Fluid Mechanics

Fluids are substances (liquids and gases) that, having no fixed shape, are easily deformable under the action of very low external stresses (forces). Fluid mechanics consists of the study of fluids at rest and fluids in motion. The applications of fluid mechanics are very significant, especially in biology.

Definition of fluid mechanics :

- ✓ Fluid mechanics studies the behavior of fluids:

at rest: hydrostatics (the statics of fluids).

in motion : hydrodynamics (the dynamics of fluids).

A fluid can be a liquid or a gas.

- ✓ Two types of fluids are distinguished :

Liquids → incompressible

Gases → compressible

Examples: liquid water, oil, gasoline (liquid), oxygen (gas), air (gas)...

a. Perfect fluids are fluids in which the frictional force within the fluids (viscosity) is zero.

b. Real fluids are fluids in which the frictional force within the fluids (viscosity) is not zero.

Part I : Hydrostatics

I -1 Pressure

The pressure p is the force exerted per unit of surface :$p = \frac{F}{A}$

In units SI : the unit of pressure (P) is the Pascal (Pa), the unit of force (F) is the Newton (N), and the unit of area (A) is the m^2

1 Pascal =1 Pa =1 kg/ms^2 = 1 N/m^2

Other units of pressure :

The atmosphere: 1atm=$1.01325.10^5$Pa

The bar : 1bar=10^5Pa

The height of mercury (the millimeter of mercury) :1mmHg=133.42Pa

Example:

Pressure of a column of a fluid of height h and of density $\boldsymbol{\rho}$ on its lower circular section :

The pressure p is due to the weight P of this column.

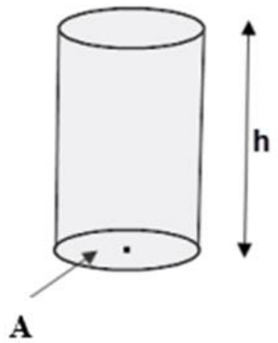
$$\boldsymbol{P = \frac{p}{A} = \frac{m.g}{A}}$$

$$\boldsymbol{\rho \rightarrow \frac{m}{v} \rightarrow m = \rho.v}$$

$$\boldsymbol{v = A.h}$$

$$\boldsymbol{m = \rho.A.h}$$

$$\boldsymbol{p = \frac{p.A.h.g}{A} \rightarrow P = \rho.g.h}$$



I -2 fundamental relation of hydrostatics:

The fundamental equation of hydrostatics can be written as :

P_A- P_B = $\boldsymbol{\rho}$gh= $\boldsymbol{\rho}$g (Z_B - Z_A)
Where :

P_A and **P_B** are the pressures at points **A** and **B**, respectively.

$\boldsymbol{\rho}$ is the fluid's density.

g is the acceleration due to gravity.

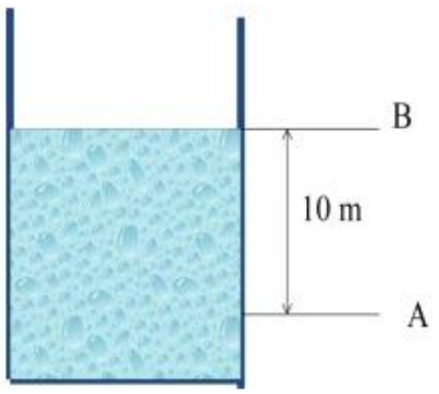
$\mathbf{Z_A}$ and $\mathbf{Z_B}$ are the depths of points **A** and **B**, respectively.

This equation represents the difference in pressure between two points in a fluid column due to the difference in height.

Example 1 :

Two points located in the water are 10 m apart vertically. The density of the water is ρ = 1000 kg·m^{-3}.

Calculate the pressure difference between these two points.



Answer:

$$P_A - P_B = \rho\, g\, h$$

$$P_A - P_B = 1\,000 \times 9{,}81 \times 10$$

$$\mathbf{P_A - P_B = 9{,}81 \times 10^{4}\ Pa}$$

Example 2 :

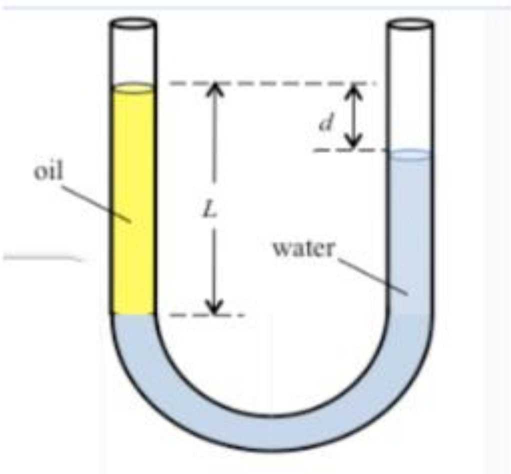
A U-tube contains Water (density ρ_w = 1000 kg/m^3) and oil is in static equilibrium as shown.

Given:

L=135mm

d=12.3mm

We are asked to calculate ρ_{oil} (the density of the oil).



Answer:

$\boldsymbol{\rho}_{oil}$=909 kg/m^3

I -3 Archimedes' Principle

When a solid body is submerged in a fluid, it experiences an upward vertical force, known as buoyant force, equal to the weight of the displaced fluid.

Thus, the weight of the body is:

$\boldsymbol{\pi} = \boldsymbol{\rho V}g$

Where ρ is the density and V is the volume

Part II : Hydrodynamics

Hydrodynamics is the study of fluids in motion. In hydrodynamics, two types of liquids are distinguished :

-Ideal fluid**,** where internal interactions are assumed to be negligible (friction is neglected). The fluid is incompressible, and its flow is steady.

-Real fluid, where internal friction is not neglected.

The path followed by a particle in a fluid is called a streamline. The velocity of the particle is tangent at each point along this streamline.

II-1Flow rate

Flow rate is the quotient of the quantity of fluid that passes through a cross-sectional area of the conduit by the duration of this flow.

a. Mass flow rate : If Δ_m is the mass of fluid that has passed through a cross-sectional area of the conduit during the time Δt, by definition the mass flow rate is:

Qm = $\Delta m/\Delta t$ with unit : $kg.s^{-1}$

b. Volumetric flow rate : If Δ_V is the volume of fluid that has passed through a cross-sectional area of the conduit during the time Δt, by definition the volumetric flow rate is:

Qv = $\Delta V/\Delta t$ with unit : $m^3.s^{-1}$

If S is the cross-sectional area of the tube (constant). The liquid moves a distance Δx during a time Δt. The volume of the liquid exiting the tube is:

ΔV = S. Δx = S.v. Δt

ΔV = Q. Δt

Q = S. V with $\boldsymbol{S} = \boldsymbol{\pi r}^2 = \boldsymbol{\pi}\frac{d^2}{4}$

II-2 Continuity equation:

For an ideal fluid : Qv=S.V= constant

where Qv is the fluid density, S is the cross-sectional area, V is the velocity vector of the particle through this section.

II-3 Bernoulli's theorem :

In an incompressible ideal fluid in steady flow, the head along a streamline is constant. The Bernoulli equation is then:

$$P + \rho . g . z + \frac{1}{2} . \rho . v^2 = constant$$

p is the pressure at the point considered (in Pa).

$\boldsymbol{\rho}$ is the density of the fluid (in kg. m^{-3}).

g is the acceleration due to gravity (in m. S^{-2}).

z is the elevation of the point considered (in m).

v is the velocity of the fluid at the point considered (m. s^{-1}).