Geometric optics

Introduction :

Optics is the branch of physics that deals with light and its properties, electromagnetic radiation, vision, and systems using or emitting light. Therefore, optics is the part of physics that studies the properties of light.

Light can be defined as a visible electromagnetic wave by the human eye.

Light propagates in a straight line with a speed of $C = 3.10^8\ m.s^{-1}$ in a vacuum.

The speed of light in a medium is given by :

$v = C/n$

where n is the refractive index of the medium ($n \geq 1$).

n for water = 1,33

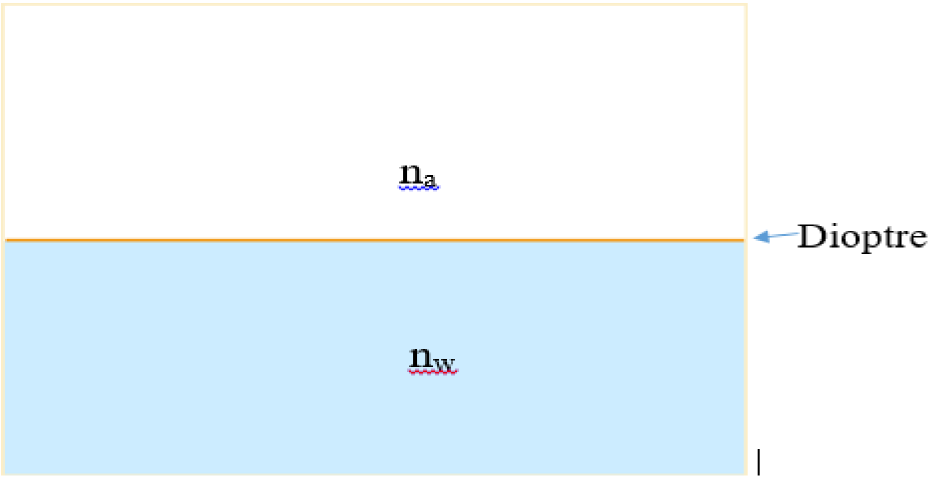
n for glass=1,52

n for air =1

I. Laws of Reflection and Refraction

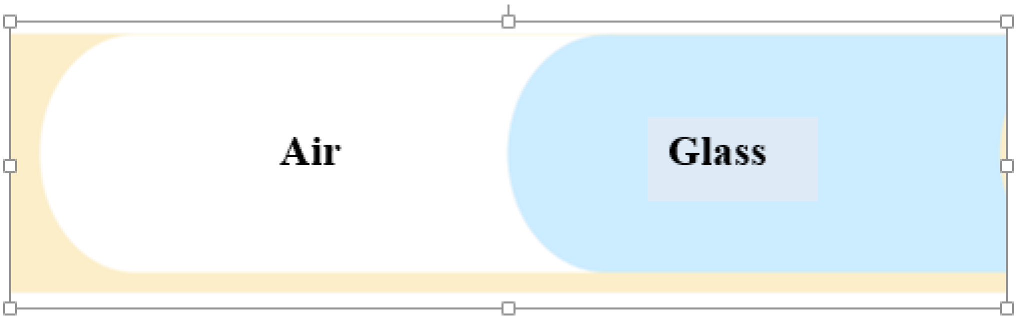
Diopter

Diopter : A diopter is a surface that separates two transparent, homogeneous mediums with different refractive indices. This surface can be flat (then called a planar diopter), spherical (spherical diopter), or of any shape



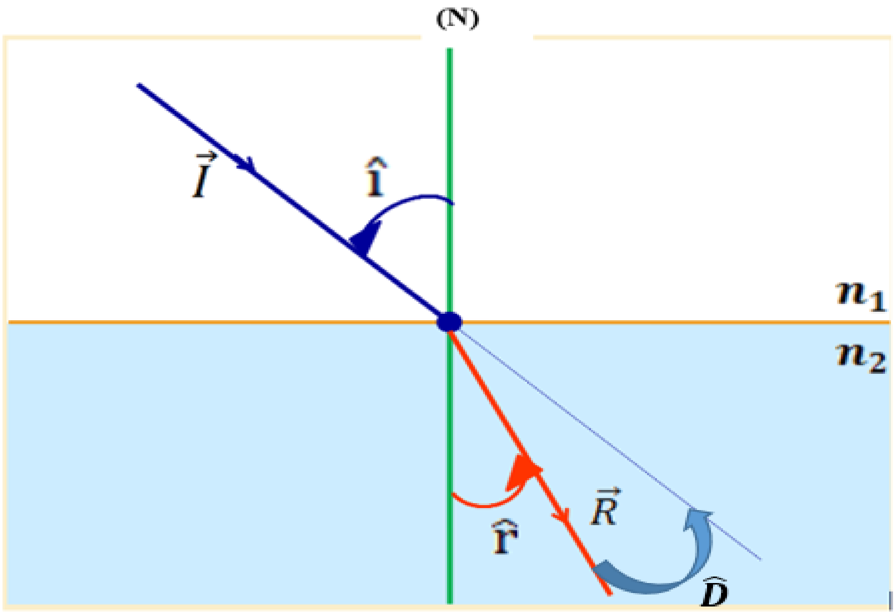
❖ Planar diopter

❖ Spherical diopter



$n_1 < n_2 \Rightarrow \hat{\imath} > \hat{r}$

$\hat{D} = \hat{\imath} - \hat{r}$



Refraction Phenomenon.

$\vec{I}$: Incident ray

$\vec{R}$: Refracted ray

$\hat{\imath}$: Angle of incidence

$\hat{r}$: Angle of refration

N: Normal

$\hat{D}$:Deviation

Laws of Refraction

- **First Law:**

The incident ray$(\overrightarrow{I})$;the reflected ray$(\overrightarrow{R})$, and(N) the normal are in the same plane, called the plane of incidence

- **Second Law:**

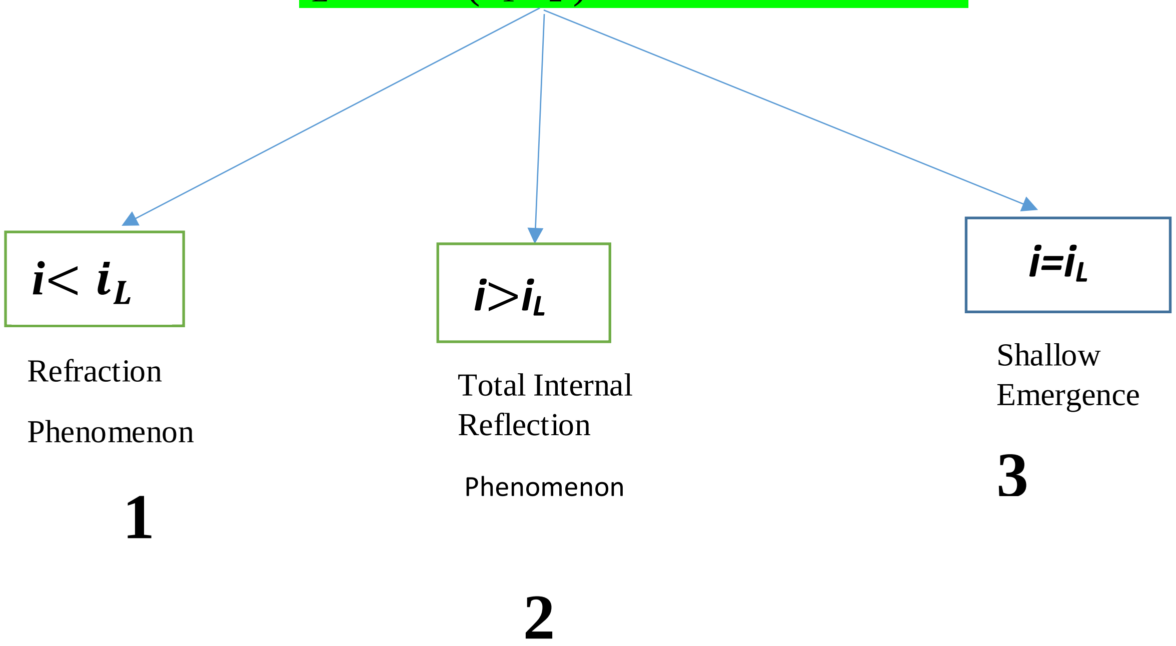
Snell's Law of Refraction: $n_1 \sin i = n_2 \sin r$

Calculations must be performed

$r = \arcsin(n_1/n_2)$:Critical Angle of Incidence($i = \pi/2$)

$n_1 > n_2$

$i_L = \arcsin(n_1/n_2)$:There are three cases

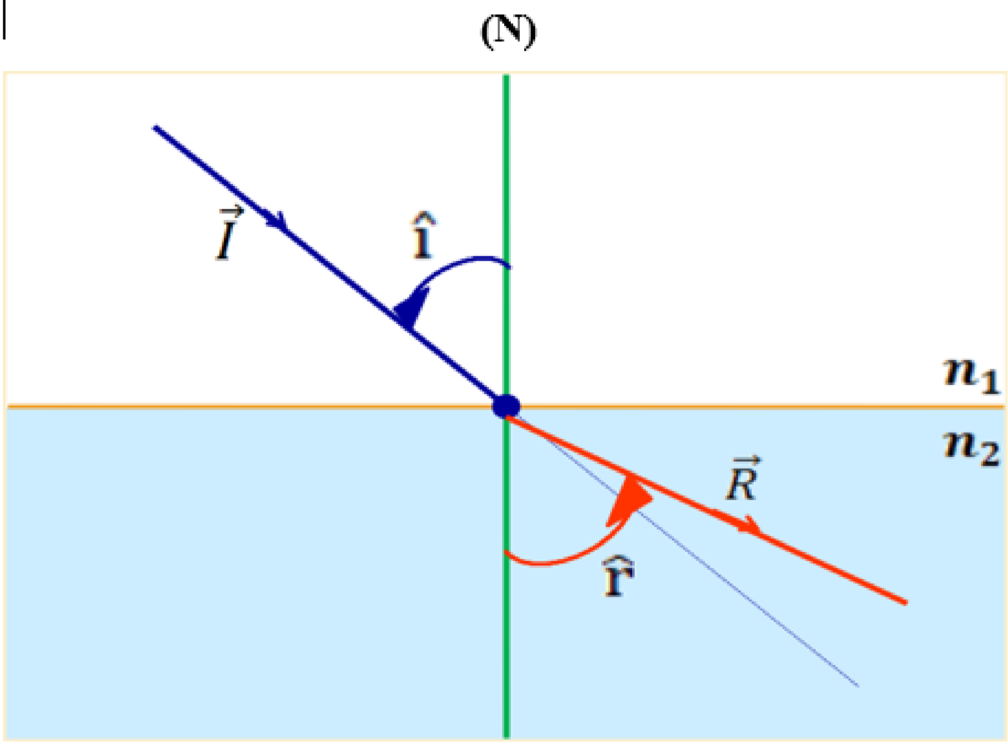


First case ($i<i_L$)

$n_1 > n_2 \Rightarrow \hat{\imath} < \hat{r}$

$\widehat{D}$: Deviation

$\widehat{D} = \hat{r} - \hat{\imath}$



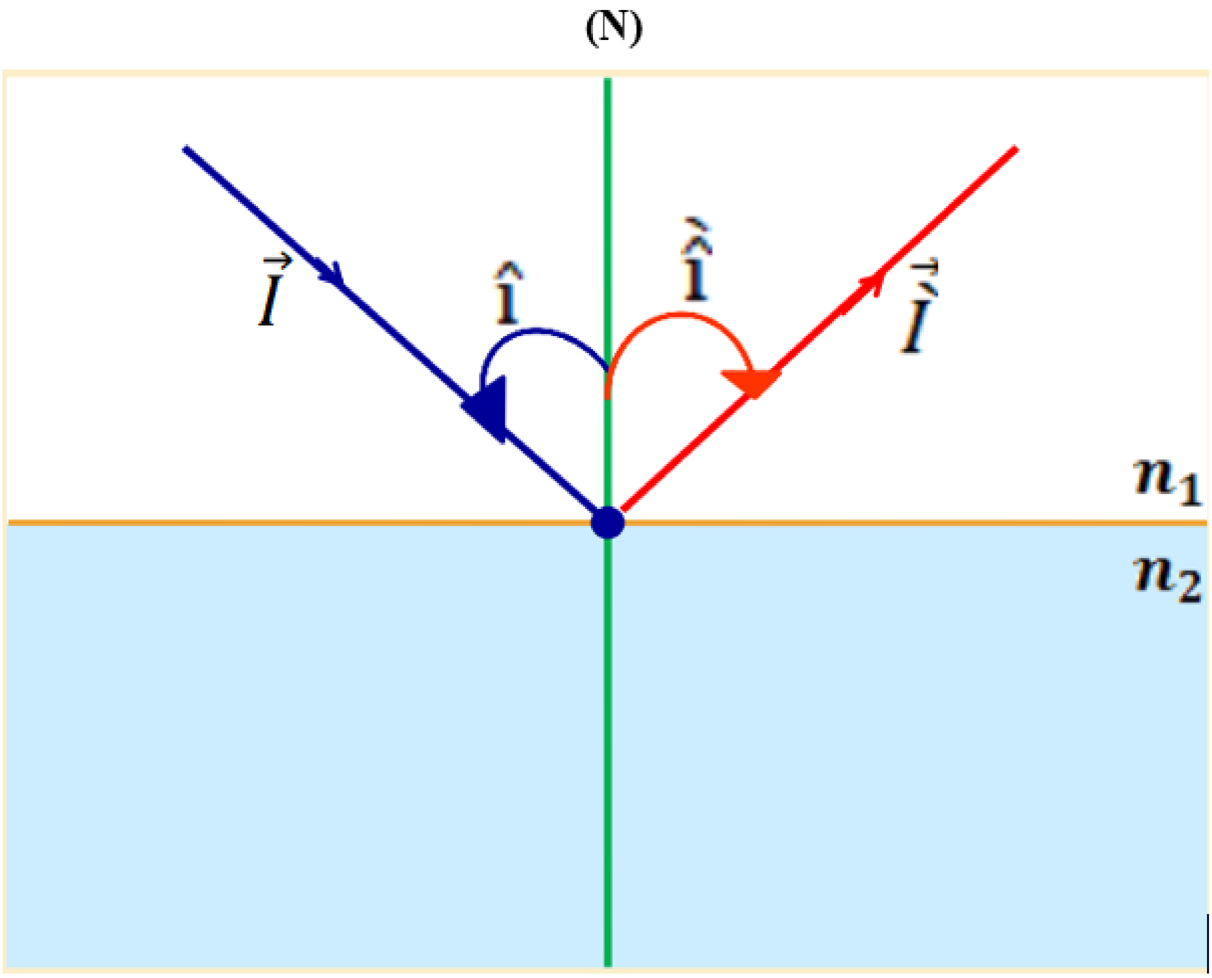
Refraction phenomenon

Second case ($i>i_L$)

$n_1 > n_2$

$\vec{I'}$:Reflected ray

$\hat{I}'$: Angle of reflection

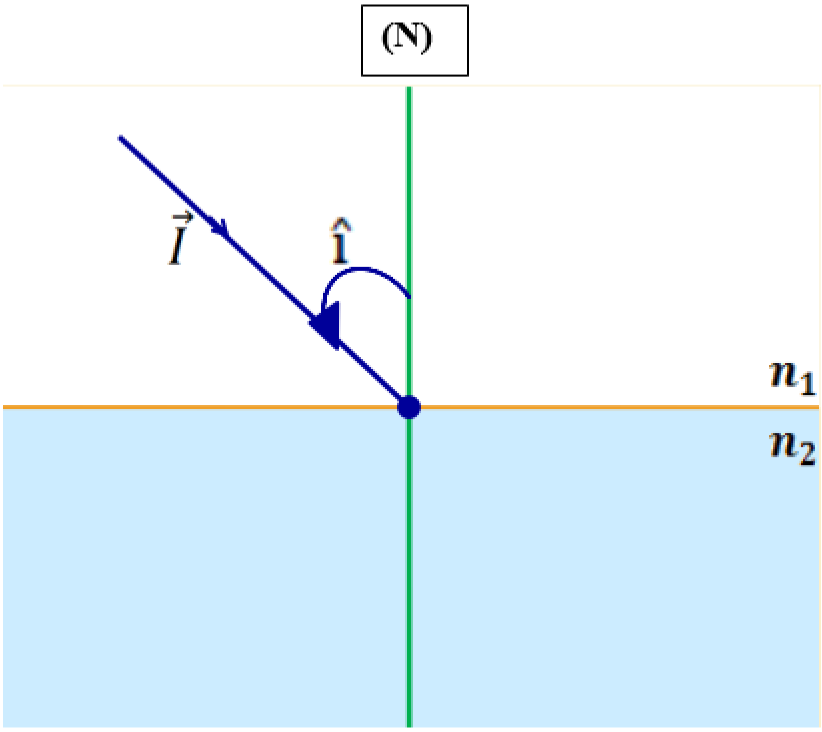


Phenomenon of Total Reflection

Third case ($i=i_L$)

$n_1 > n_2$

$\hat{r} = \frac{\pi}{2}$



$\hat{r} = \frac{\pi}{2}$ **Shallow Emergence**

I.1. Refraction

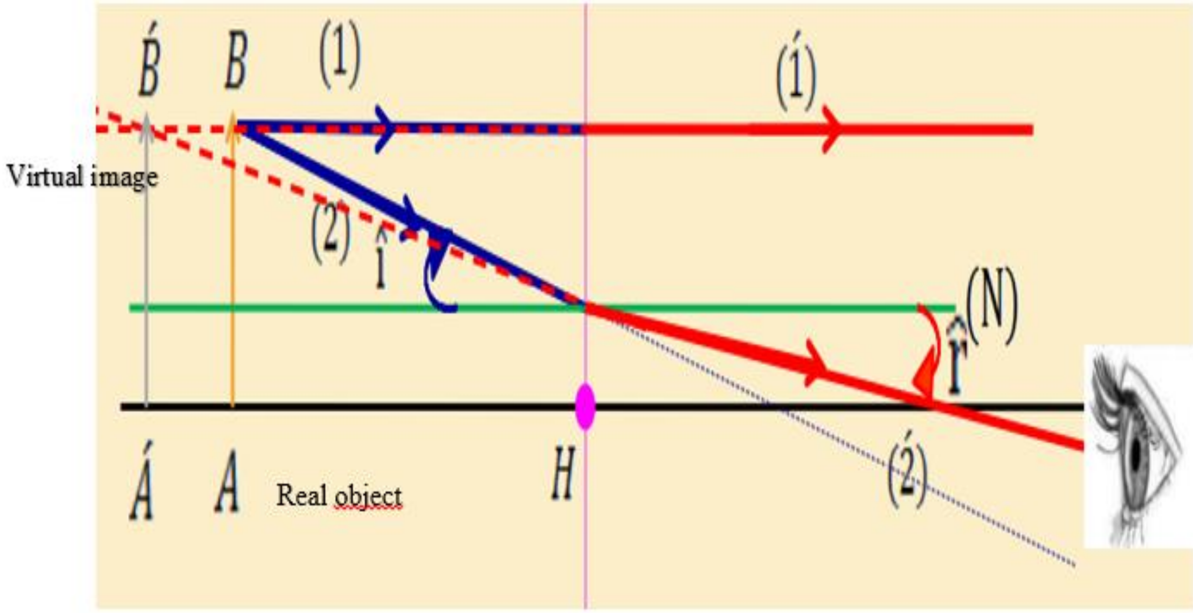
I.1. 1 Planar diopter

Two transparent media separated by a flat surface. The flat surface is the plane diopter

$$n_1 < n_2 \Longrightarrow \hat{\imath} > \hat{r}$$

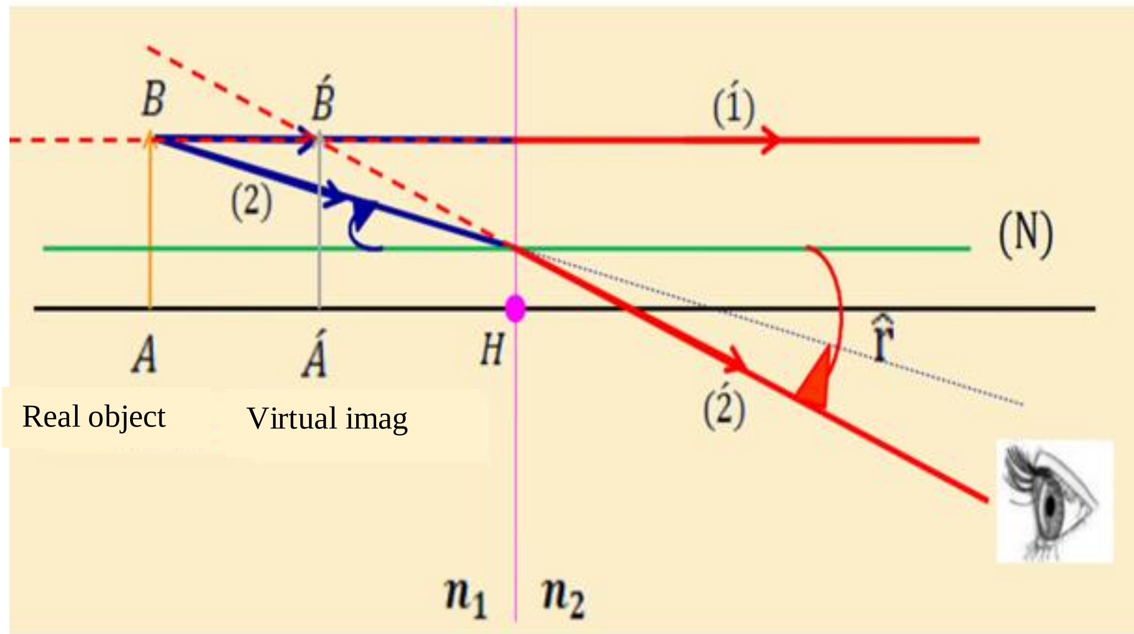
AB :object

A'B' : image



Refraction phenomenon

$n_1 > n_2 \longrightarrow \hat{\imath} < \hat{r}$



Refraction phenomenon

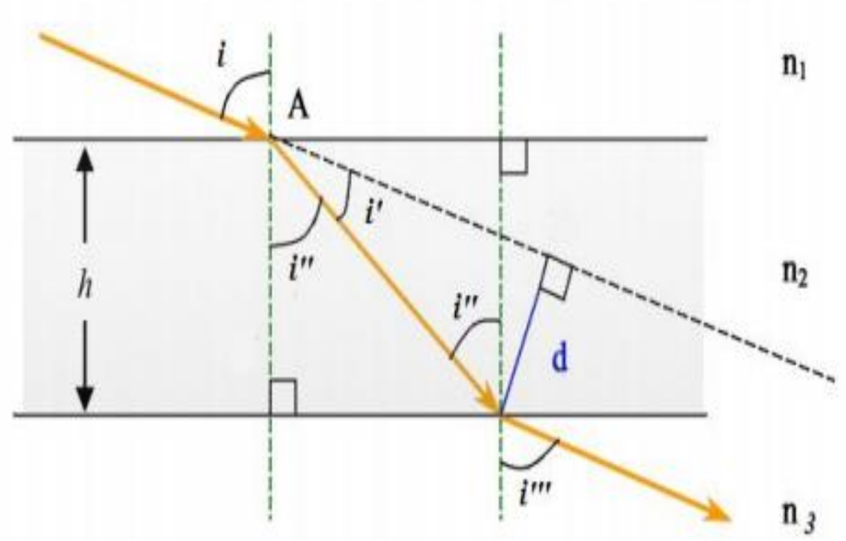
- **Conjugate Formula**

It is the relationship between the object position, image position, and the characteristics of the optical system.

$\overline{HA}/n_1 = \overline{H'A}/n_2$

- **The parallel-faced blade**

Medium with refractive index n confined by two parallel planes



$$n_1 \sin i = n_2 \sin i' = n_3 \sin i'''$$

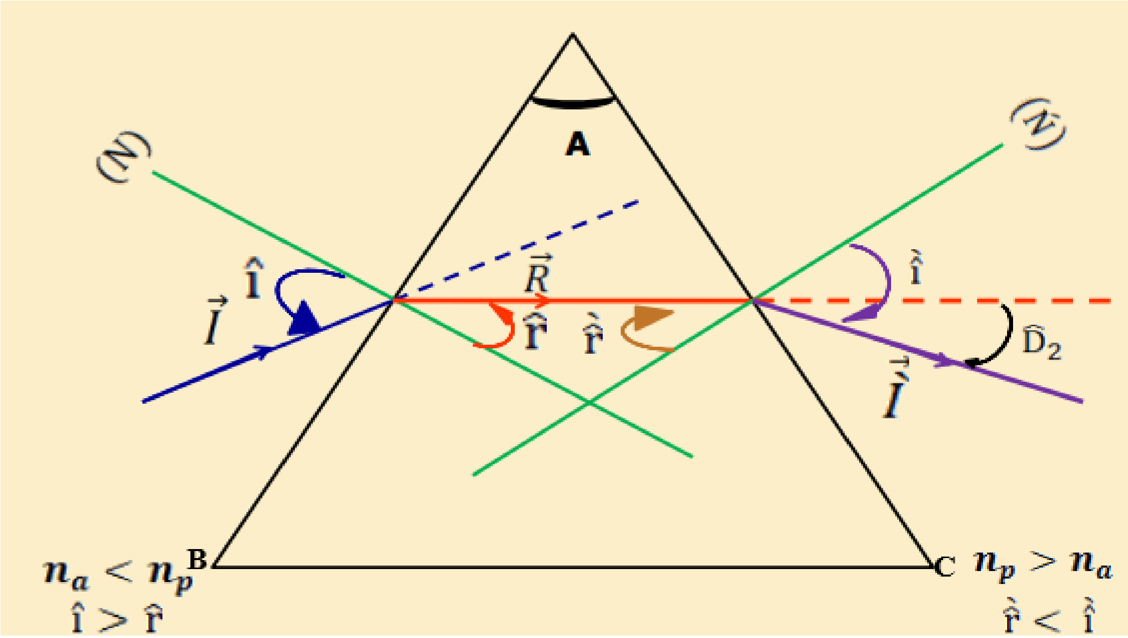
The direction of the emerging ray is independent of the index of the blade; the incident and emerging rays are parallel if the extreme media are identical.

- **Prism**

Definition:

A prism is a set of three transparent, homogeneous, and isotropic media separated by two flat diopters. The flat diopters are, in practice, limited to segments AB and AC and form a triangle in a cutting plane or main section plane. The angle Â = (AB, AC) is called the apex angle.

If n is the index of the prism, Snell-Descartes laws at I and I' impose the following two relations."



Deviation : $\widehat{D_2}=\widehat{\imath'}-\widehat{r'}$

Total deviation

- Diopter (air-prism): $\widehat{D}_1=\hat{\imath}-\hat{r}$ (a)
- Diopter (prism-air): $\widehat{D}_2=\widehat{\imath'}-\widehat{r'}$ (b)

$$\widehat{A}=\hat{r}+\widehat{r'}$$

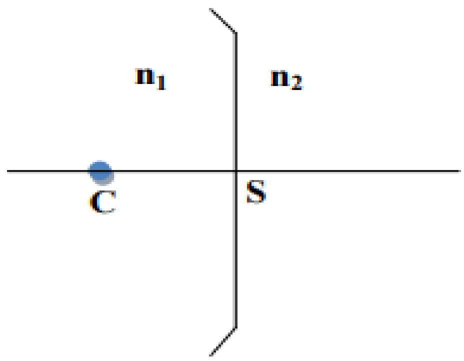
The total deviation : $\widehat{D_T}=\widehat{D_1}+\widehat{D_2}$

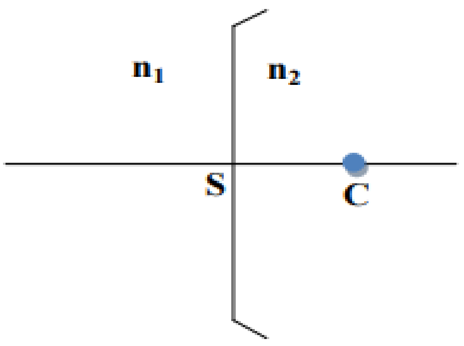
(a) and (b) $\widehat{D_T}=\hat{\imath}+\widehat{\imath'}-\widehat{A}$

I.1.2 Spherical diopter

A spherical diopter is a spherical surface with center C separating two media with different refractive indices. It is characterized by the vertex, the center, and the optical axis that passes through S and C.

Concave diopter ($R = \overline{SC} < 0$) **Convex diopter ($R = \overline{SC} > 0$)**





❖ **Conjugate Formula**

$$\frac{n_2}{\overline{SA'}} - \frac{n_1}{\overline{SA}} = \frac{n_2 - n_1}{\overline{SC}} = V$$

$\overline{SA}$: Object position

$\overline{SA'}$: Image position

With: V is the vergence or the power of the diopter (unit: δ = Diopter = m^{-1})

If V > 0: Converging diopter

If V < 0: Diverging diopter

❖ **The magnification**

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2}\frac{\overline{SA'}}{\overline{SA}}$$

$\overline{AB}$: Object size

$\overline{A'B'}$: Image size

γ>0: Upright image

γ <0: Inverted image

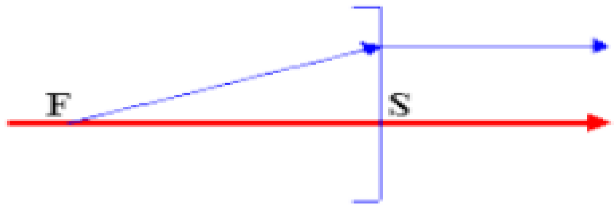
$|\gamma|$<1: Reduced image

$|\gamma|$ >1: Enlarged image

- **Foci**

Object focus F: the point on the axis that corresponds to an image at infinity; SF is the object focal distance.

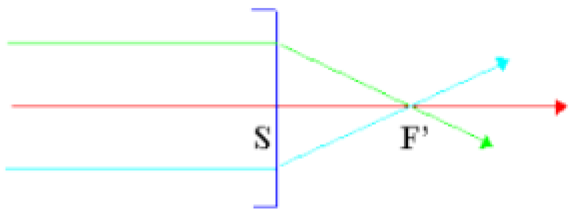
$$\overline{SF} = -\frac{n_1\overline{SC}}{n_2-n_1} = -\frac{n_1}{V}$$



An incident ray passing through the object focus of the lens will refract into a ray parallel to the optical axis of the dioper

Image focus F': the point on the image axis of an object point located at infinity; SF' is the image focal distance.

$$\overline{SF'} = \frac{n_2\overline{SC}}{n_2-n_1} = \frac{n_2}{V}$$

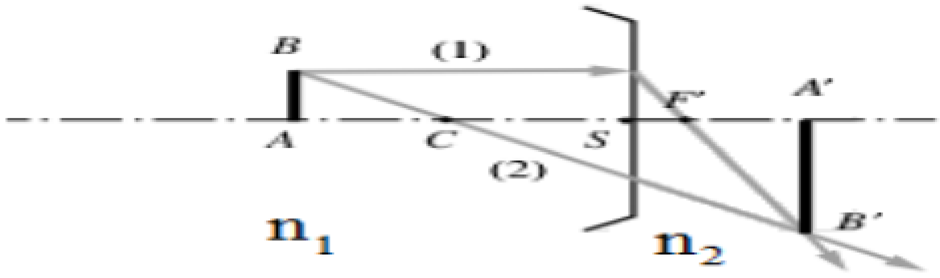


It follows from this definition that any incident ray parallel to the optical axis refracts through the image focus F'.

- **Image struction:**

Concave spherical diopter ($\overline{SC}$< 0)

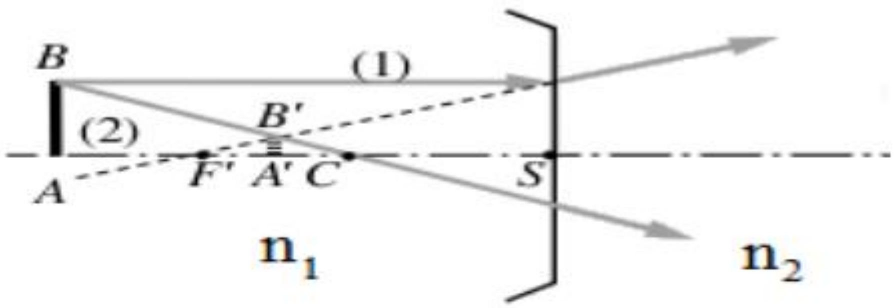
Case $n_1 > n_2$ ⟹ $\overline{SF} < 0$ and $\overline{SF'} > 0$: converging diopter



AB : real object

A'B' : Real ,inverted and enlarged image

- Case $n_1 < n_2$ ⟹ $\overline{SF} > 0$ *et* $\overline{SF'} < 0$: diopter diverging

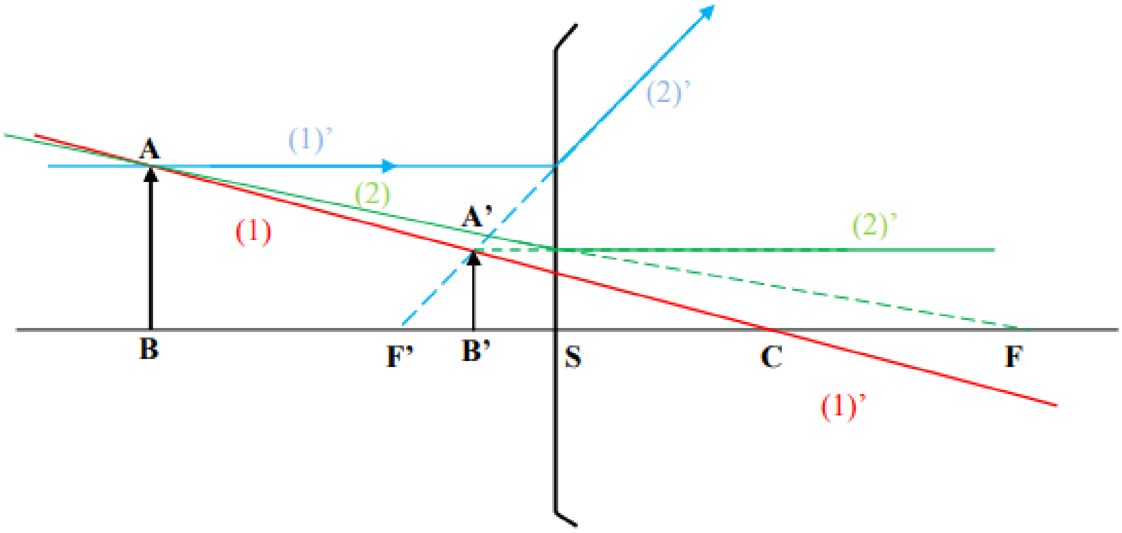


AB : real object

A'B' : virtual, Upright, and reduced image

- **Convex spherical diopter ($\overline{SC} > 0$).**

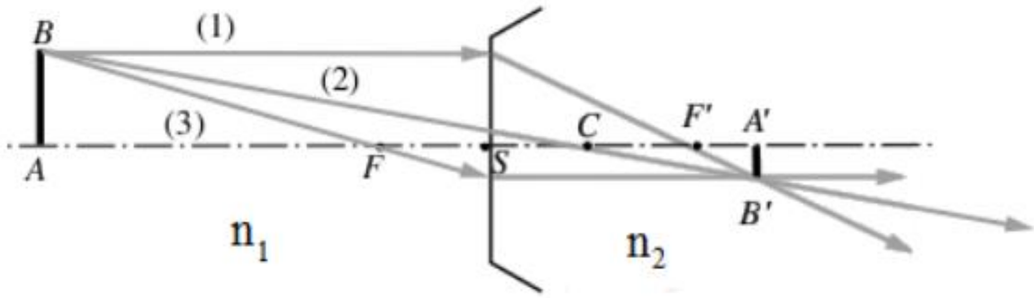
Case $n_1 > n_2$ ⟹ $(\overline{SF}) > 0$ *et* $(\overline{SF'}) < 0$: diopter diverging



AB : real object

A'B' : virtual, Upright, and reduced image

- Case $n_1 < n_2$ ⟹ $(\overline{SF}) < 0$ and $(\overline{SF'}) > 0$: converging diopter



AB : real object

A'B' : real, inverted and reduced image

We will use three specific rays for this construction :

- A ray originating from B and passing through the object focal point F: it is refracted along a line parallel to the principal axis.
- A ray passing through the center of the lens, which is not deviated upon passing through it.

- A ray originating from B and parallel to the principal axis: it is refracted along a line that passes through the image focal point.
-

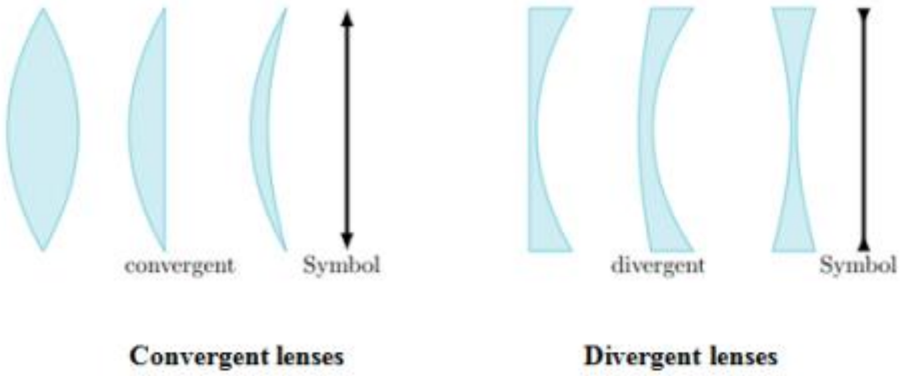
I.1.3 Thin lenses

❖ **The different types of lenses**

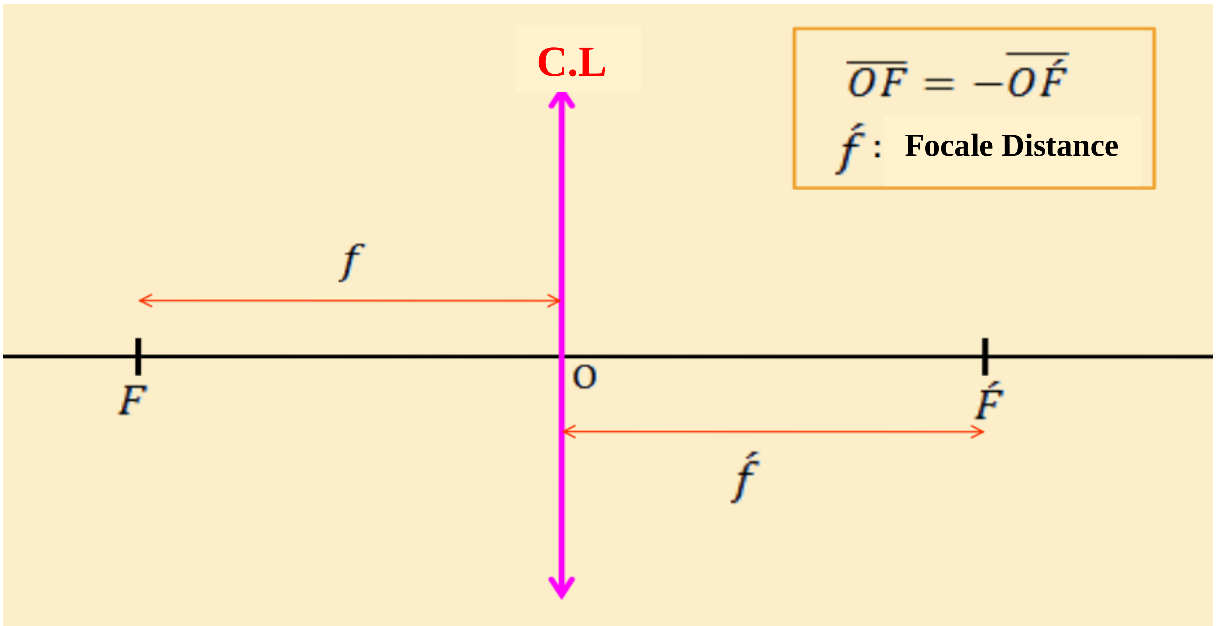
There are two families of lenses:

1. Lenses with thin edges, they are **convergent.**

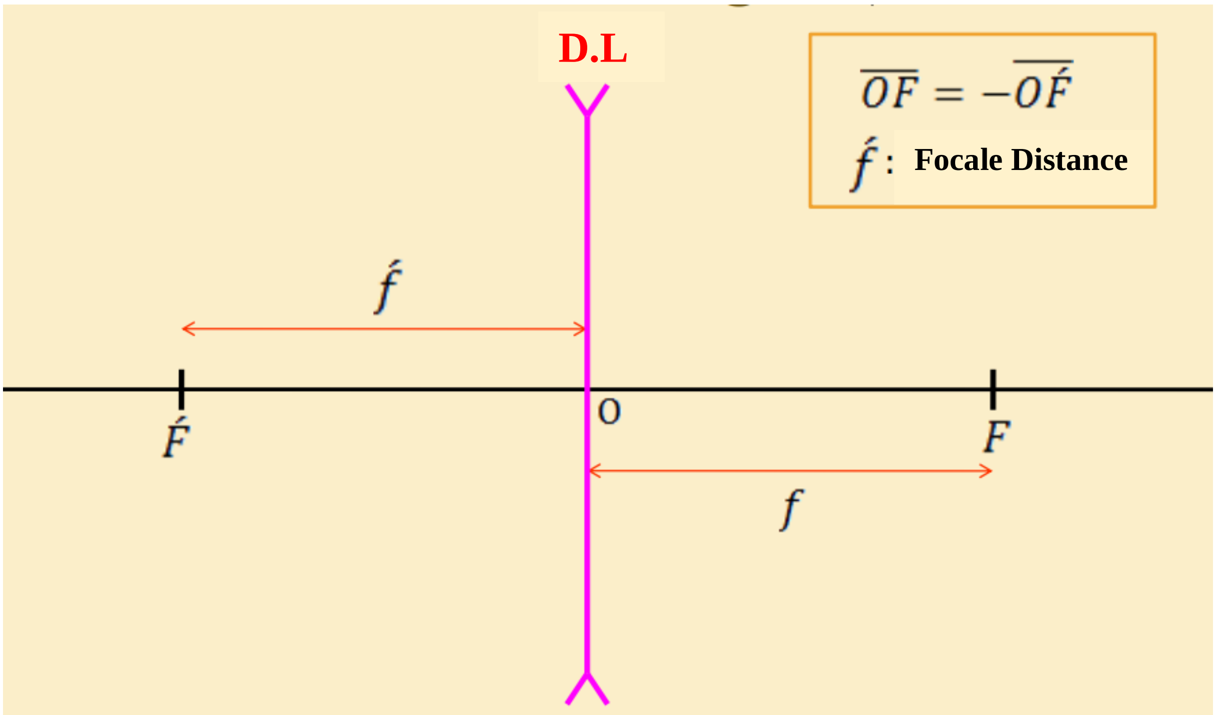
2. Lenses with thick edges, they are **divergent.**



Convergent lenses: C. L :$\acute{f}$ >0



Divergent lenses C. L :f´<0



❖ **The conjugate focal formula**

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\acute{f}}$$

$\overline{OA}$: Object position

$\overline{O\acute{A}}$: Image position

$\acute{f}$: Focal distance

❖ **Magnification γ**

$$\gamma = \frac{\overline{\acute{A}\acute{B}}}{\overline{AB}} = \frac{\overline{O\acute{A}}}{\overline{OA}}$$

$\overline{AB}$:Object size

$\overline{\acute{A}\acute{B}}$:Image size

$\boldsymbol{\gamma}$>0: Upright image

$\boldsymbol{\gamma}$ <0: Inverted image

|γ|<1: Reduced image

|γ| >1: Enlarged image

- **Vergence :**

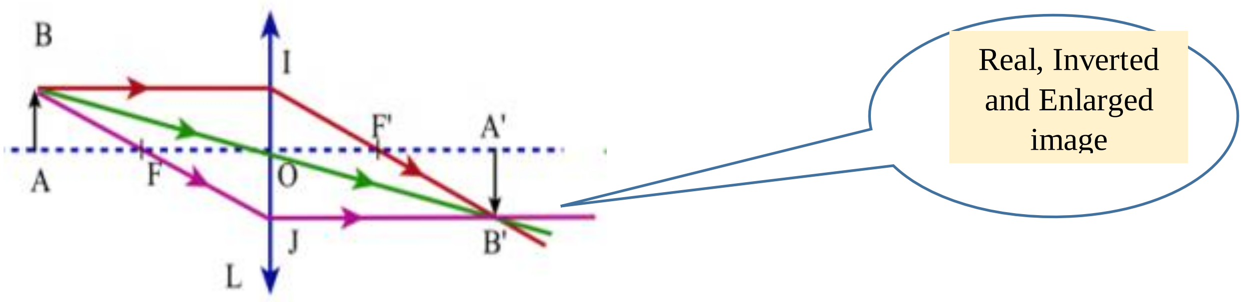
V= $\frac{\mathbf{1}}{\acute{\boldsymbol{f}}}$ (Unit: v in Diopters $\boldsymbol{\delta}$: $1\boldsymbol{\delta}= m^{-1}$)

- **Image formation**

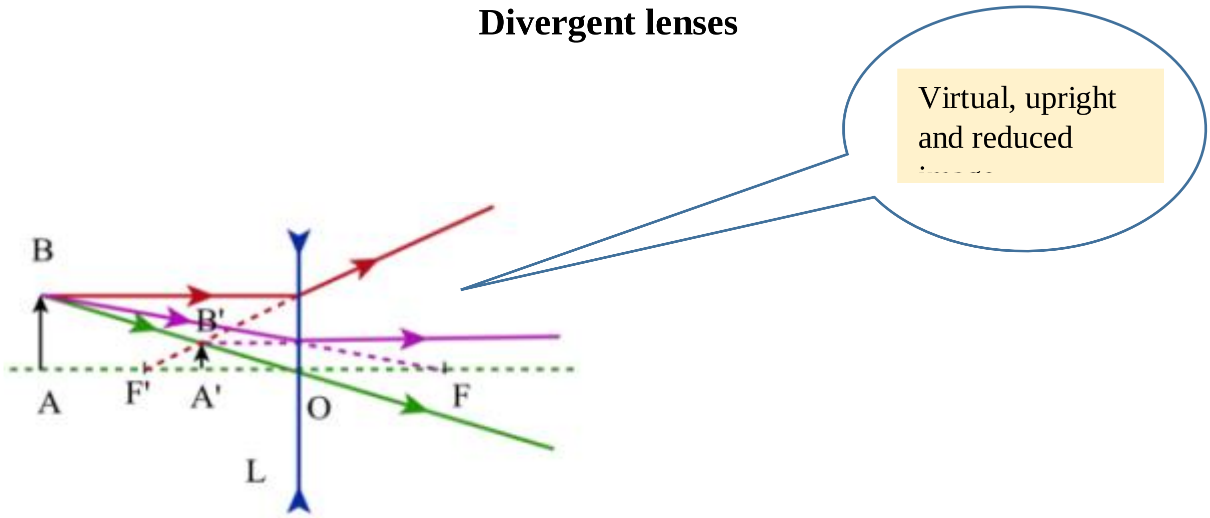
Ray construction : the construction of the image of an extended object obeys the following rules:

- We will place ourselves in the Gaussian approximation : stigmatism and planetism approximated.
- To find the image of a point, simply consider two rays coming from this point.
- The image of a point on the optical axis is also on the optical axis
- If the object is real, it is necessarily to the left of the lens.
- If the object is virtual, it is located to the right of the lens.
- A horizontal ray arriving at a lens will converge at F' if it is convergent and will diverge appearing to come from F' if the lens is divergent.
- A ray passing or extending at F will emerge horizontally.
- A ray passing through O is not deflected.
- Once the rays have been traced, we determine whether the image is real or virtual.

Convergent lenses



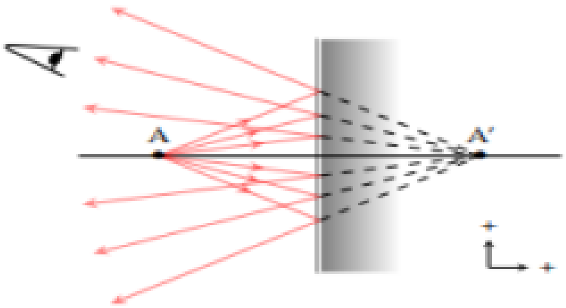
Divergent lenses



I.2 Reflection

I.2. 1 Plane mirror

Consider a source point A sending light rays onto a plane mirror. A simple construction of the reflected rays shows that all the emerging rays seem to come from a point A', virtual image of A. In the same way, if we reverse the direction of the light, we see that the image of a virtual object placed in A' is a real image placed in A. The plane mirror is therefore strictly stigmatic.



Conjugation relationship

The position of the image relative to the mirror equals the position of the object relative to the mirror. The image **A'** is symmetrical to the object **A** in relation to the mirror

$$\overline{\mathbf{HA}} = -\overline{\mathbf{HA'}}$$

Where H is the orthogonal projection of A on the mirror: the image of A is the orthogonal symmetric of A

The object and the image are of different natures :

- Real Object-Virtual Image

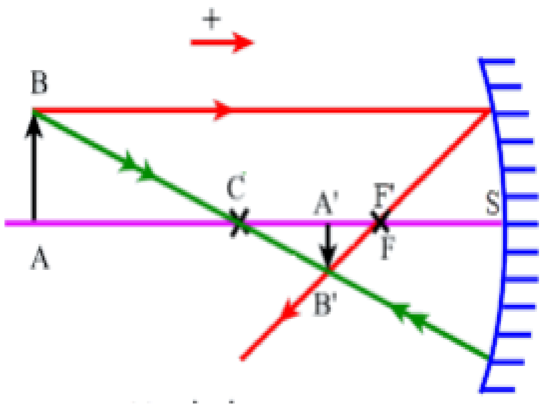
- Virtual Object-Real Image

The size of the image equals the size of the object :

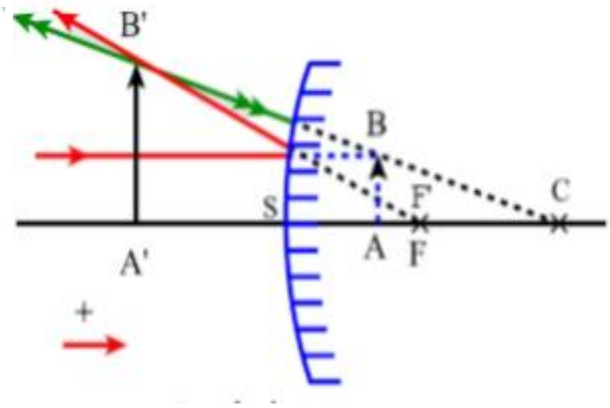
$$\overline{\mathbf{AB}} = \overline{\mathbf{A'B'}}$$

I.2.2 Spherical Mirror

A spherical mirror is a reflective spherical surface with a center **C**, a vertex **S**, and a radius of curvature **R**. There are two types of spherical mirrors : Concave ($\boldsymbol{R}$ = ($\overline{SC}$) < 0) and Convex ($\boldsymbol{R}$ = ($\overline{SC}$) > 0)



Concave mirrors



Convex mirrors

The conjugate relationship for a spherical Mirror:

$$\frac{1}{\overline{SA}} + \frac{1}{\overline{SA'}} = \frac{2}{\overline{SC}}$$

The magnification $\boldsymbol{\gamma}$of a spherical mirror :

$$\Upsilon = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

The focal distances $\overline{\overline{SF}}$et $\overline{S\acute{F}}$ have the following expressions :

$$\overline{SF} = \overline{SF'} = \frac{\overline{SC}}{2}$$

I.3 Optical Instruments

I.3. 1 the eye

- **Eye modeling**

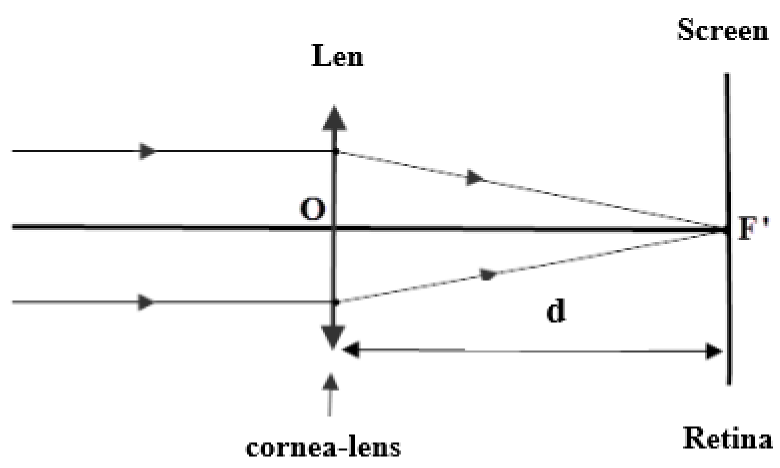
The eye can be modeled as a converging lens (comprising the cornea and lens) with a variable focal length to accommodate the position of objects and form their images on the retina. The distance between this lens and the retina is called the depth of the eye. For a normal or emmetropic eye, the focal length is equal to the depth of the eye. The furthest point of distinct vision (the Punctum Remotum) is at infinity. The closest point of distinct vision (the Punctum Proximum) is at the minimum distance $d_m = 25\ cm$ from the lens.

The amplitude of accommodation is defined A by:

$$A = \frac{1}{\overline{PR}} - \frac{1}{\overline{PP}}$$

$\overline{PP}$ is the distance between the lens and the Punctum Proximum PP.

$\overline{PR}$ is the distance between the lens and the Punctum Remotum PR.



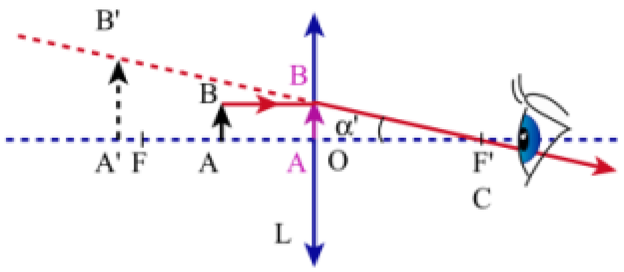
❖ **Eye Defects:**

Among the eye defects, those resulting from refractive errors are corrected by appropriate lenses. Among these defects:

- ✓ **Myopia:** a nearsighted eye is an eye that is too convergent, in this case $O\acute{F} < d$ ($\acute{F}$ is in front of the retina.
- ✓ **Hypermetropia:** A hypermetropic eye is an eye that is not convergent enough OF $\acute{>}$ d($\acute{F}$ is behind the retina).
- ✓ **Presbyopia:** is the loss of accommodation power with age.
- ✓ **Astigmatism:** the focal length is not the same in all directions of observation.

I.3. 2The magnifying glass :

The magnifying glass is a converging lens that enlarges an object **AB**. For this purpose, **AB** it must be placed between the optical center **O** and the focal point F of the lens.



The algebraic distance $\overline{\acute{O}A}$of an image $\acute{A}\acute{B}$ from an object **AB** is calculated using the conjugate relationship:

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}}$$

The magnification γ of the magnifying glass is calculated by the equation:

$$\Upsilon = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$

The power **P** of the magnifying glass is defined by the equation:

$$P = \frac{tan\alpha'}{AB}$$

$\acute{\alpha}$ is the angle with which the observer sees the image $\acute{A}\acute{B}$. The unit of P is the diopter (δ).

The magnification of the magnifying glass is :

$$G = \frac{tan\alpha'}{tan\alpha}$$

α The angle under which we observe the image is A'B'.

$\acute{\alpha}$ is the angle under which we observe the object AB from a distance d_m where d_m represents the minimum distinct vision distance : $d_m = 25cm = 0{,}25m$

Relation between G and P: $\boldsymbol{G = p.d_m}$

I.1. 3 Microscope

The optical microscope consists of two converging thin lenses.

- The objective with a focal distance $\mathbf{O_1\acute{F}_1}$ of a few millimeters produces a real, enlarged, and inverted image $\mathbf{A_1B_1}$ of the object. The magnification of the objective **AB** is defined by

$$\Upsilon = \frac{\overline{A_1B_1}}{\overline{AB}} = \frac{\overline{O_1A_1}}{\overline{O_1A}}$$

- The eyepiece (ocular) with a focal distance $\boldsymbol{O_2\acute{F}_2}$ of a few centimeters functions like a magnifying glass. It allows observing a virtual, enlarged image $\boldsymbol{\acute{A}\acute{B}}$ projected to infinity from $\mathbf{A_1B_1}$. For this to happen, the image $\boldsymbol{A_1B_1}$ of the object **AB** must be located in the object focal plane of the eyepiece. The point $\mathbf{A_1}$ must, therefore, coincide with the object focal $\mathbf{F_2}$ point of the eyepiece.
- The eyepiece power of an optical microscope is defined by the equation :

$$p_O = \frac{1}{O\acute{f}_2}$$

❖ The power of the microscope

$$P_m = \gamma . P_0$$

❖ The magnification of the microscope

$$G = P_m . d_m$$