## Exercise 1

0,5

| X(i) | 0 | 0.5 | 1 | 2 |
|---|---|---|---|---|
| F(i) | -1 | -0.25 | 0 | 0.2 |

$$P_3(x) = L_0 \times f(x_0) + L_1 \times f(x_1) + L_2 \times f(x_2) + L_3 \times f(x_3)$$

$$P_3^1(x) = \frac{(x-0.5)(x-1)(x-2)}{(0-0.5)(0-1)(0-2)} \times (-1)$$

$$= x^3 - \frac{7}{2}x^2 + \frac{7}{2}x - 1$$ 1

$$P_3^2(x) = \frac{(x-0)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)} \times (-0.25)$$

$$= -\frac{2}{3}x^3 + 2x^2 - \frac{4}{3}x$$ 1

$$P_3^3(x) = \frac{(x-0)(x-0.5)(x-2)}{(1-0)(1-0.5)(1-2)} \times 0$$

$$= 0$$ 0,5

$$P_3^4(x) = \frac{(x-0)(x-0.5)(x-1)}{(2-0)(2-.5)(2-1)} \times (0.2)$$

$$= \frac{1}{15}x^3 - \frac{1}{10}x^2 + \frac{1}{30}x$$ 1

$$P_3(x) = \frac{2}{5}x^3 - \frac{8}{5}x^2 + \frac{11}{5}x - 1$$ 0,5

$$P(1.5) = \frac{1}{20}, \qquad f(1.5) = \frac{1}{8}$$

$$\varepsilon = \frac{|f(1.5) - p(1.5)|}{f(1.5)} = 0.6$$ 0,5

We cannot evaluate $P(2.5)$ because it is out of the interval of interpolation.

## Exercise 2

$$\frac{dy}{y} = 2tdt \rightarrow \frac{dy}{dt} = 2ty$$ 0,5

### Euler method

$$\begin{cases} k = f(t_n, y_n) \\ y_{n+1} = y_n + hk \end{cases}$$ 0,25 0,25

| t | k | y |
|---|---|---|
| 0 | | 1 |
| 0.5 | 0 | 1 |
| 1 | 1 | 1.5 |
| 1.5 | 3 | 3 |

0,25 0,25 0,25 0,25

### Mid point method

$$\begin{cases} k_1 = f(t_n, y_n) \\ k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) \\ y_{n+1} = y_n = hk_2 \end{cases}$$ 0,25 0,25 0,25

| t | y_mid |
|---|---|
| 0 | 1 |
| 0.5 | 1.25 |
| 1 | 2.421875 |
| 1.5 | 6.962891 |

0,25 0,25 0,25 0,25

### RK4 method

$$\begin{cases} y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ k_1 = f(t_n, y_n), \\ k_2 = f(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1) \\ k_3 = f(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2) \\ k_4 = f(t_n + h, y_n + hk_3) \end{cases}$$ 0,25 0,25 0,25 0,25 0,25

| t | K1 | K2 | K3 | K4 | Y(i) |
|---|---|---|---|---|---|
| 0 | | | | | 1 |
| 0.5 | 0 | 0.5 | 0.56 | 1.28 | 1.28 |
| 1 | 1.28 | 2.4 | 2.83 | 5.4 | 2.71 |
| 1.5 | 5.43 | 10.17 | 13.14 | 27.85 | 9.37 |

**Exercise 3**

- **Gauss**

We write the system in enlarged matrix

A =

$$\begin{matrix} 2 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 3 & 0 & 2 & 1 & 2 \\ 4 & 1 & 1 & 1 & -1 \end{matrix}$$

A =

$$\begin{matrix} 2 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -5/2 & -5 & -1 \\ 4 & 1 & 1 & 1 & -1 \end{matrix}$$

A =

$$\begin{matrix} 2 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -5/2 & -5 & -1 \\ 0 & 1 & -5 & -7 & -5 \end{matrix}$$

A =

$$\begin{matrix} 2 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -5/2 & -5 & -1 \\ 0 & 1 & -5 & -7 & -5 \end{matrix}$$

A =

$$\begin{matrix} 2 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -5/2 & -5 & -1 \\ 0 & 0 & -5 & -8 & -6 \end{matrix}$$

A =

$$\begin{matrix} 2 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -5/2 & -5 & -1 \\ 0 & 0 & 0 & 2 & -4 \end{matrix}$$

$$x = \begin{pmatrix} -\frac{8}{5} \\ 3 \\ \frac{22}{5} \\ -2 \end{pmatrix}$$

- **Sholesky**

$a^t = a$ , the matrix is symmetric.

$$\mathrm{Det}(A) = 2*\begin{bmatrix} 0 & 3 & 4 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} - 0 + 3*\begin{bmatrix} 0 & 1 & 1 \\ 3 & 0 & 1 \\ 14 & 1 & 1 \end{bmatrix} - 4*\begin{bmatrix} 0 & 1 & 1 \\ 3 & 0 & 2 \\ 4 & 1 & 1 \end{bmatrix}.$$

❖ $2*\begin{bmatrix} 0 & 3 & 4 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ not strictly positive

| |
|---|
| The matrix is symmetric but its determinant is not positive, then we cannot apply Cholesky method to resolve this equation system. |