Chapter 1

Mathematical review

1.1 Introduction

- Physics is based on observations and experimental quantitative measurements.
  - Observation: the story of Newton's apple.
  - Measurement: measure of falling time of the apple.
- The main objective of physics is to find a limited number of fundamental laws that govern natural phenomena and use them to develop theories that can predict the results of future experiments.
- Physics laws are mathematical equations that relate physical quantities to each other.

1.2 Quantities we call quantity anything that can be measured like mass, density, temperature, electric field, etc.

- In physics, there are seven fundamental quantities.
  1. Length: the distance between two points.
  2. Mass:
     - a number we attach to each particle or body and that it is obtained by comparing the body with a standard body, using the principle of an equal arm balance. Mass, therefore, is a coefficient that distinguishes one particle from another.
     - measures the inertia of body, meaning the resistance to acceleration (change of velocity) when a net force is applied.
  3. Time: the continued sequence of events that occurs in an apparently irreversible succession from the past, through the present, into the future.
  4. Electric current: Movement of charged body.
  5. Thermodynamic temperature: represent the measure of the average total internal energy of an object (measure of its kinetic energy, energy of motion).
  6. Amount of substance: a quantity proportional to the number of elementary entities of a substance.
  7. Luminous intensity: the quantity of visible light that is emitted in unit time per unit solid angle.
- All other quantities in physics can be expressed in terms of these basic quantities.

Note: In physics, there are two type of quantities:

- **Scalar quantities** refers to number like length.
- **Vector quantities** refers to the magnitude and direction like force.

1.3 Dimension: The dimension of a quantity is simply its physical nature. A quantity can have a dimensions of mass, speed, energy, etc.

1.4 Unities: The same length can be quantified differently from one country to another. For example, a distance between two points can be given in fingers, feet, yards, miles, and meters..., it is, therefore, necessary to define a universal unit so that the quantities have meaning for everyone.

In 1960, an international committee established standard units for the basic quantities. The system is called System of Units (SI). The abbreviation (IS) is refers to "International System").

Dimension	Symbol	Unity
Length	L	Meter (m)
Mass	M	Kilogram (Kg)
Time	T	Second (s)
Electric current	A	Ampere (A)
Thermodynamic temperature	Ɵ	Kelvin (k)
Amount of substance	n	Mole (mol)
Luminous intensity	J	Candela (cad)

If X is a given quantity, the dimension of X will be denoted by [X]. for example, if X represent volume, so $[V] = [L] \times [L] \times [L] = [L]^3$.

1.5 Dimensional analysis

As we said before, any quantity can be expressed in terms of the seven basic quantities. The dimension of any quantity X can be written by:

$$[X] = [L]^{\alpha}.[M]^{\beta}.[T]^{\gamma}.[A]^{\varepsilon}.[\Theta]^{\mu}.[N]^{\chi}.[J]^{\omega} = L^{\alpha}.M^{\beta}.T^{\gamma}.A^{\varepsilon}.\Theta^{\mu}.N^{\chi}.J^{\omega}$$

The two member the above equation must have the same dimension, so we can find the values of α, β, γ, ε, μ, , and ω.

Examples :

- The time equation for uniform rectilinear motion is $x = v.t$ where x is distance, v is speed and t is time. So the dimension of velocity will be written as,

$$[Velocity] = \left[\frac{x}{t}\right] = \frac{L}{T} = LT^{-1} \quad (1)$$

- The kinetic energy E of an object of mass m moving at speed v is written as $Ec = \frac{1}{2}mv^2$. Then,

$$[Energy] = [E] = \left[\frac{1}{2}\right].[m].[v]^2 = 1.M.[L.T^{-1}]^2 = ML^2.T^{-2} \quad (2)$$

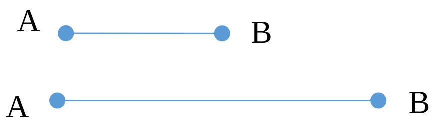
Note: the dimension of any number is equal to 1.

1.6 Vectors

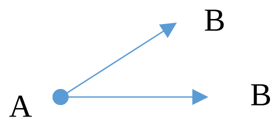
1.6.1 Definition

Suppose that an object moves in a straight line from an initial point A to a final point B. This movement is characterized by two properties;

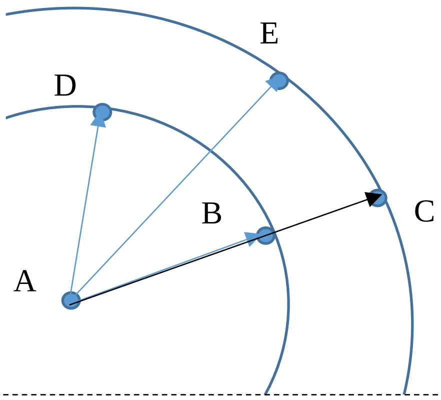
1 - The length of the segment AB and,



2- The direction from A to B.



This quantity is represented by a vector $\overrightarrow{AB}$.



The figure in front illustrates

$\overrightarrow{AB} \neq \overrightarrow{AC}$: they have the same direction but do not have the same module

$\overrightarrow{AB} \neq \overrightarrow{AD}$: they have the same module but do not have the same direction

$\overrightarrow{AB} \neq \overrightarrow{AE}$: they do not have neither the module nor the direction

1.6.2 Unit vector:

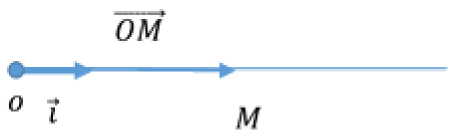
Unit vector only indicates the direction of the vector; its length is equal to one unity. Generally, the unit vector is represented by $\vec{u}$. Then, we can write $\overrightarrow{AB} = \|AB\|\vec{u}$.

$$\vec{u} = \frac{\overrightarrow{AB}}{\|AB\|},$$

$$\|\vec{u}\| = 1$$

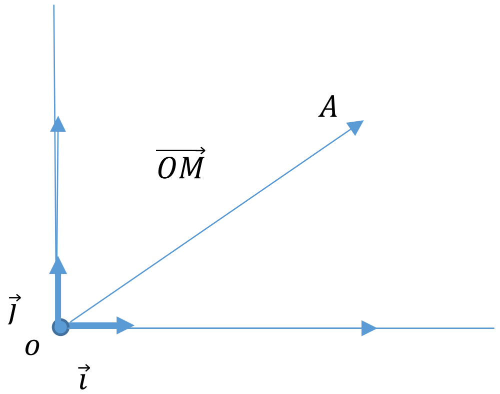
1.6.3 Graphical Representation of a vector in the Cartesian Coordinate (System $(\vec{\imath}, \vec{\jmath}, \vec{k})$)

(1) one dimension



$$\overrightarrow{OM} = \|OM\| \times \vec{\imath}$$

(2) two dimensions



Vector in plane can be written in terms of two independent linear vectors

$$\overrightarrow{OM} = \overrightarrow{A_x} + \overrightarrow{A_y}$$

$$\overrightarrow{OM} = \|A_x\| \times \vec{\imath} + \|A_y\| \times \vec{\jmath}$$

$$\overrightarrow{OM} = \quad x\vec{\imath} + y\vec{\jmath}$$

(3) three dimensions

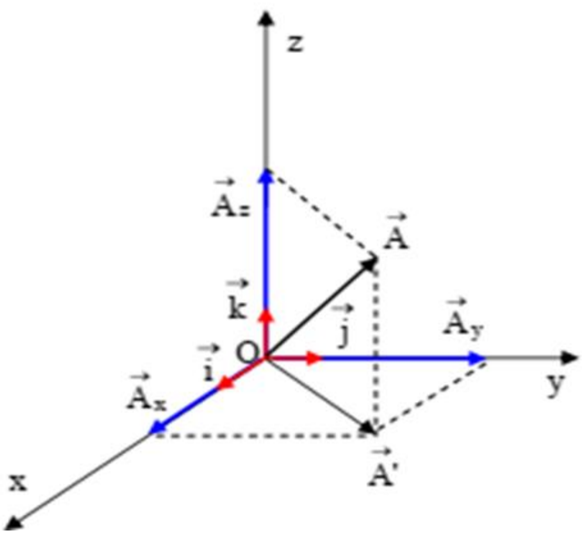
Vector in space can be written in three non-parallel vectors, two by two, and not all of them in the same plane.

$$\overrightarrow{OM} = \overrightarrow{A_x} + \overrightarrow{A_y} + \overrightarrow{A_z}$$

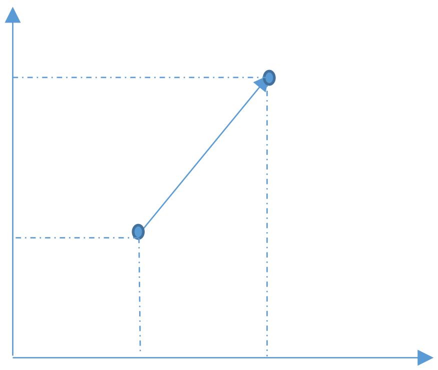
$$\overrightarrow{OM} = \|A_x\| \times \vec{\imath} + \|A_y\| \times \vec{\jmath} + \|A_z\| \times \vec{k}$$

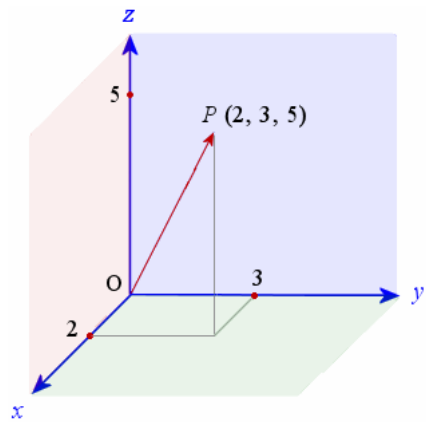
Usually the $\overrightarrow{OM}$ will be written as,

$$\overrightarrow{OM} = \quad x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

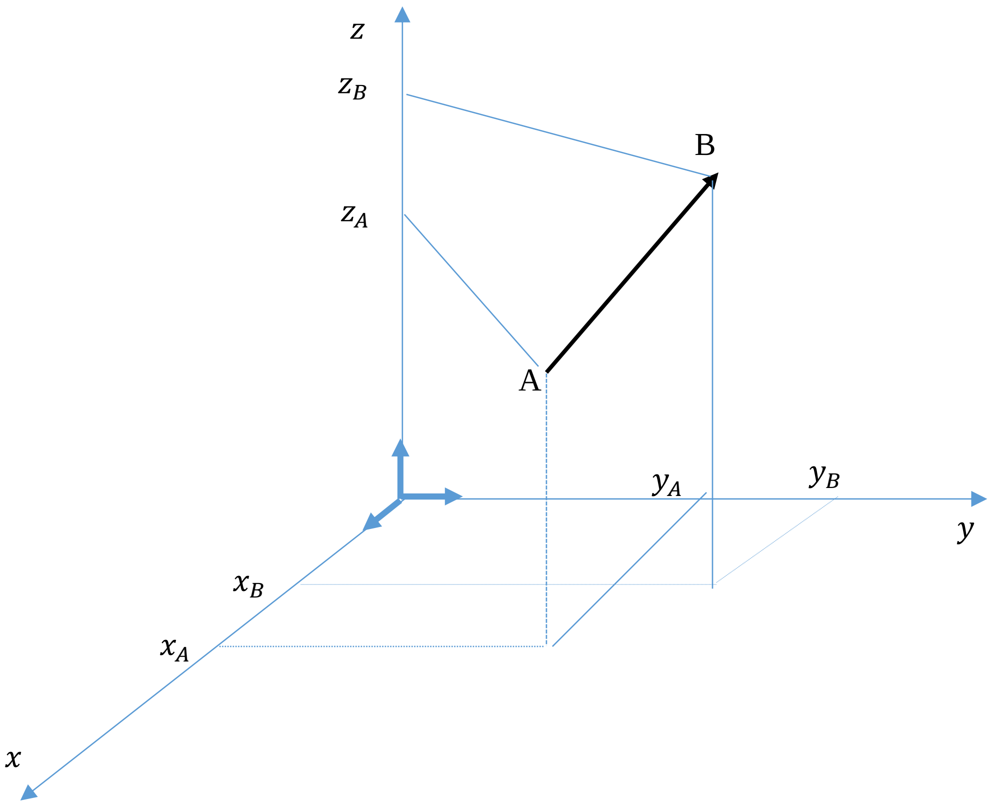


1.6.4 Representation of points in two and three dimensions





In space a vector can be defined by the starting points A$(x_{A,} y_{A,}, z_A)$ and the ending B$(x_{B,} y_B, z_B)$, then $\overrightarrow{AB} = (x_B - x_{A)}\vec{i} + (y_B - y_{A)}\vec{j} + (z_B - z_{A)}\vec{k}$



1.3.5 Operations on vectors

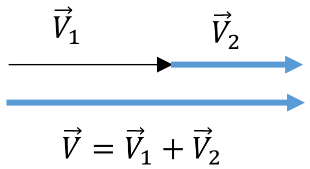
a. Addition:

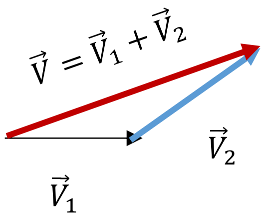
Consider that we have two vectors $\vec{V}_1, and\ \vec{V}_2$

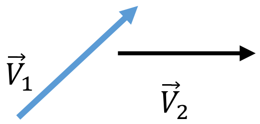
$$\vec{V}_1 = \quad x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k},$$

$$\vec{V}_2 = \quad x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k},$$

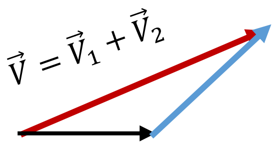
The sum of the two vector is $\vec{V} = \vec{V}_1 + \vec{V}_2 = \quad (x_1 + x_2)\vec{\imath} + (y_1 + y_2)\vec{\jmath} + (z_1 + z_2)\vec{k}$





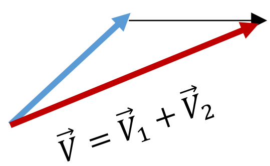


Move v_1 until the start of v_1 coincides with the end of v_2



Or

Move v_2 until the start of v_2 coincides with the end of v_1



b. Some properties

- $\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$
- $\vec{V}_1 + (\vec{V}_2 + \vec{V}_3) = \overline{(\vec{V}_1 + \vec{V}_2)} + \vec{V}_3$
- $\alpha\vec{V}_1 = \vec{V}_2$, $\vec{V}_2$ has the same direction and magnitude $\|\vec{V}_2\| = \alpha\|\vec{V}_1\|$
- $\alpha(\vec{V}_1 + \vec{V}_2) = \alpha\vec{V}_1 + \alpha\vec{V}_2$

c. Subtraction:

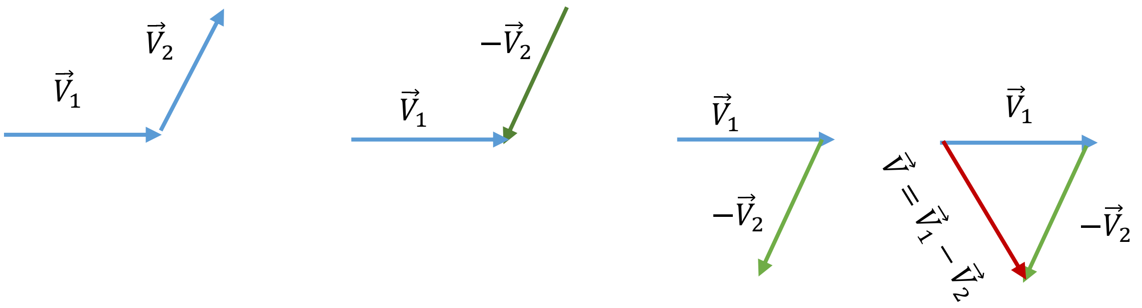
Consider that we have two vectors $\vec{V}_1, and\ \vec{V}_2$

$$\vec{V}_1 = \quad x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k},$$

$$\vec{V}_2 = \quad x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k},$$

$$\vec{V} = \vec{V}_1 - \vec{V}_2 = (x_1 - x_2)\vec{\imath} + (y_1 - y_2)\vec{\jmath} + (z_1 - z_2)\vec{k}$$

Subtraction is the same operation as addition, except that the subtracted vector must be returned to its negative direction.



$$\vec{V} = \vec{V}_1 - \vec{V}_2 \rightarrow \vec{V} = \vec{V}_1 + (-\vec{V}_2)$$

d. Product

There are two types of vectors product;

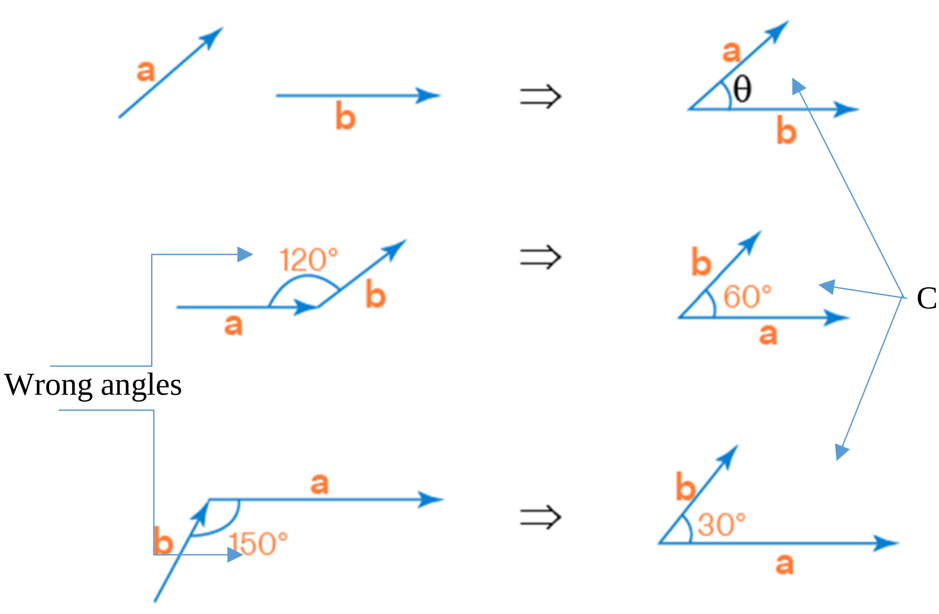
1. **Scalar product (Dot product)**

The result of scalar product of two vectors is a scalar quantity (number), and is defined by this formula,

$$\vec{V}_1 \cdot \vec{V}_2 = \|\vec{V}_1\| \cdot \|\vec{V}_2\| cos(\alpha)$$

$$\vec{V}_1 \cdot \vec{V}_2 = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_1$$

α is the **angle between the two vectors**, which is defined as the angle formed at the intersection of their tails (of their beginnings). If it is not the case, we have to connect them tail to tail to find the angle between them.



Some Properties

- $\vec{V}_1 \cdot \vec{V}_2 = \vec{V}_2 \cdot \vec{V}_1$
- $\vec{V}_1 \cdot (\vec{V}_2 + \vec{V}_3) = \vec{V}_1 \cdot \vec{V}_2 + \vec{V}_1 \cdot \vec{V}_3$
- $\alpha(\overrightarrow{V_1} \cdot \vec{V}_2) = (\alpha\vec{V}_1) \cdot \vec{V}_2 = \vec{V}_1 \cdot (\alpha\vec{V}_2)$
- $\vec{V}_1 \cdot \vec{V}_2 = 0$ only if one of the two is equal to zero or the two vectors are perpendicular.

2. Cross product

The result of the cross product of two vectors ($\vec{V}_1$ *and* $\vec{V}_2$) is a third vector $\vec{V}_3$ that is perpendicular to both of them.

$$\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2$$

The magnitude of the resulting vector is given by

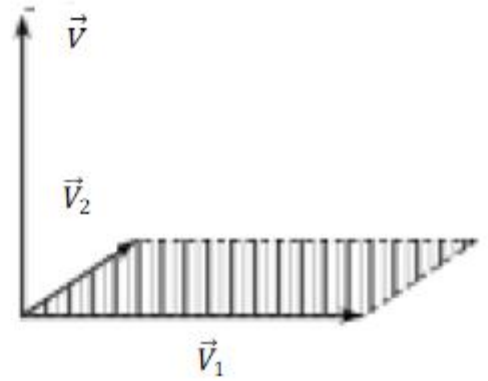
$$\|\vec{V}_3\| = \|\vec{V}_1\| \,.\, \|\vec{V}_2\| \sin(\alpha)$$

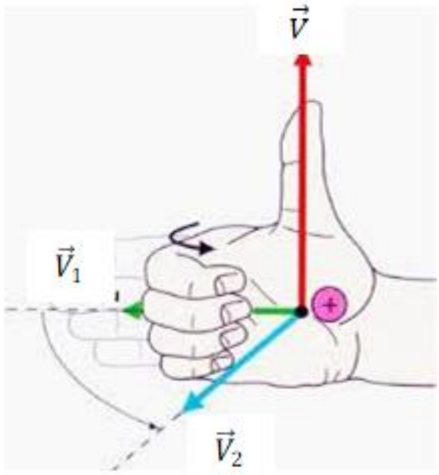
➢ The module of $\vec{V}_3$ represents the surface constructed by the two vectors $\vec{V}_1$ and $\vec{V}_2$.

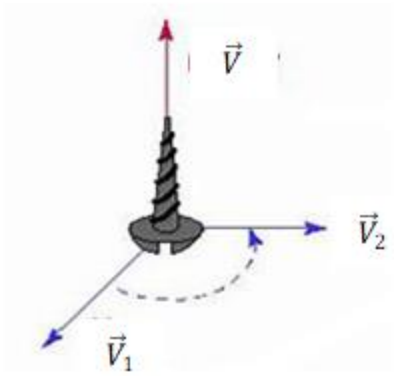
$$S = \|\vec{V}_3\| = \|\vec{V}_1\| . \| \vec{V}_2\| \sin(\alpha)$$

- We conclude the surface of the triangle constructed by these vectors is

$$S = \frac{1}{2}\|\vec{V}_3\| = \frac{1}{2}\|\vec{V}_1\| . \| \vec{V}_2\| \sin(\alpha)$$







The direction of the resulting vector is deduced by the right hand rule the crew rule

Consider that we have two vectors, $\overrightarrow{V_1} = x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k}$, and $\overrightarrow{V_2} = x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k}$ then,

$$\vec{V} = \overrightarrow{V_1} \times \overrightarrow{V_2} = \begin{vmatrix} \vec{\imath} & -\vec{\jmath} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

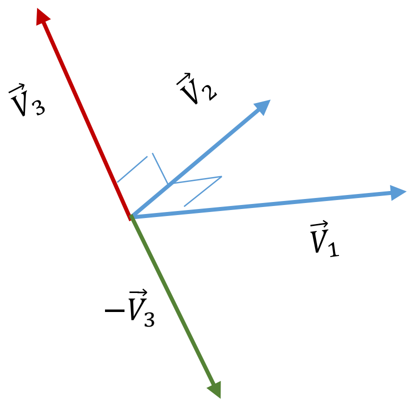
$$\vec{V} = \overrightarrow{V_1} \times \overrightarrow{V_2} = (y_1 \cdot z_2 - z_1 \cdot y_2)\vec{\imath} - (x_1 \cdot z_2 - z_1 \cdot x_2)\vec{\jmath} + (x_1 \cdot y_2 - y_1 \cdot x_2)\vec{k}$$

Some properties

- $\vec{V}_1 \wedge (\vec{V}_2 + \vec{V}_3) = \vec{V}_1 \wedge \vec{V}_2 + \vec{V}_1 \wedge \vec{V}_3$
- $\alpha(\overrightarrow{V_1 \wedge V_2}) = (\alpha\vec{V}_1)\wedge\vec{V}_2 = \vec{V}_1 \wedge (\alpha\vec{V}_2)$

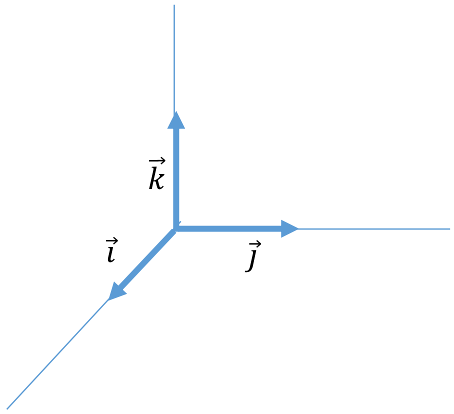
- $\vec{V}_1 \wedge \vec{V}_2 = \vec{0}$ only if one of the two is equal to zero or the two vectors are parallels.

- If $\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2$, then $\vec{V}_2 \wedge \vec{V}_1 = \overline{-\vec{V}_3}$



- In Cartesian Coordinate System $(\vec{\imath}, \vec{\jmath}, \vec{k})$

$$\vec{i} \wedge \vec{j} = \vec{k} \qquad \vec{i} \wedge \vec{k} = -\vec{j}$$
$$\vec{j} \wedge \vec{k} = \vec{i} \qquad \vec{k} \wedge \vec{j} = -\vec{i}$$
$$\vec{k} \wedge \vec{i} = \vec{i} \qquad \vec{j} \wedge \vec{i} = -\vec{k}$$



$\wedge$

d. Mixed product

The mixed product is defined by,

$\vec{V} = \vec{V}_1 \cdot (\vec{V}_2 \wedge \vec{V}_3)$

The results of the mixed product is a scalar quantity and represents the volume of the parallelepiped constructed with $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$.