Series of exercises N 3

Exercise 1

Determine the domain of definition of each of the following functions

❶ $f(x,y) = \sqrt{1 - x^2 - y^2}$

❷ $f(x,y) = \dfrac{x^2 + y^2}{x + y}$

❸ $f(x,y) = \dfrac{x^2y^2}{x^2 + y^2}$

❹ $f(x,y) = x^2 + y + \ln(x^2 + y^2)$

Exercise 2

Calculate the partial derivatives of order 1 of the following functions

❶ $f(x,y) = e^x \tan y$

❷ $f(x,y) = x^3 + y^3 - 3xy$

❸ $f(x,y) = (x^2 + y^2)\sin(xy)$

❹ $f(x,y) = \sqrt{1 + x^2y^2}$.

Exercise 3

We consider the real function of two variables f defined by

$$f(x,y) = \frac{x^2}{y - 2x^2}$$

❶ Determine the domain of definition of f

❷ Calculate the gradient of f at the point $(1,1)$.

Exercise 4

Calculate the partial derivatives of order 2 of the following functions

❶ $f(x,y) = xe^{xy}$

❷ $f(x,y) = x^2(x - y)$

❸ $f(x,y) = \ln\left(x + \sqrt{x^2 + y^2}\right)$

❹ $f(x,y) = x^4 + y^3 + 2y\cos(x) + 5y$.

Exercice 5

Calculate the following double integrals

❶ $I_1 = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x+y)\,dx\,dy$

❷ $I_2 = \int_1^2 \int_{-1}^1 \frac{x^2}{y}\,dx\,dy$

❸ $I_3 = \int_0^2 \int_0^1 y\,\frac{e^{2x+y^2}}{1+e^x}\,dx\,dy$

❹ $I_4 = \iint_D (x+y)\,dx\,dy$ with $D = \left\{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le y, \quad x^2+y^2 \le 1\right\}$

❺ $I_5 = \iint_D \frac{1}{(x+y)^3}\,dx\,dy$ with $D = \left\{(x,y) \in \mathbb{R}^2 \mid x \ge 1, \;\; y \ge 1, \quad x+y \le 3\right\}$.

Exercise 6

Calculate the following triple integrals

❶ $I_1 = \iiint_P \frac{x^2 y}{z}\,dx\,dy\,dz$ with $P = [0,1] \times [0,1] \times [1,2]$

❷ $I_2 = \iiint_P f(x,y,z)\,dx\,dy\,dz$

with $f(x,y,z) = 1$ and $P = \left\{(x,y,z) \in \mathbb{R}^3 \mid x,y,z \ge 0, \;\; x+y+2z \le 1\right\}$.