University Center Abdelhafid Boussouf Mila
Institute of Mathematics and Computer Science
1st Year Mathematics (Algebra 1)

Series of Tutorial No. 4
Rings of Polynomials

Exercise 1.
Find the polynomial P of degree less than or equal to 3 such that: $P(0) = 1$, $P(1) = 0$, $P(-1) = -2$, and $P(2) = 4$.

Exercise 2.
Perform the Euclidean division of A by B for the following cases:

1. $A = 3X^5 + 4X^2 + 1$ *and* $B = X^2 + 2X + 3$.
2. $A = 3X^5 + 2X^4 - X^2 + 1$ *and* $B = X^3 + X + 2$.
3. $A = X^4 - X^3 + X - 2$ *and* $B = X^2 - 2X + 4$.

Exercise 3.
Let $P, Q, R, S \in A[X]$.

1. *If $P|Q$ and $Q|R$ then $P|R$.*
2. *If $P|Q$ and $P|R$ then $P|Q + R$.*
3. *If $P|Q$ and $Q \neq 0$ then $\deg(P) \leq \deg(Q)$.*
4. *If $P|Q$ and $R|S$ then $PR|QS$.*
5. *If $P|Q$ then $P^n|Q^n$ for all $n \geq 1$.*

Exercise 4.
Let $P, Q, R, S \in A[X]$.

1. *If $P|Q$ and $Q|P$ then P and Q are associated.*
2. *If P is associated to R and Q is associated to S then $P|Q \Leftrightarrow R|S$.*

Exercise 5.
Find the gcd of the following polynomials:

1. $X^3 - X^2 - X - 2$ *and* $X^5 - 2X^4 + X^2 - X - 2$.
2. $X^4 + X^3 - 2X + 1$ *and* $X^3 + X + 1$.

Exercise 6.

1. *Reducible polynomials in $\mathbb{K}[X]$ have degree greater than or equal to 2.*
2. *All polynomials of degree 1 are irreducible.*