Abdelhafid Boussouf University Center, Mila
Institute of Mathematics and Computer Sciences
First year of Computer Science License 2024/2025

Algebra I, Worksheet 1

Exercise n°1

I. Express symbolically, using quantifiers, the following assertions :

1. The square of every real number is a positive real number.
2. There is a positive integer whose square is equal to itself.
3. There are two different real numbers that have the same square.
4. Any real number has a cube root.
5. For every real number x, there exists at least one natural number greater than or equal to x.

II. In the library, let S denote the set of subscribers and B denote the set of books. Let P be the assertion : "the subscriber s likes the book b, denoted sLb". Translate each of the symbolic assertions into English sentence.

1. $\forall s \in S, \exists b \in B : sLb$.
2. $\forall b \in B, \exists s \in S : sLb$.
3. $\exists b \in B, \forall s \in S : sLb$.
4. $\exists s \in S, \forall b \in B : sLb$.

Exercise n°2 : Translate each of the following symbolic assertions into English sentences and indicate the truth value of each assertion. Then, write the negations of these assertions :

1. $\forall m \in \mathbb{Z}, \forall n \in \mathbb{Z} : m + n = 0$.
2. $\forall m \in \mathbb{Z}, \exists n \in \mathbb{Z} : m + n = 0$.
3. $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z} : m + n = 0$.
4. $\exists m \in \mathbb{Z}, \exists n \in \mathbb{Z} : m + n = 0$.

Exercise n°3 :

Using a truth table, prove the following logical equivalences for any three assertions P, Q and R.

(a). $P \wedge (Q \vee R) \Longleftrightarrow (P \wedge Q) \vee (P \wedge R)$ (This is the distributive law of conjunction over disjunction).

(b). $P \vee (Q \wedge R) \Longleftrightarrow (P \vee Q) \wedge (P \vee R)$ (This is the distributive law of disjunction over conjunction).

(c). $\overline{(P \wedge Q)} \Longleftrightarrow \overline{P} \vee \overline{Q}$ and $\overline{(P \vee Q)} \Longleftrightarrow \overline{P} \wedge \overline{Q}$ De Morgan's Laws.

Exercise n°4 :Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function. Negate each of the following assertions.

$(a) \forall x \in \mathbb{R} : f(x) \neq 0.$
$(b) \forall M > 0, \exists A > 0, \forall x \geq A : f(x) > M.$
$(c) \forall x \in \mathbb{R} : f(x) > 0 \Longrightarrow x \leq 0.$
$(d) \forall \varepsilon > 0, \exists \delta > 0, \forall x, y \in I : \left|x - y\right| \leq \delta \Longrightarrow \left|f(x) - f(y)\right| \leq \varepsilon.$

Exercise **n**°5 : Let $n \in \mathbb{Z}$ be an integer. Prove the following assertion using a proof by contrapositive : " If n^2 is an even integer, then n is even".

Exercise **n**°6 : Show using a proof by contradiction that the square root of 2 is an irrational number, i.e $\sqrt{2} \notin \mathbb{Q}$.