

Solution TD: 3

$$a) \frac{5x^2 + 17x + 15}{(x+2)^2(x+1)} = \frac{a}{(x+2)} + \frac{b}{(x+2)^2} + \frac{c}{x+1} = \frac{2}{x+2} + \frac{-1}{(x+2)^2} + \frac{3}{x+1}$$

$$= \frac{a(x+2)(x+1) + b(x+1) + c(x+2)^2}{(x+2)^2(x+1)}$$

$$= \frac{(a+c)x^2 + (3a+b+4c)x + (2a+b+4c)}{(x+2)^2(x+1)}$$

$$\begin{cases} a+c=5 \\ 3a+b+4c=17 \\ 2a+b+4c=15 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=-3 \\ c=3 \end{cases}$$

$$b) \frac{x}{(x-3)^2(2x+1)} = \frac{a}{x-3} + \frac{b}{(x-3)^2} + \frac{c}{2x+1}$$

$$\Rightarrow \frac{a(x-3)(2x+1) + b(2x+1) + c(x-3)^2}{(x-3)^2(2x+1)}$$

$$= \frac{(2a+c)x^2 + (-5a+2b-bc)x + (-3a+b+9c)}{(x-3)^2(2x+1)}$$

$$\begin{cases} 2a+c=0 \\ -5a+2b-bc=1 \\ -3a+b+9c=0 \end{cases} \Rightarrow \begin{cases} a=\frac{1}{49} \\ b=\frac{21}{49} \\ c=-\frac{2}{49} \end{cases}$$

$$\Rightarrow \frac{x}{(x-3)^2(2x+1)} = \frac{1}{49} \left(\frac{1}{x-3} + \frac{21}{(x-3)^2} + \frac{-2}{2x+1} \right)$$

$$\text{Exo 4: } \frac{x^2 - 3x - 7}{(x^2 + x + 2)(2x - 1)} = \frac{ax + b}{x^2 + x + 2} + \frac{c}{2x - 1}$$

$$= \frac{(2a+c)x^2 + (-a+2b+c)x + (-b+2c)}{(x^2 + x + 2)(2x - 1)}$$

$$\Rightarrow \begin{cases} 2a+c = 1 \\ -a+2b+c = -3 \\ -b+2c = -7 \end{cases} \Rightarrow \begin{cases} a = \frac{36}{11} \\ b = \frac{-45}{11} \\ c = \frac{-61}{11} \end{cases}$$

$$\Rightarrow \frac{x^2 - 3x - 7}{(x^2 + x + 2)(2x - 1)} = \frac{1}{11} \left(\frac{36x - 45}{x^2 + x + 2} + \frac{-61}{2x - 1} \right)$$

you can complete the rest like the same way.

$$\text{Exo 5: a) } \frac{x^3 + 1}{x^2 + 1} = C(x) + \frac{R(x)}{x^2 + 1}$$

$$\begin{array}{r} x^2 + 1 \overline{) x^3 + 1} \\ \underline{-x^3 - x} \\ -x + 1 \end{array}$$

$$\Rightarrow \frac{x^3 + 1}{x^2 + 1} = x + \frac{-x + 1}{x^2 + 1}$$