Conductors in Electrostatic Equilibrium

A good electrical conductor contains electrons that are not bound to any atom and therefore are *free to move* about within the material

When no net motion of charge occurs within a conductor, the conductor is said to be in ***electrostatic equilibrium***

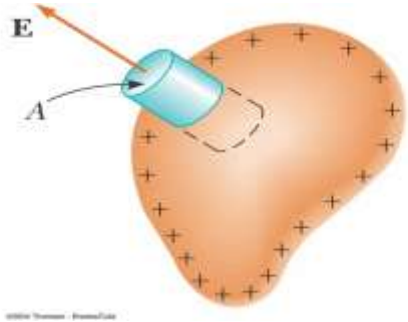
Properties of a Conductor in Electrostatic Equilibrium

1. The E field is zero everywhere inside the conductor

2. If an isolated conductor carries a charge, the charge resides on its surface

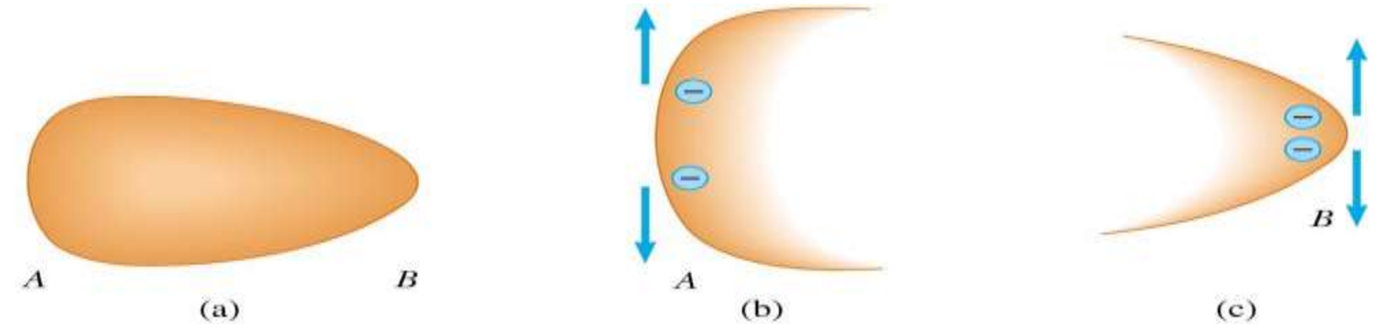
3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude **σ/ε_0** , where is the surface charge density at that point.

$$\phi_C = \oint E_n \, dA = E_n A = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{\varepsilon_0}$$



4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest

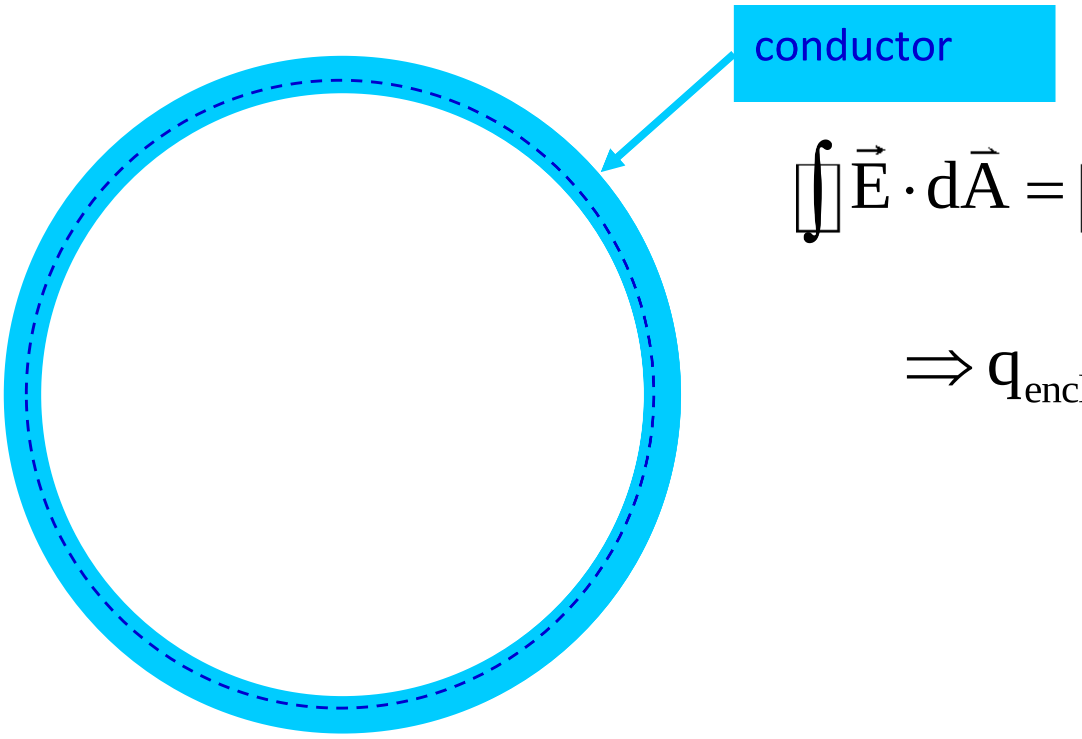
Property 4



- Any excess charge moves to its surface
- The charges move apart until an equilibrium is achieved
- The amount of charge per unit area is less at the flat end
- The forces from the charges at the sharp end produce a larger resultant force away from the surface

Example 01:

If there is an empty nonconducting cavity inside a conductor, Gauss' Law tells us there is no net charge on the interior surface of the conductor.

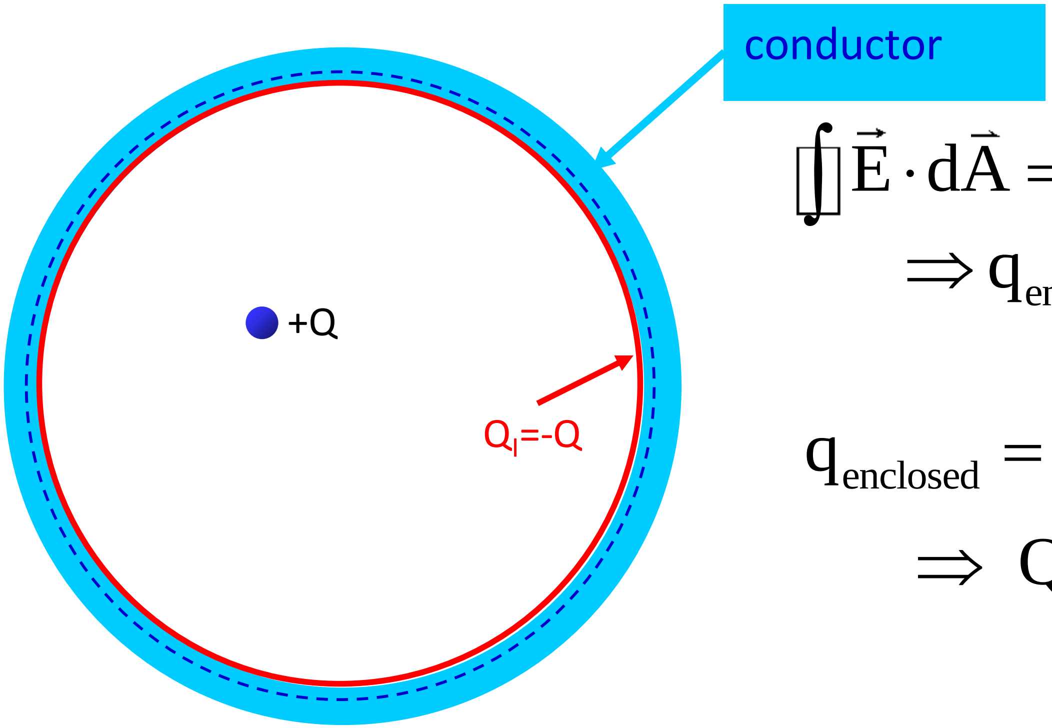


$$\oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot dA = 0$$

$$\Rightarrow q_{enclosed} = 0$$

Example 02:

If there is a nonconducting cavity inside a conductor, with a charge inside the cavity, Gauss' Law tells us there is an equal and opposite induced charge on the interior surface of the conductor.



$$\oint \vec{E}\cdot d\vec{A} = \oint 0\cdot dA = 0$$

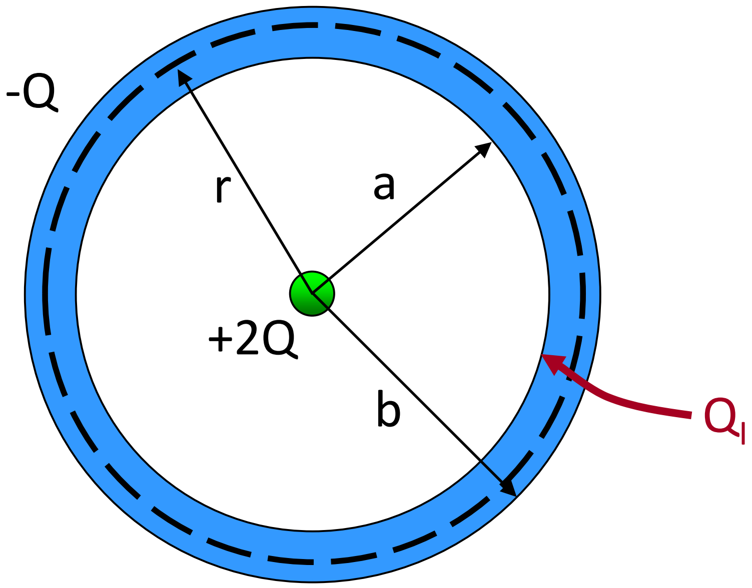
$$\Rightarrow q_{enclosed} = 0$$

$$q_{enclosed} = 0 = +Q + Q_I$$

$$\Rightarrow Q_I = -Q$$

Example 03:

a conducting spherical shell of inner radius ***a*** and outer radius ***b*** with a net charge ***-Q*** is centered on point charge ***+2Q***.
Use Gauss's law to show that there is a charge of ***-2Q*** on the inner surface of the shell, and a charge of ***+Q*** on the outer surface of the shell

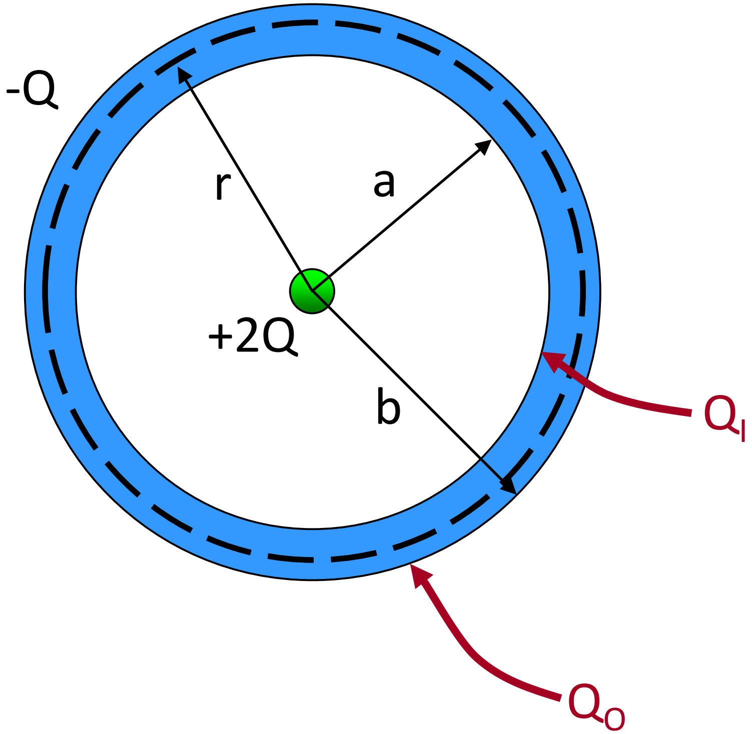


$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

E=0 inside the conductor!

Let ***r*** be infinitesimally greater than ***a***.

$$0 = \frac{q_{enclosed}}{\varepsilon_0} = \frac{Q_I + 2Q}{\varepsilon_0} \Rightarrow Q_I = -2Q$$



$$Q_I = -2Q$$

From Gauss' Law we know that excess charge on a conductor lies on surfaces.

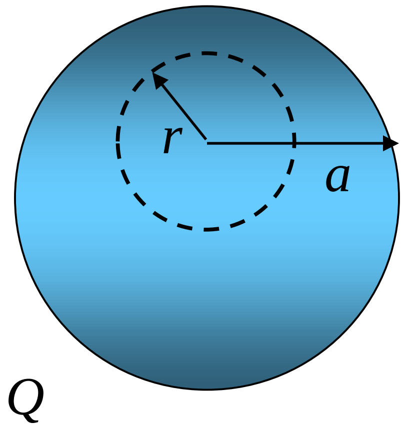
Electric charge is conserved:

$$Q_{shell} = -Q = Q_I + Q_O = -2Q + Q_O$$

$$-Q = -2Q + Q_O \Rightarrow Q_O = +Q$$

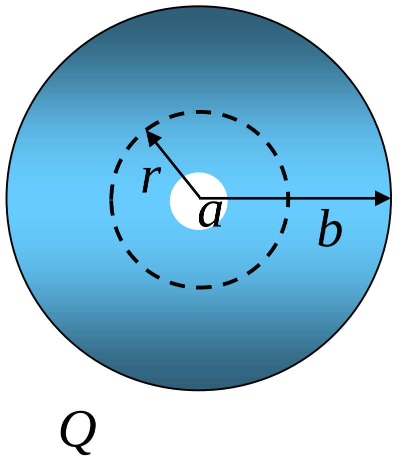
Example 04:

an insulating sphere of radius a has a uniform charge density ***ρ*** and a total positive charge ***Q***. Calculate the electric field at a point inside the sphere.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho V_{enclosed}}{\varepsilon_o}$$

$$E\left(4\pi r^2\right) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_o}$$



$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$q_{enclosed} = \rho V_{enclosed} = \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)$$

Calculate the electric field at a point outside the sphere.

$$q_{enclosed} = \rho V_{enclosed} = \rho\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)$$

A conductor in **electrostatic equilibrium** has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on the conductor resides entirely on its surface.
3. The electric field just outside the conductor is perpendicular to its surface and has a magnitude σ/ϵ_0, where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.