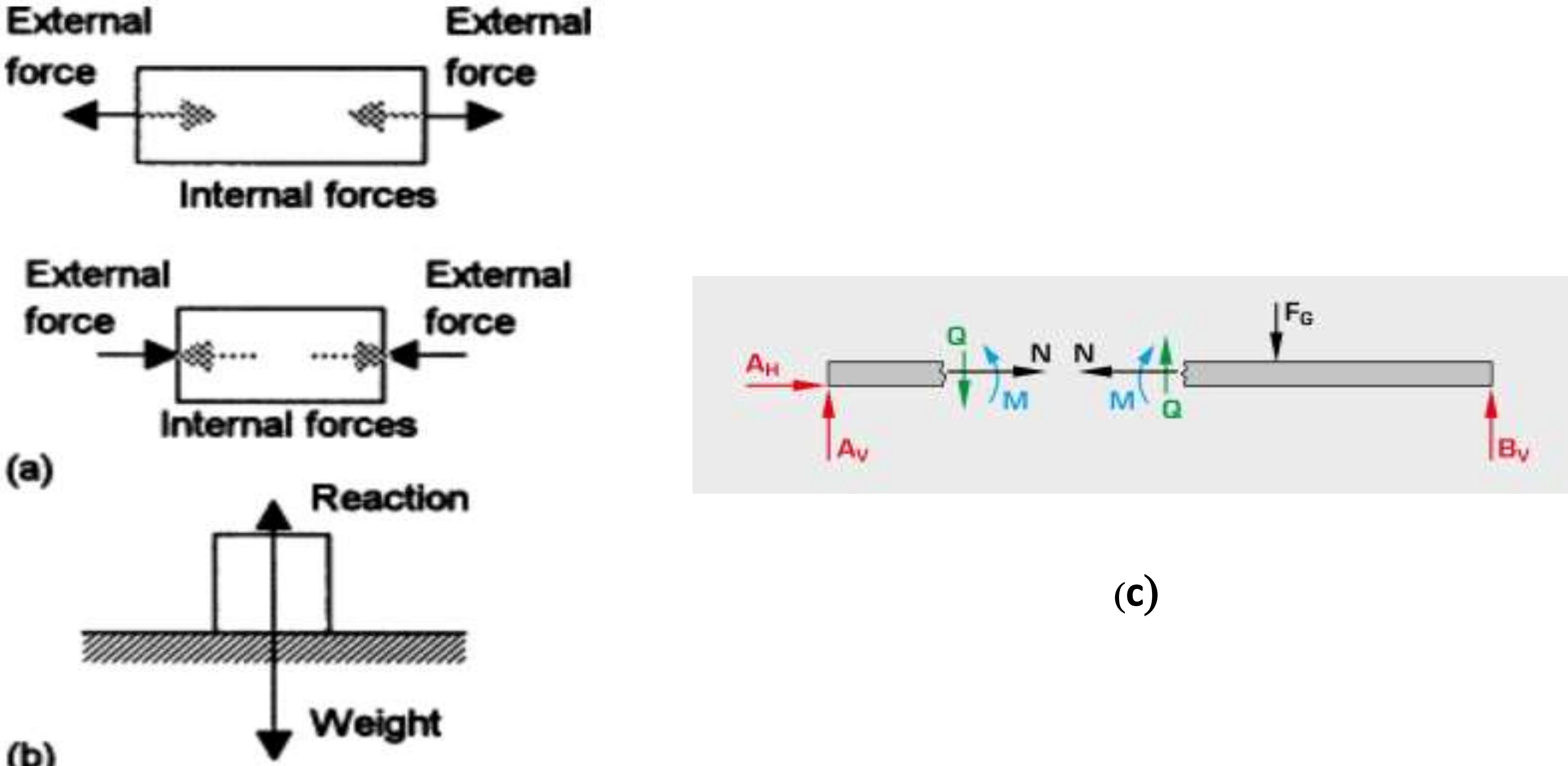

External force
External force
Internal forces
External force
External force
Internal forces
(a)
Reaction
Weight
(b)
A_H
A_V
Q
M
N
N
M
Q
F_G
B_V
(c)

Fig.11

Types of forces acting on a body

Forces on a system can be applied either by direct physical contact or by remote action. According to Newton′s third law, force applied is always accompanied by reactive force. Both applied force and reactive force may be either ***concentrated*** or ***distributed*** . We can be the following three types of forces acting on a body in equilibrium.

1. **Applied forces** There are the forces of push or pull applied externally to the body. Each force has got a point of contact with the body.
2. **Self –Weight** The weight W (= *mg*) of the body acts vertically downward.
3. **Reactions** These are the forces developed by other bodies in contact with the body under consideration. If the body under equilibrium presses (called *action* force) against another body,it is also pressed (called *reaction* force) by the other body. As per Newton′s third law, the reaction is equal and opposite to the action. The reactions ajust

themselves to keep the body in equilibrium. It is of utmost importance to correctly identifiy the action-reaction pair of forces. The reaction forces are supplied by supplied by ground, walls, rollers, knife-edges, cables pins, or other means. A roller or knife-edges support gives a reaction perpendicular to the member.However, a pin connection can give the reaction at any angle.

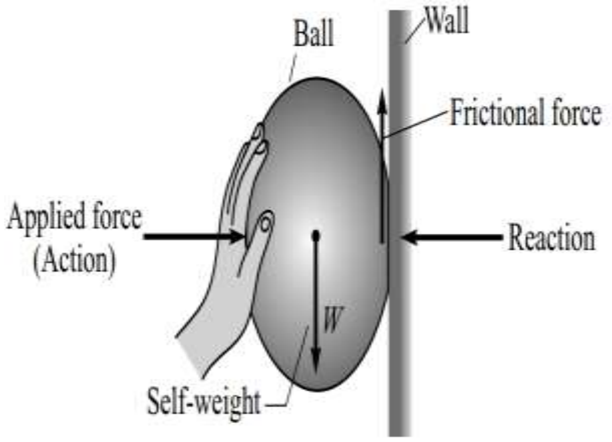


Fig.1. Types of forces acting on a body.

Force Classification

Forces are classified as either *contact* or *body* forces. A contact force is produced by direct physical contact; an example is the force exerted on a body by a supporting surface.
On the other hand, a body force is generated by virtue of the position of a body within a force field such as a gravitational, electric, or magnetic field. An example of a body force is your weight. Forces may be further classified as either ***concentrated or distributed***

Concentrated force/point load

when the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an *area*, as in the case of mechanical contact, over a *volume* when a body force such as weight is acting, or over a *line*, as in the case of the weight of a suspended cable.

Distributed force

Every contact force is actually applied over a finite area and is therefore really a distributed force (a).
Uniformly distributed force. This load is a system of forces having continuous and uniformly distribution along a straight line (b).

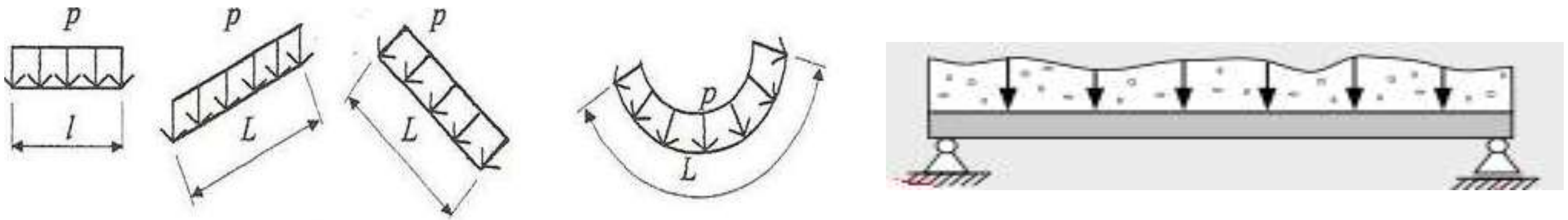
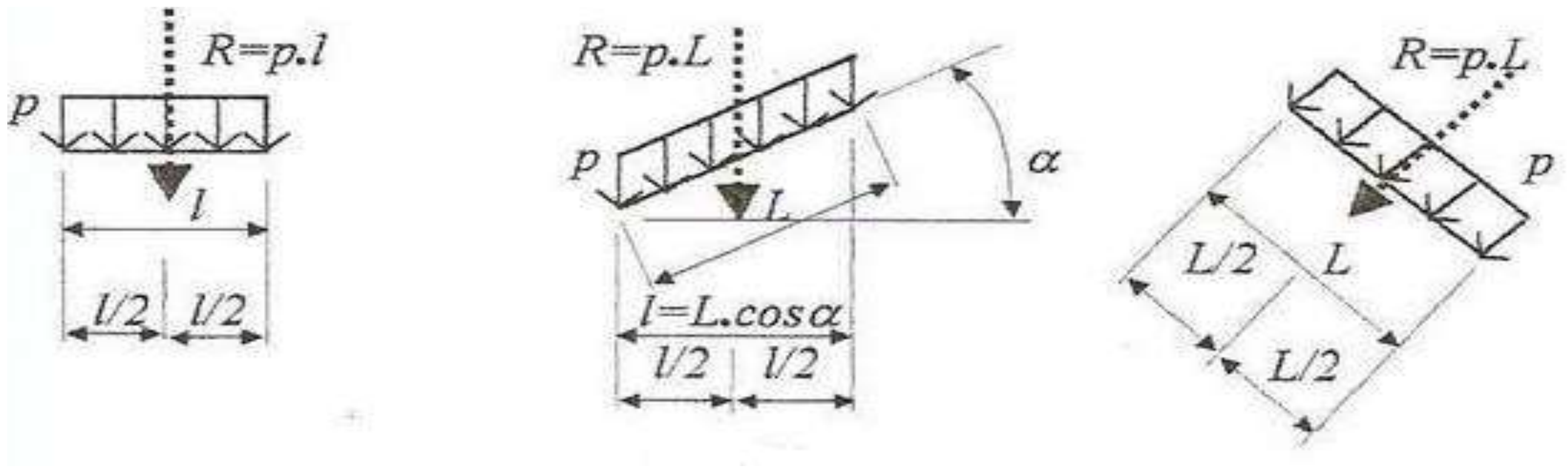


Fig.2. The uniformly distributed force .

Being a system of forces, the uniformly distributed force can be replaced with a resultant force having as magnitude the product between the intensity of it and the length of distribution: $R = p\,.\,l$ or $R = p\,.\,L$

The direction and the sense of the resultant is the same of the components of the system and the position is in the middle of the distribution length.



Physical force or active force:

acts in the normal direction on a body (e.g. weight, wind pressure and snow)

Reaction force

acts in the opposite direction to the physical force and causes the body to remain in equilibrium (e.g. normal force F_N, support force and adhesive force).

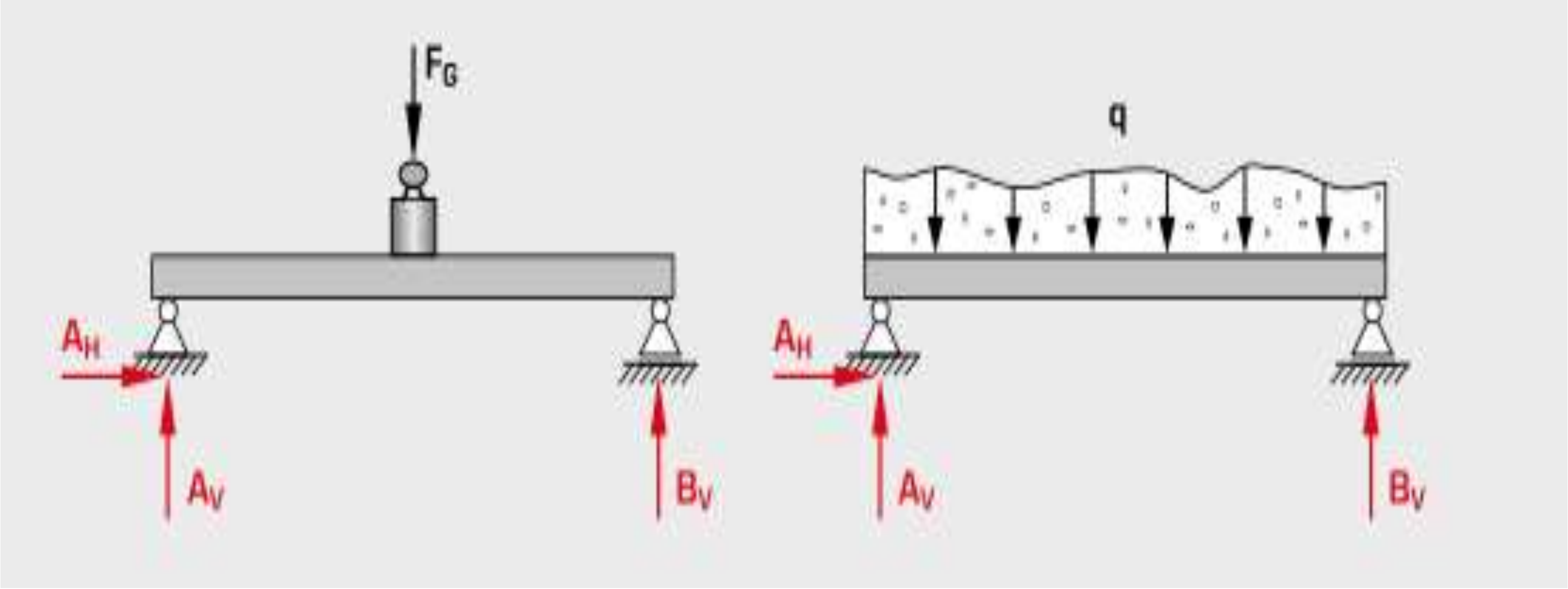


Fig.3. Physical force.

Definition of Moment of a Force about a point

A force acting on a body has two kinds of effects. In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This *rotational* or *turning* effect of the force is known as the *moment* **M** of the force. Moment is also referred to as *torque*.

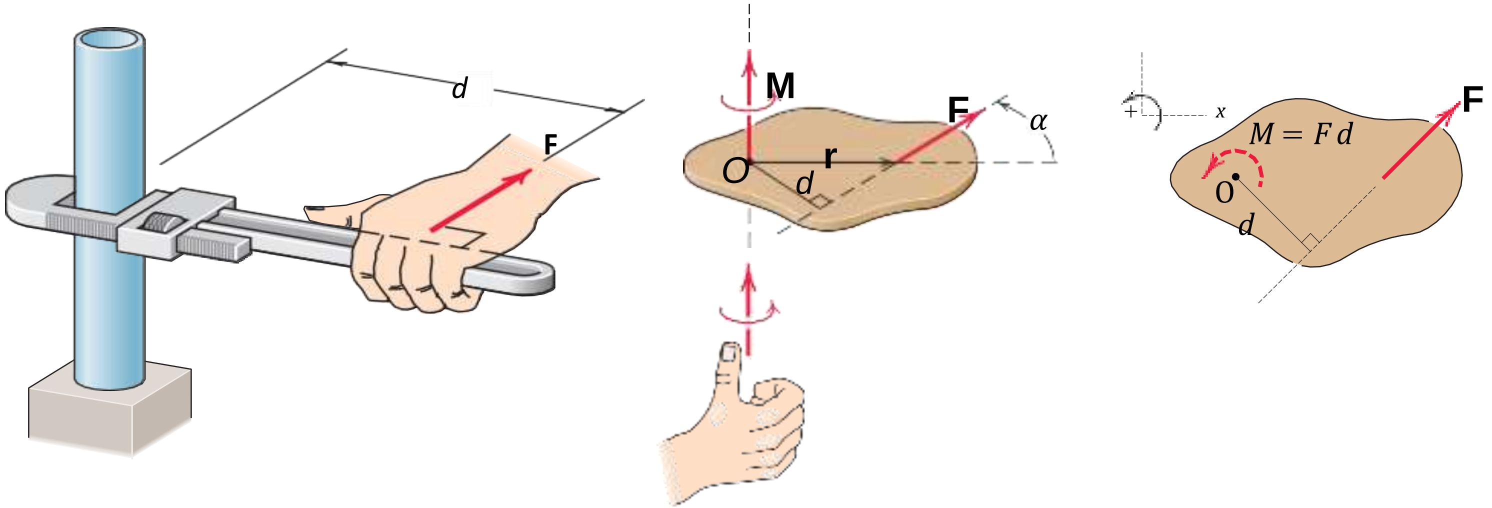
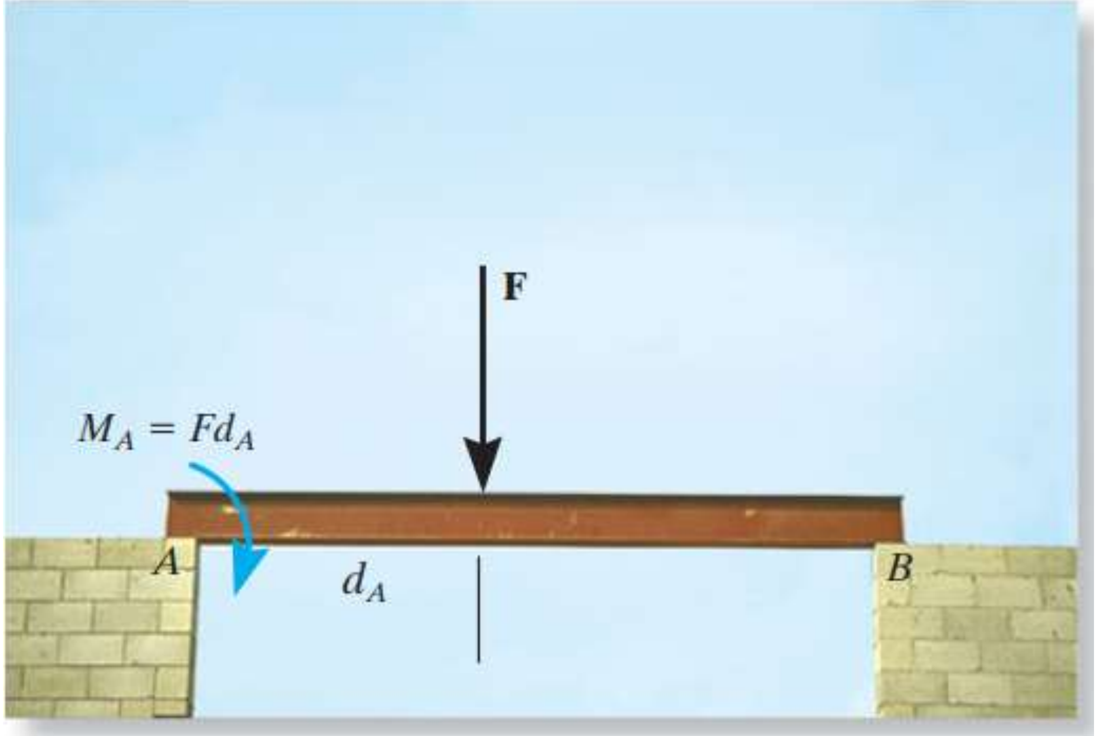
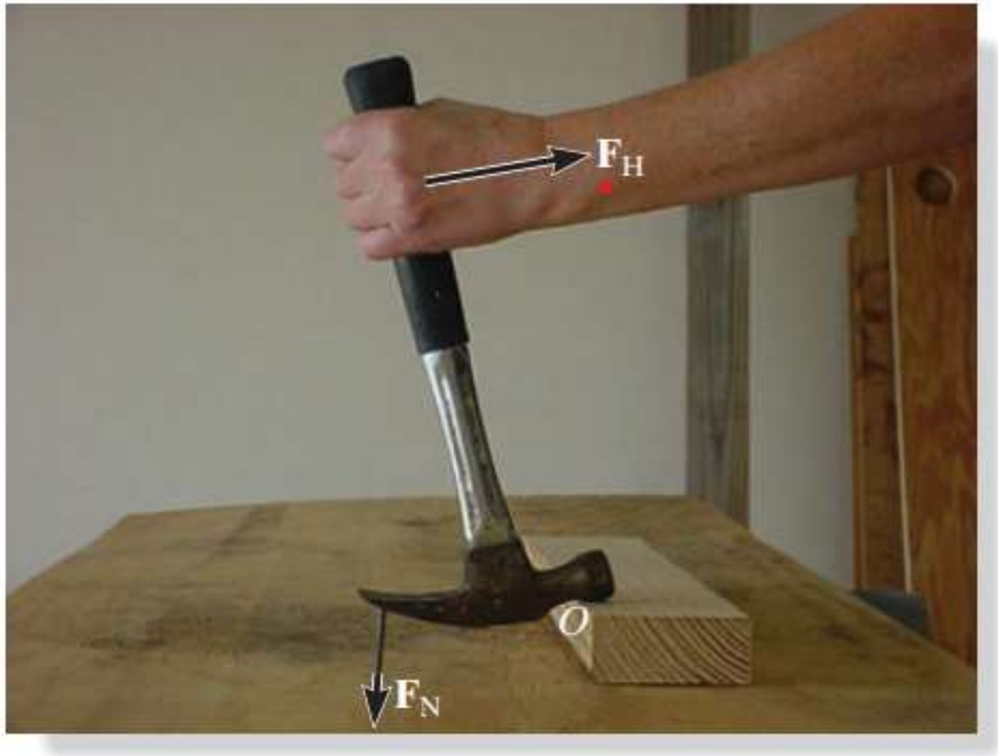


Fig.4. The moment of the force.

Fig.4. shows a two-dimensional body acted on by a force ***F*** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm* ***d***, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as:

$$M_F^{(O)} = \pm Fd$$



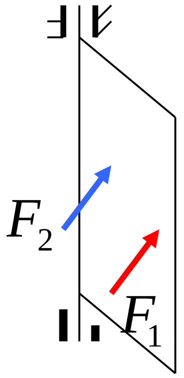


As illustrated by the example problems, the moment of a force does not always cause a rotation. For example, the force **F** tends to rotate the beam clockwise about its support at *A* with a moment The actual rotation would occur if the support at *B* were removed.

The ability to remove the nail will require the moment of about point *O* to be larger than the moment of the force **F**N about *O* that is needed to pull the nail out

Moment of Force

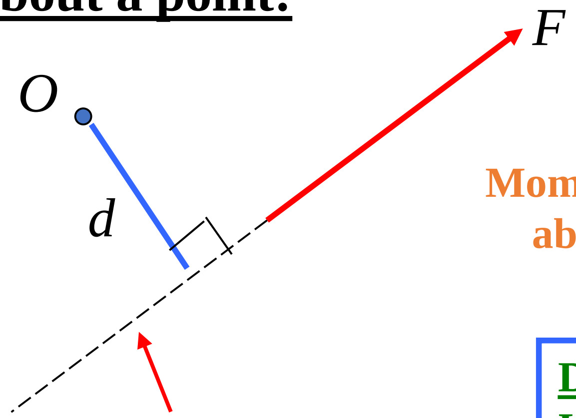
(1) By basic definition



Forces distributed over a rigid body may cause rotation.
Effects of a force on rotation depend not only on magnitude and direction of the force but also on where it applies.

Concept of moment ← **To open the door where is better to apply the force?**

Definition of Moment of a Force about a point:



Line of action of the force

$$M_F^{(O)} = \pm F d$$

Magnitude of the force (F)

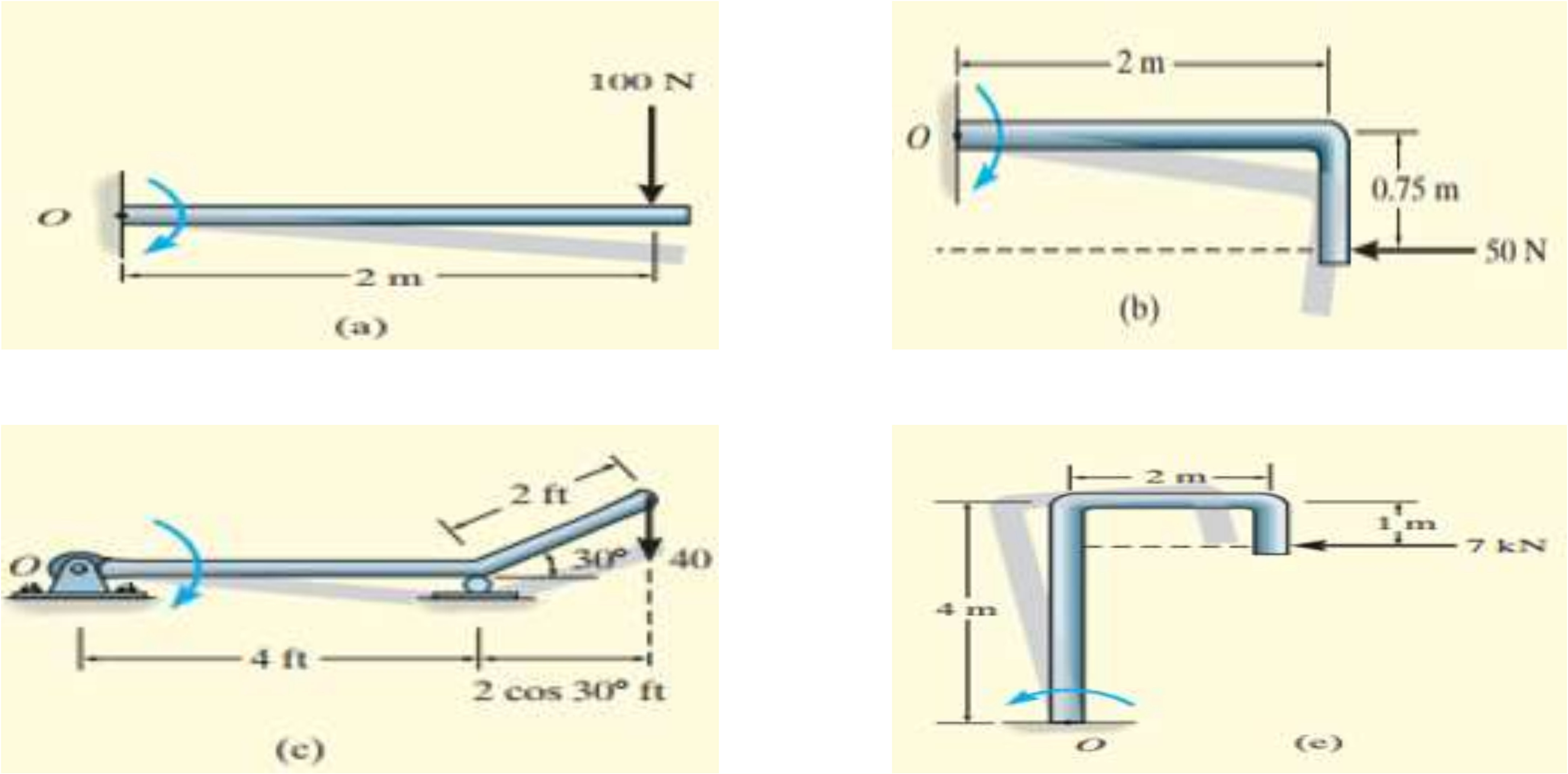
Distance of the force to its line of action (d)

Moment of a force about point O ($M_F^{(O)}$)

Sign convention:
"+" ---- counterclockwise
"-" --- clockwise

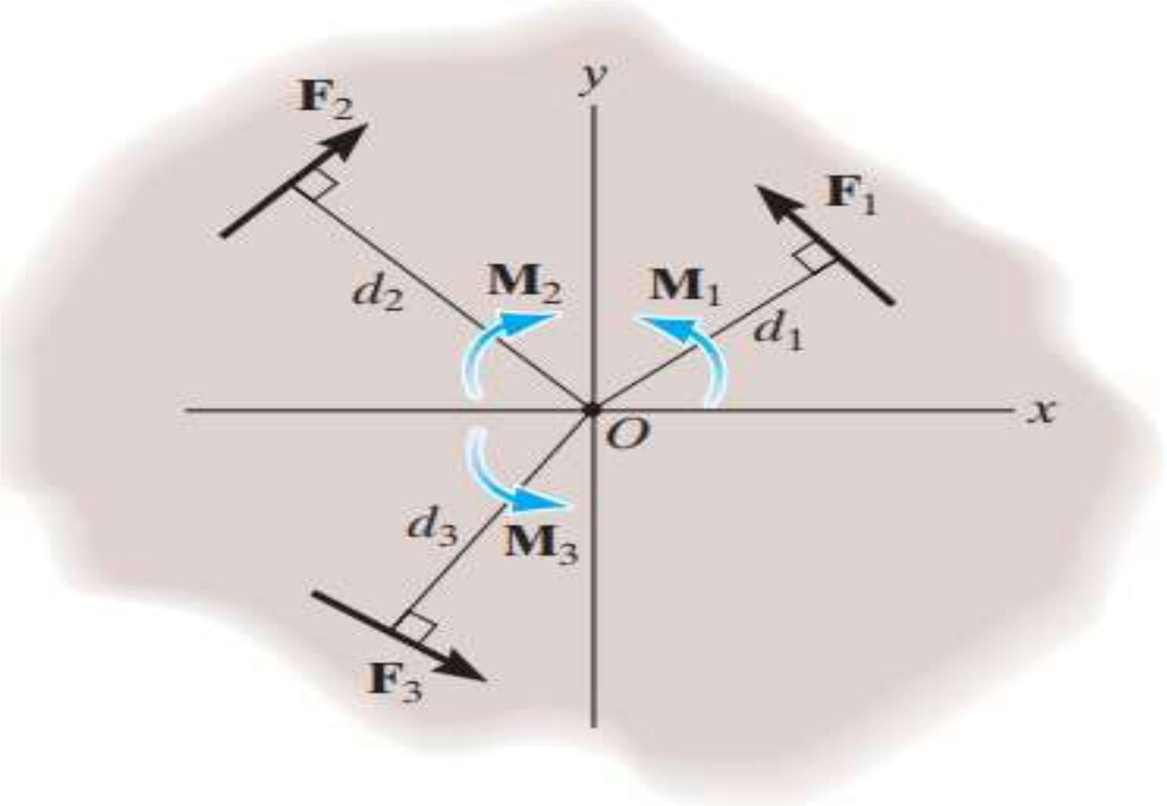
Dimension: [M}=[F}{L]
Unit: N-m; lb-ft

Example.1. Determine the moment of the force about point *O* for each case illustrated in Fig.5.



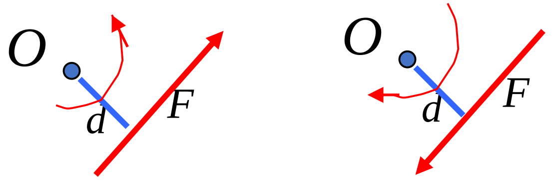
Resultant Moment. For two-dimensional problems, where all the forces lie within the *x*–*y* plane. Using this sign convention, the resultant moment in Fig. is therefore

$$\circlearrowleft + (M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$



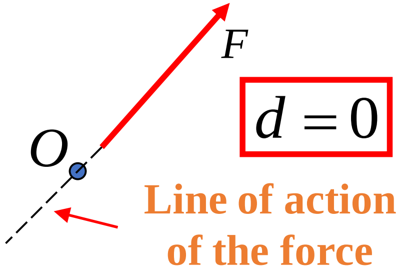
General Comments:

Sign convention:
"+"---- counterclockwise
"-" --- clockwise



$$M_F^{(O)} = +Fd \qquad M_F^{(O)} = -Fd$$

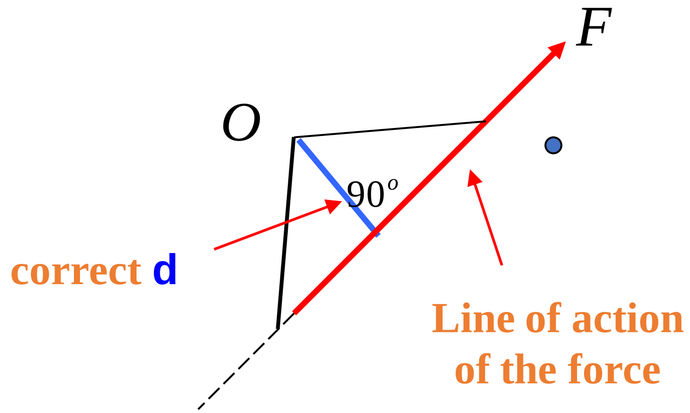
Moment of a force about a point is zero if the point is on the line of action of the force.

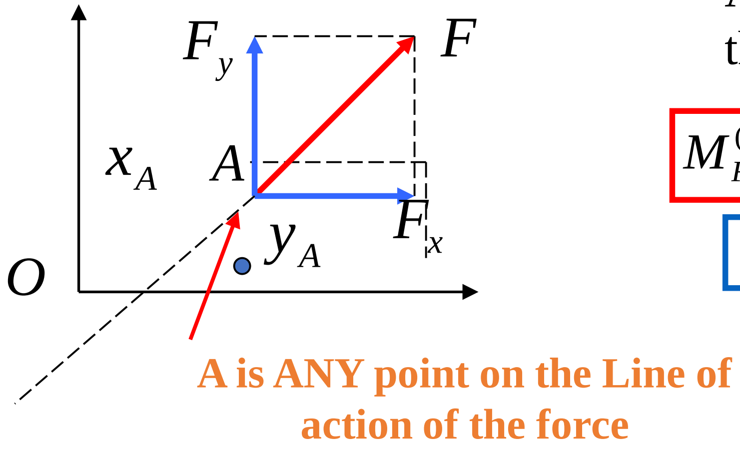


d must be the perpendicular distance to the line of action of the force

(2) By Cartesian components

This method is generally easier than finding the same moment using Mo = F d.





Alternative formula based on the additive property of moment

$$M_F^{(O)} = F_y x_A - F_x y_A$$

Equivalence?

(3) By the cross-product definition

In some two-dimensional and many of the three-dimensional problemsto follow, it is convenient to use a vector approach for moment calculations. The moment of ***F*** about point ***O*** of Fig.6. may be represented by the cross-product expression

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Magnitude. where r is a position vector which runs from the moment reference point ***O*** to any point on the line of action of ***F***. The magnitude of this expression is given by defined from.

$$\mathbf{M}_O = F\,(r\ \sin\theta) = F.d$$

Direction.

The direction and sense are determined by the right-hand rule as it applies to the cross product.

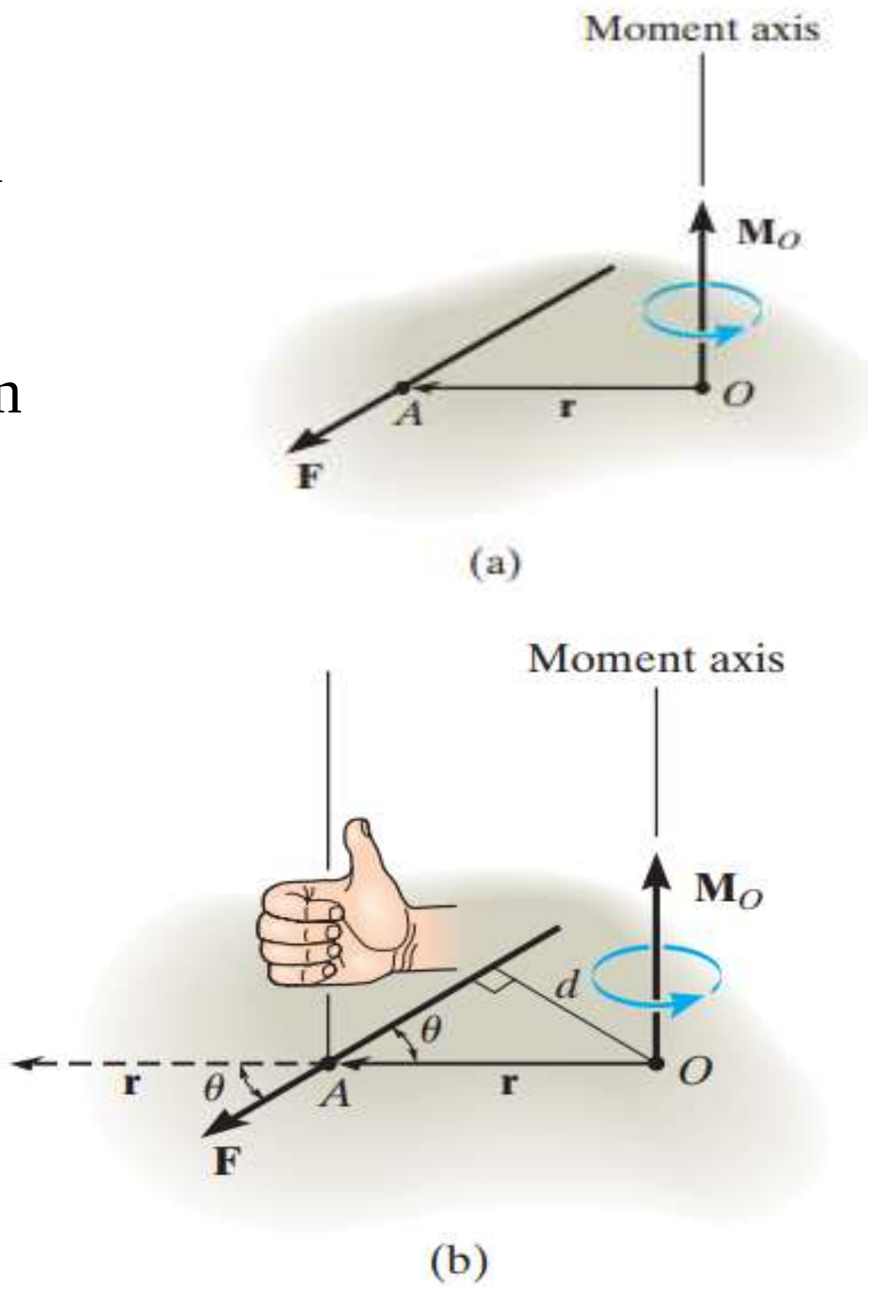
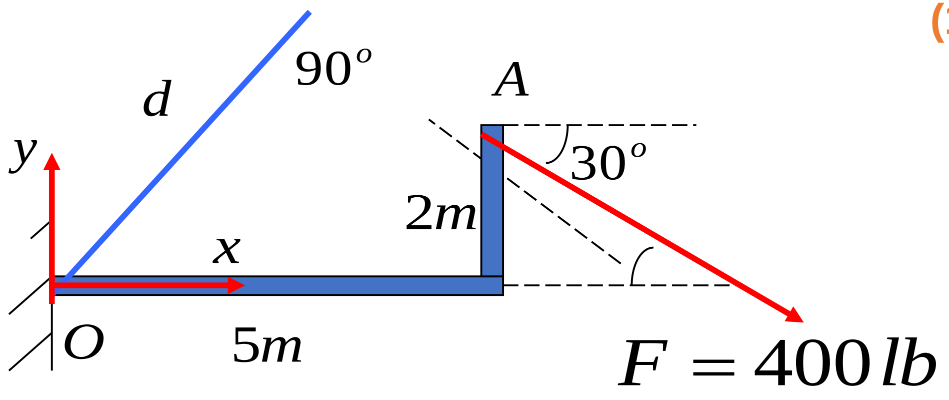


Fig.6. The moment axis.

Example.2. Find the moment of the force shown about point O.



(1) By basic definition

$$d = (5 + 2/\tan 30^o)\sin 30^o$$
$$= 4.23 \ \text{(in m)}$$
$$\Rightarrow M_F^{(O)} = -Fd$$
$$= -(400)(4.23) = -1.69 \ (\text{in } kN - m)$$

Clockwise

(2) By Cartesian compopnents

$$\vec{F} = 400(\cos 30^o \vec{i} - \sin 30^o \vec{j})$$
$$\vec{r}_A = 5\vec{i} + 2\vec{j}$$
$$M_F^{(O)} = F_y x_A - F_x y_A$$
$$= (-400\sin 30^o)(5) - (400\cos 30^o)(2)$$
$$= -(1000 + 400\sqrt{3})$$
$$= -1690 \ \text{(in N - m)}$$

(3) By the cross-product definition

$$\vec{F} = 400(\cos 30^o \vec{i} - \sin 30^o \vec{j}) \qquad \vec{r}_A = 5\vec{i} + 2\vec{j}$$
$$\vec{M}_F^{(O)} = \vec{r}_A \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 400\cos 30^o & -400\sin 30^o & 0 \\ 5 & 2 & 0 \end{vmatrix}$$
$$= [(-400\sin 30^o)(5) - (400\cos 30^o)(2)]\vec{k}$$
$$= -1690\,\vec{k} \ \text{(in N - m)}$$

Moment of a Couple

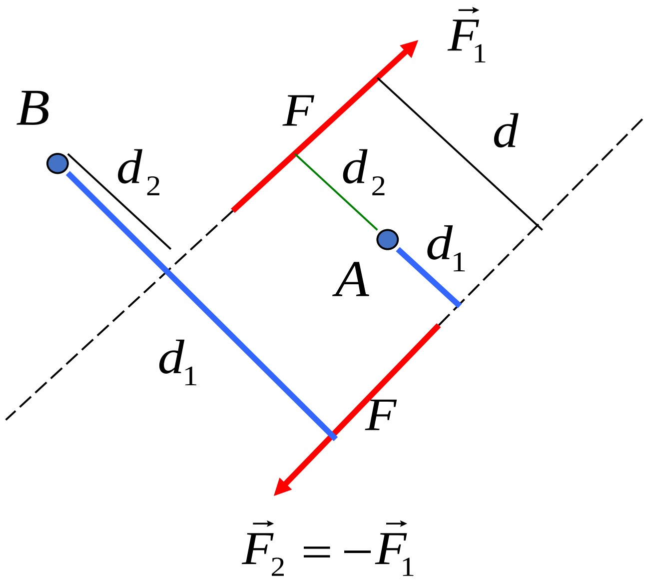
The moment produced by two equal, opposite, and noncollinear forces is called a couple. Couples have certain unique properties and have important applications in mechanics.

Couple ----- a pair of forces with the same magnitude and opposite direction

Moments of a Couple:

$$M = \pm Fd$$

- Distance between lines of action of the couple of force
- Moment of the couple
- Sign convention
- Magnitude of force



$$M^{(A)} = -Fd_1 - Fd_2 = -F(d_1 + d_2) = -Fd$$

$$M^{(B)} = -Fd_1 + Fd_2 = -F(d_1 - d_2) = -Fd$$

Moment of a couple about ANY point is the same!!

Moments in Three Dimensions

Cartesian Vector Formulation. If we establish *x*, *y*, *z* coordinate axes, then the position vector **r** and force **F** can be expressed as Cartesian vectors, Fig.7.*a*. Applying Eq. we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where r_x, r_y, r_z represent the *x*, *y*, *z* components of the position vector drawn from point *O* to *any point* on the line of action of the force rep F_x, F_y, F_z, *y*, *z* components of the force vector. If the determinant is expanded, we have

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} - (r_x F_z - r_z F_x)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

The scalar magnitudes of the moments of these forces about the positive *x*-, *y*-, and *z*-axes through *O* can be obtained from the moment arm rule, and are

$$M_x = r_y F_z - r_z F_y \qquad M_y = r_z F_x - r_x F_z \qquad M_z = r_x F_y - r_y F_x$$

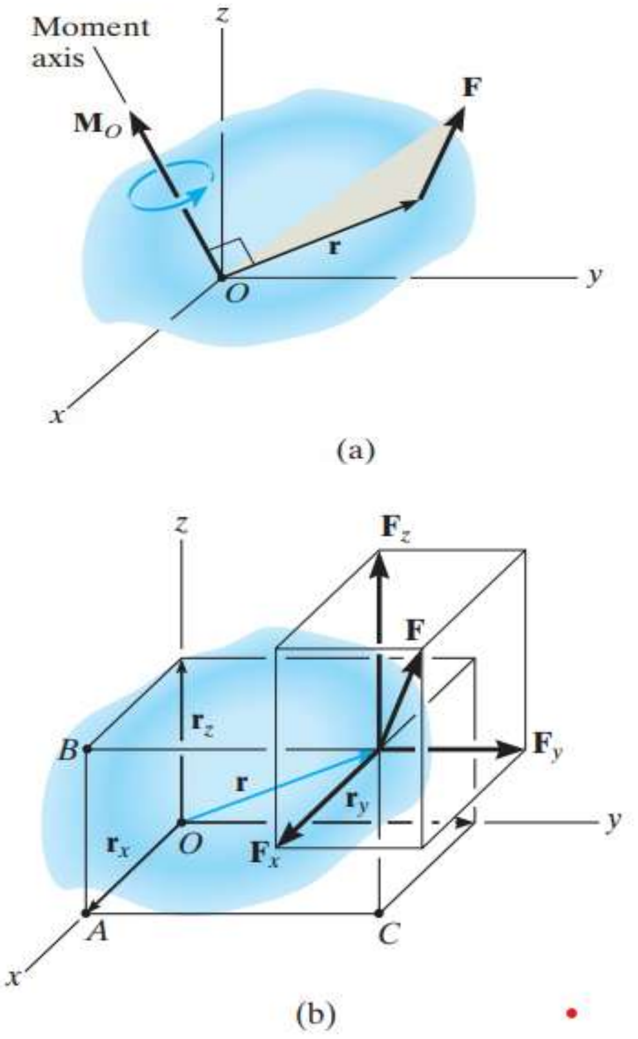
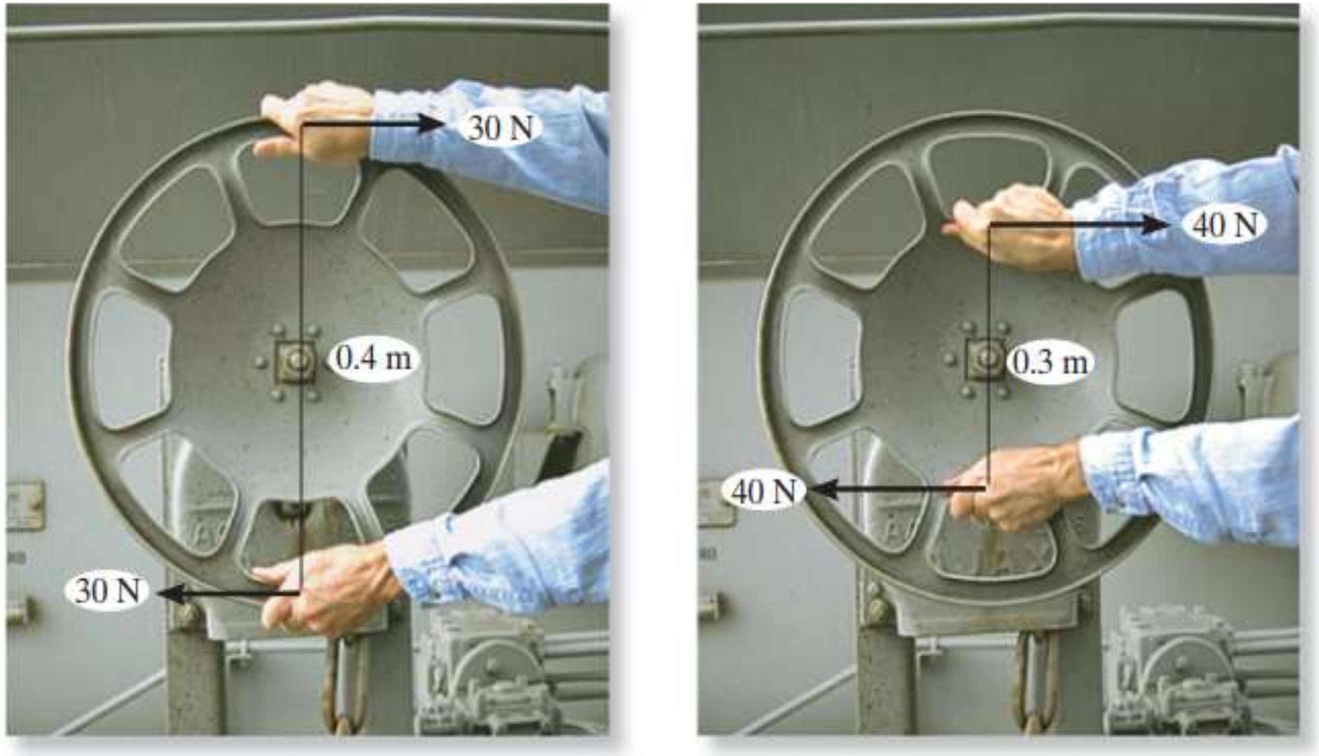


Fig.7. The moment in three dimensions.

Equivalent Couples. If two couples produce a moment with the *same magnitude and direction*, then these two couples are *equivalent*. For example, the two couples shown in Fig. are *equivalent* because each couple moment has a magnitude of
M = 30 N(0.4 m) = 40 N(0.3 m) = 12 N.m.



Example.3. Replace the two couples acting on the pipe column in Fig. *a* by a resultant couple moment.

Solution (vector analysis)

The couple moment developed by the forces at *A* and *B*, can easily be determined from **a scalar formulation**

$M_1 = F.d = 150\ N\ (0.4\ m) = 60\ N.m$

By the right-hand rule, acts in the direction +**i** , Fig. *b*. Hence, $\mathbf{M}_1 = 60\mathbf{i}$ N.m

Vector analysis will be used to determine $\mathbf{M}_2$, caused by forces at *C* and *D*. If moments are computed about point *D*, Fig.b,

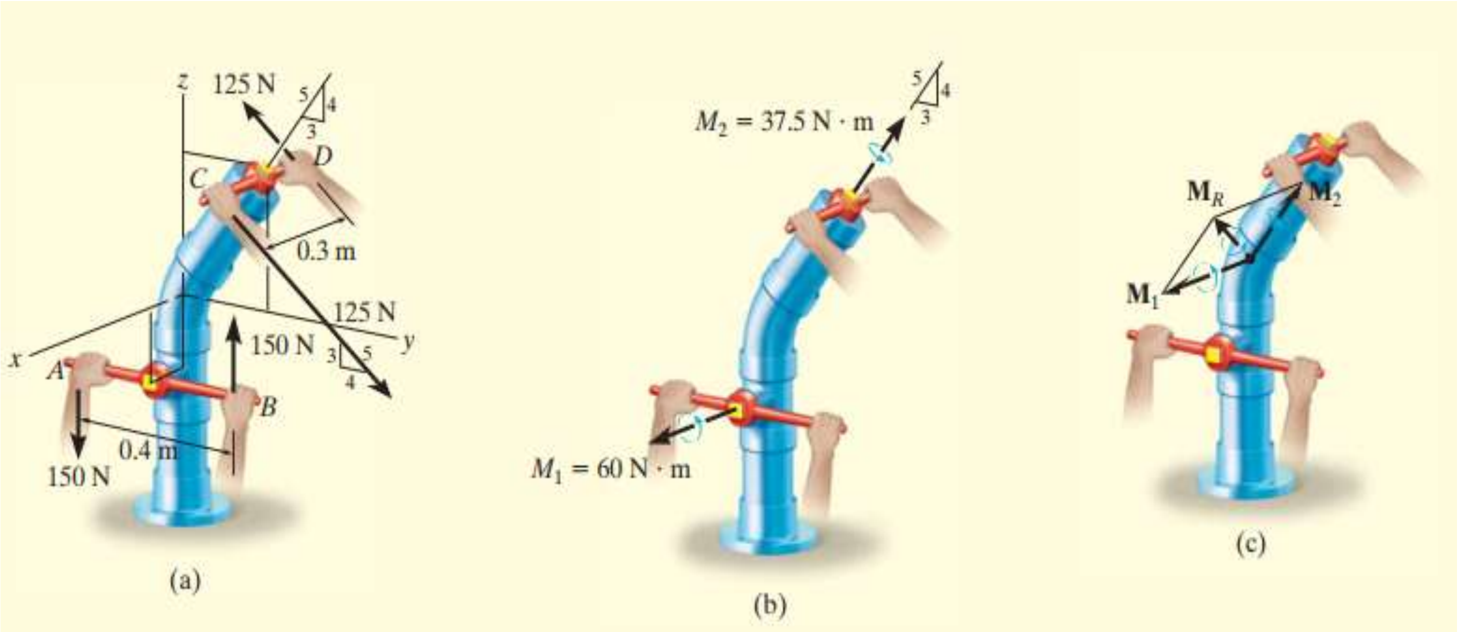
$\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C$ then $\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C = 0.3\mathbf{i} \times [125\ (4/5)\ \mathbf{j} - 125\ (3/5)\ \mathbf{k}$

$= 0.3\mathbf{i} \times [100\mathbf{j} - 75\mathbf{k}] = 30\ (\mathbf{i} \times \mathbf{j}) - 22.5\ (\mathbf{i} \times \mathbf{k})$

$= \{-22.5\ \mathbf{j} + 30\ \mathbf{k}\}$ N.m

Since and are free vectors, they may be moved to some arbitrary point and added vectorially, Fig. *c*. The resultant couple moment becomes

$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = 60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k}$ N.m



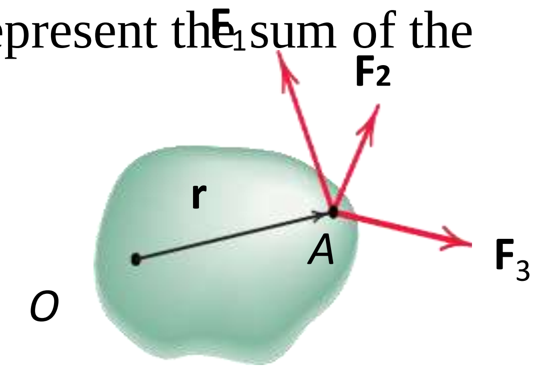
Varignon's Theorem in Three Dimensions

The theorem is easily extended to three dimensions. Figure 8 shows a system of concurrent forces F_1, F_2, F_3, ... The sum of the moments about O of these forces is

$$\mathbf{r} \times \mathbf{F_1} + \mathbf{r} \times \mathbf{F_2} + \mathbf{r} \times \mathbf{F_3} + \cdot\cdot\cdot = \mathbf{r} \times (\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + \cdot\cdot\cdot) = \mathbf{r} \times \Sigma\mathbf{F}$$

where we have used the distributive law for cross products. Using the symbol $\boldsymbol{M_O}$ to represent the sum of the moments on the left side of the above equation, we have

$$\mathbf{M_O} = \Sigma\,(\mathbf{r} \times \mathbf{F}) = \mathbf{r} \times \mathbf{R}$$



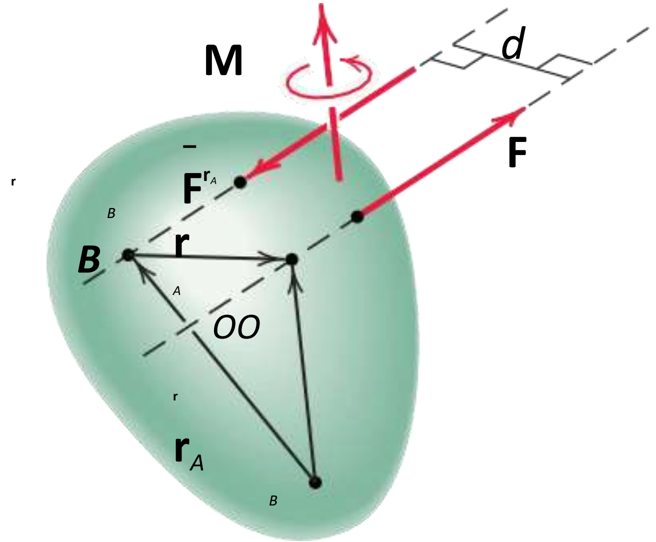
Couples in Three Dimensions

The concept of the couple is easily extended to three dimensions. Figure 2 shows two equal and opposite forces ***F*** and ***−F*** acting on a body. The vector r runs from any point ***B*** on the line of action of ***−F*** to any point ***A*** on the line of action of ***F***. Points ***A*** and ***B*** are located by position vectors r_A and r_B from any point ***O***. The combined moment of the two forces about ***O*** is

$$\mathbf{M} = \mathbf{r_A} \times \mathbf{F} + \mathbf{r_B} \times (-\mathbf{F}) = (\mathbf{r_A} - \mathbf{r_B}) \times \mathbf{F}$$

However, $\mathbf{r_A} - \mathbf{r_B} = \mathbf{r}$, so that all reference to the moment center ***O*** disappears, and the moment of the couple becomes

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



Resultant Moment of a System of Forces.

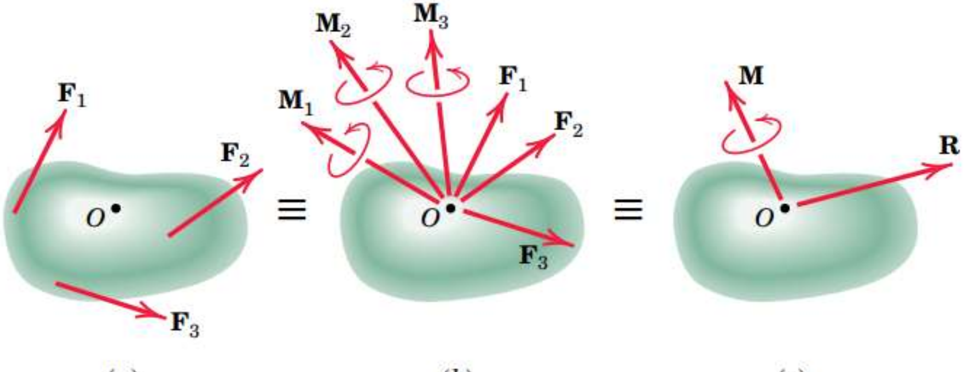
The system of forces **F**1, **F**2, **F**3 . . . acting on a rigid body in **Fig. *a***, we may move each of them in turn to the arbitrary point *O*, provided we also introduce a couple for each force transferred. Thus, for example, we may move force **F**1 to *O*, provided we introduce the couple $\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1$, where $\mathbf{r}_1$ is a vector from *O* to any point on the line of action of $\mathbf{F}_1$. When all forces are shifted to *O* in this manner, we have a system of concurrent forces at *O* and a system of couple vectors, as represented in part *b* of the figure. The concurrent forces may then be added vectorially to produce a resultant force **R**, and the couples may also be added to produce a resultant couple **M**, **Fig.*c***. The general force system, then, is reduced to

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$

One of the two Golden Jubilee Bridges in London, England, adjacent to the Hungerford Bridge. The cables of this bridge exert a three-dimensional system of concentrated forces on each bridge tower.

The couple vectors are shown through point *O*, but because they are free vectors, they may be represented in any parallel positions. The magnitudes of the resultants and their components are



$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \qquad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \qquad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$