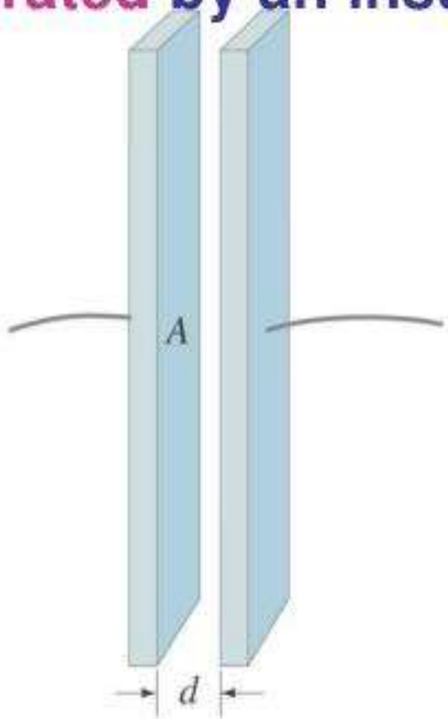
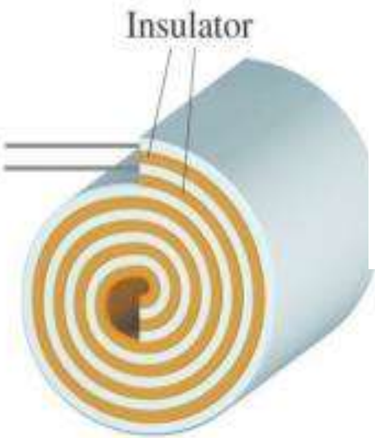
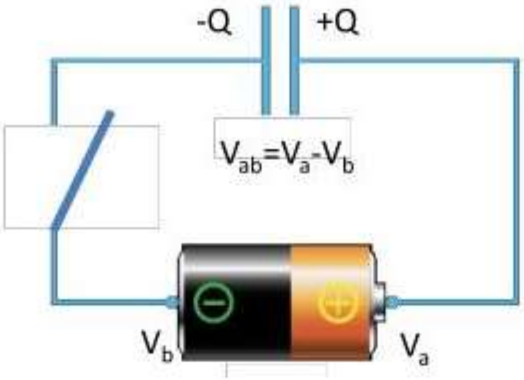
Capacitors

- A capacitor is a device that **stores** charge and electrical energy.
- A capacitor consists of **two** conductors **separated** by an insulator.

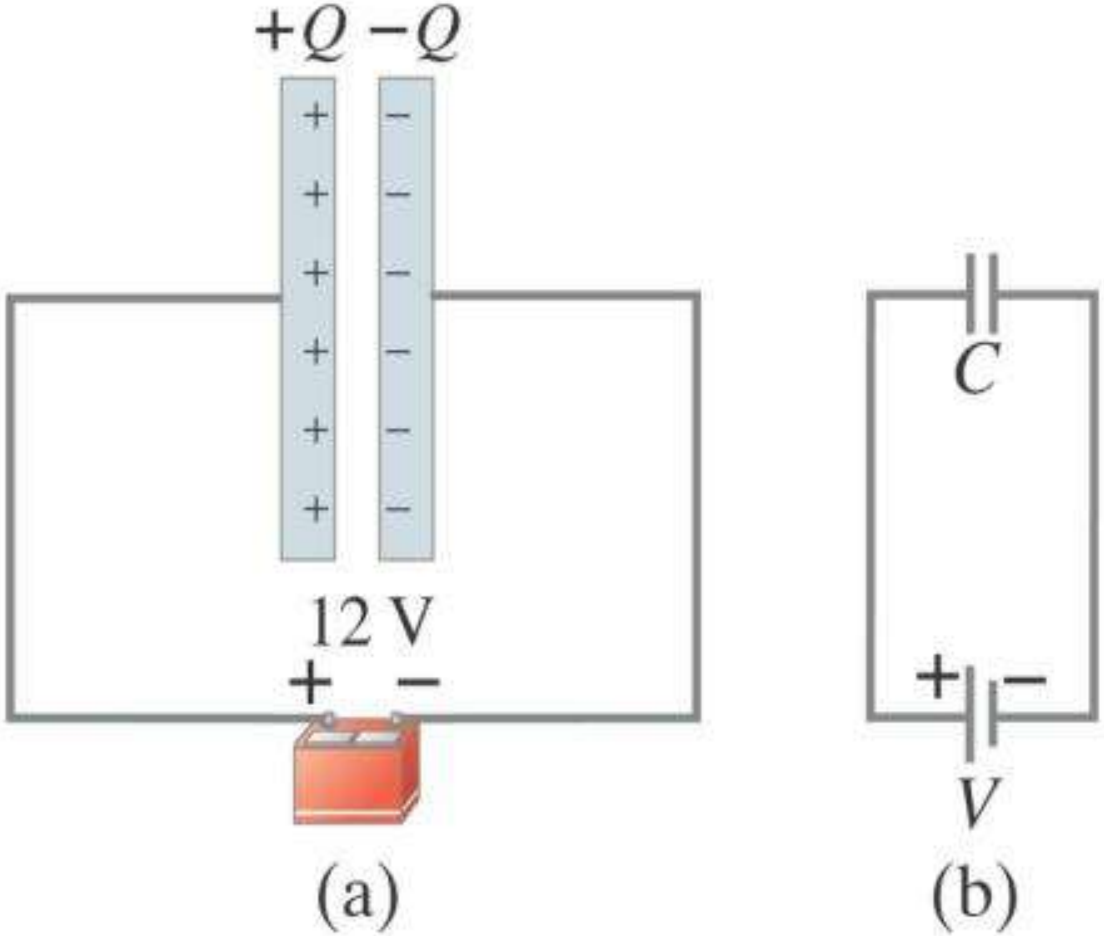






Capacitors

Parallel-plate capacitor connected to battery. (b) is a circuit diagram.



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1. Definition of Capacitance

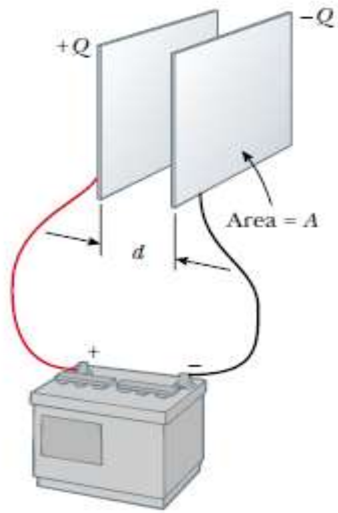
When **a capacitor** is connected to a battery, the charge on its plates is proportional to the potential difference between the conductors:

$$Q = C.\Delta V$$

The quantity ***C*** is called the *capacitance.* It is always a positive quantity. Furthermore, the charge Q and the potential difference ΔV are positive quantities. Because the potential difference increases linearly with the stored charge, the ratio $Q / \Delta V$ is constant for a given capacitor.

The SI unit of capacitance is the farad (F),

$$1\ \text{F} = 1\ \text{C/V}$$



Capacitance

- Charge Q is measured in coulombs, C.
- Potential difference, V, is measured in volts, V.
- Capacitance, C, is measured in farads, F.
- 1 farad is 1 coulomb per volt: $1\ F = 1\ C\ V^{-1}$

- 1 farad is a very large unit. It is much more common to use the following:

  $mF = 10^{-3}\ F$

  $\mu F = 10^{-6}\ F$

  $nF = 10^{-9}\ F$

  $pF = 10^{-12} F$

2. Calculating Capacitance

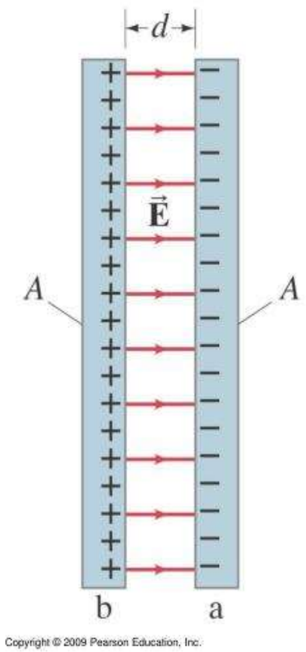
The capacitance of an isolated charged sphere

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

This expression shows that the capacitance of an isolated charged sphere is proportional to its radius and is independent of both the charge on the sphere and the potential difference.

Parallel-Plate Capacitors

Two parallel metallic plates of equal area *A are separated by a distance d*, One plate carries a charge +*Q , and the other carries a charge -Q .*



Capacitance

For a parallel-plate capacitor as shown, the field between the plates is

$$E = Q/\varepsilon_0 A.$$

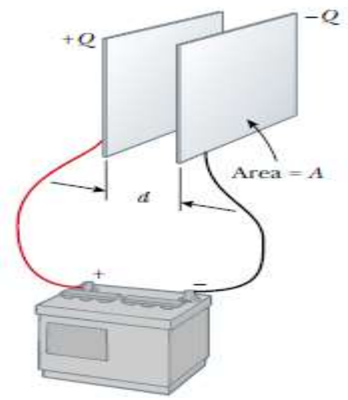
In a uniform field, $V=Ed$:

$$V_{ba} = Qd/\varepsilon_0 A.$$

This gives the capacitance:

$$C=Q/V= \varepsilon_0 A/d$$

That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation



Example .1 **Parallel-Plate Capacitor**

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-4}\ \text{m}^2)}{1.00 \times 10^{-3}\ \text{m}}$$

$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$

Combinations of Capacitors

Parallel Combination

• Capacitors in parallel are the same and are equal to the potential difference applied across each one.

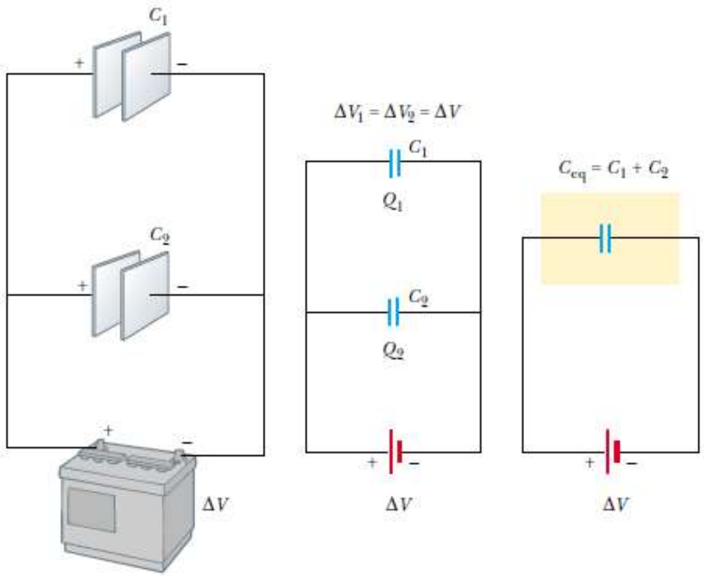
• The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors.

$$Q = Q_1 + Q_2$$

$$Q_1 = C_1\,\Delta V \qquad Q_2 = C_2\,\Delta V$$

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$



for the equivalent capacitor

$$Q = C_{eq}\,\Delta V$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

Series Combination

• The charges on capacitors connected in series are the same.

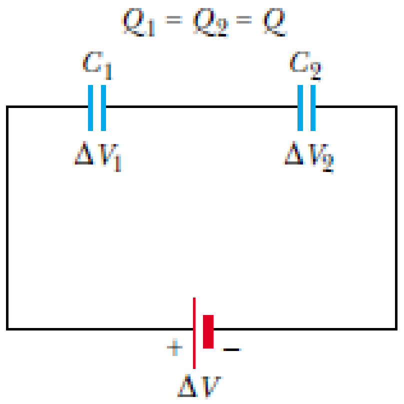
• The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = \frac{Q}{C_{eq}}$$

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

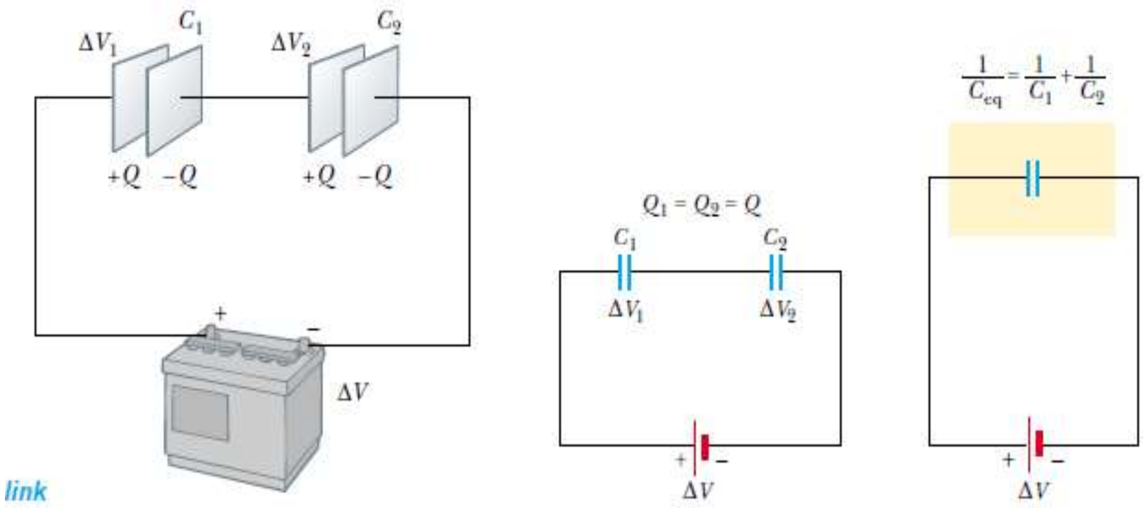


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series combination)}$$

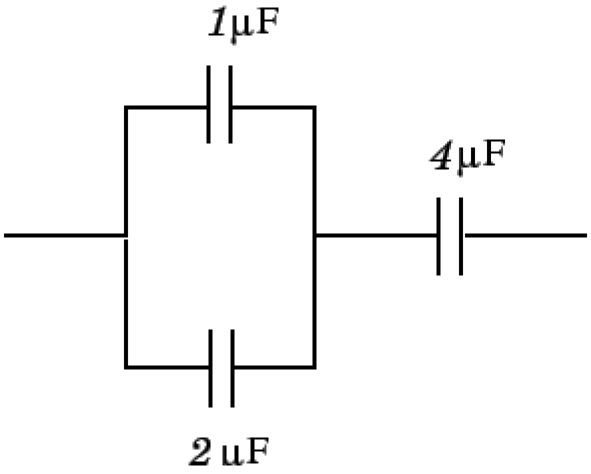
When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \text{(series combination)}$$

The equivalent capacitance of a series combination is always less than any individual capacitance in the combination.



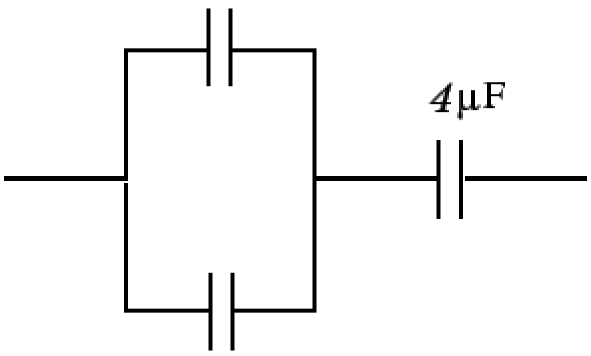
Question: A $1\ \mu F$ and a $2\ \mu F$ capacitor are connected in parallel, and this pair of capacitors is then connected in series with a $4\ \mu F$ capacitor, as shown in the diagram. What is the equivalent capacitance of the whole combination? What is the charge on the $4\ \mu F$ capacitor if the whole combination is connected across the terminals of a 6V battery? Likewise, what are the charges on the $1\ \mu F$ and $2\ \mu F$ capacitors?



Solution:

The equivalent capacitance of the $1\,\mu\text{F}$ and $2\,\mu\text{F}$ capacitors connected in parallel is

$$1 + 2 = 3\,\mu\text{F}$$



When a $3\,\mu\text{F}$ capacitor is combined in series with a $4\,\mu\text{F}$ capacitor, the equivalent capacitance of the whole combination is given by

$$\frac{1}{C_{eq}} = \frac{1}{(3 \times 10^{-6})} + \frac{1}{(4 \times 10^{-6})} = \frac{(7)}{(12 \times 10^{-6})}\ \text{F}^{-1},$$

4. Energy Stored in a Charged Capacitor

- Suppose that *q* is the charge on the capacitor at some instant during the charging process.
- At the same instant, the potential difference across the capacitor is

$$\Delta V = q/C$$

- The work necessary to transfer an increment of charge *dq* from the plate carrying charge -*q* to the plate carrying charge *q*

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

- The total work required to charge the capacitor from *q*=0 to some final charge *q*=*Q* is

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

The potential energy stored in a charged capacitor is:

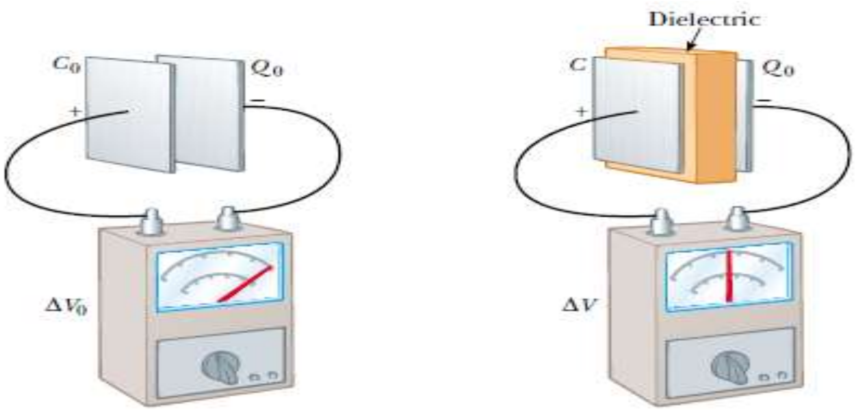
$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

Example:

A 10 000μF capacitor is described as having a maximum working voltage of 25 V. Calculate the energy stored by the capacitor.

$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 10{,}000 \times 10^{-6} \times 25^2 = 3.125$ J

Capacitor with a Dielectric



$$\Delta V = \frac{\Delta V_0}{\kappa}$$

The dielectric constant

The surface charges on the dielectric reduce the electric field inside the dielectric. This reduction in the electric field is described by the ***dielectric constant*** k, which is the ratio of the field magnitude E_0 without the dielectric to the field magnitude E inside the dielectric:

$$\kappa = \frac{E_0}{E}$$

Every dielectric material has a characteristic *dielectric strength*, which is the maximum value of the electric field that it can tolerate without breakdown

Some properties of dielectrics

Material	Dielectric Constant	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, $\kappa = \text{unity}$