Mathematics 2

Work Sheet 02

Exercise 1. Which of the following equations are linear?

① $2x - 3y + 4z = 10.$

② $\frac{1}{\sqrt{2}}x + 4^3y = \sin\left(\frac{\pi}{3}\right).$

③ $x^2 + y^2 + z^2 = 1.$

④ $2.123x_1 + 5.541x_2 - 9.101x_3 = 11.012.$

⑤ $x + yz = 3.$

Exercise 2.

$$\begin{aligned} x + 2y + 3z + 4w &= 4 \\ x + y + z + w &= 2. \\ x + 2y - +2z + w &= 2 \end{aligned}$$

For each of the following tuples (x, y, z, w) of real numbers, determine whether it is a solution of the first equation, second equation, and/or third equation. Which ones are solutions to the system of equations?

ⓐ $(2, 0, -2, 2)$ ⓒ $(1, 1, -1, 1)$ ⓔ $(2, -2, 2, 0)$

ⓑ $(2, 2, -2, 0)$ ⓓ $(3, 0, -1, 1)$

Exercise 3. Using Cramer's rule, find the solutions of the next system

$$\begin{aligned} x + y - z &= 6 \\ 3x - 2y + z &= -5. \\ x + 3y - 2z &= 14 \end{aligned}$$

Exercise 4. Using Gaussian elimination, find the solutions of the next systems, then calculate the determinant of each coefficient matrix.

①

$$S:\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 1 \\ x_1 - 3x_2 - 6x_4 = -1 \\ 2x_2 - x_3 + 2x_4 = 3 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 1 \end{cases}.$$

②

$$S':\begin{cases} 2x_1 + 4x_2 - x_3 + 5x_4 = -10 \\ x_1 + 2x_2 + 7x_4 = -13 \\ x_1 + x_2 + 3x_3 + x_4 = 4 \\ 2x_1 + x_2 + 2x_3 + 4x_4 = -5 \end{cases}$$

Exercise 5. Solve the next system, where x, y and z are positive real numbers.

$$\begin{cases} x^3y^2z^6 = 1 \\ x^4y^5z^{12} = 2 \\ x^2y^2z^5 = 3. \end{cases}$$

Exercise 6. Find the real numbers α, β, γ such that for every polynomial P of degree $\leqslant 3$ we have

$$\int_2^4 P(x)dx = \alpha P(2) + \beta P(3) + \gamma P(4).$$

Exercise 7. Find the characteristic polynomial, eigenvalues, and basic eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 3 & -1 \\ -2 & 4 & -2 \\ 2 & -3 & 3 \end{bmatrix}.$$

Exercise 8. Let

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}.$$

Find A^{50}.