Institute of Science and Technologies 2nd year: Engineering of Proceeding

Numerical methods 2023/2024

Series N°4: Solving of 1^{rst} order differential ordinary equations

Exercise 1

Consider the following differential equation,

$$f(t,y) = \frac{t}{y}$$

If at $t = 0$ $y = 1$ find $y(1)$ with :

1) Euler method. 2) Heun (Modified Euler) method.

Exercise 2

1. Show that the Cauchy problem $\begin{cases} \dot{y} = 1 + y & t \in [0.1] \\ y(0) = 0 \end{cases}$

Has a unique solution.

2. Calculate an approximate value of $y(1)$ using the Euler method with a step equal to 0.1.
3. Find the exact (analytical) solution. What is the approximation error of y(1).

Exercise 3

Perform three iterations with h = 0.1 of the Euler and Runge-Kutta methods of order 4 for solving the following differential equation:

$$\dot{y}(t) = tsin(y) \qquad y(0) = 2$$

Exercise 4

Perform three iterations with h = 0.1 of the Modified-Euler and RK4 for solving the following differential equation:

$$\dot{y}(t) = t^2 + (y(t))^2 + 1 \qquad y(1) = 0$$

Exercise 5

Consider the differential equation $f(t,y) = \frac{t}{y}(y-1)$. if $at\ t = 0, y = 2$ find $y(1)$ by using Heun Euler method ($h = 0.1$) and Mid-point ($h = 0.5$). What do you conclude?