Exercises on Hypothesis Testing

Exercice 1. *Let $X \sim$ Geometric(θ). We observe X and we need to decide between*

$H_0 : \theta = \theta_0 = 0.5$,

$H_1 : \theta = \theta_1 = 0.1$.

Design a level 0.05 test ($\alpha = 0.05$) to decide between H_0 and H_1. Find the probability of type-II error β.

Exercice 2. *Let X_1, X_2, X_3, X_4 be a random sample from a $N(\mu, 1)$ distribution, where μ is unknown. Suppose that we have observed the following values $2.82, 2.71, 3.22, 2.67$. We would like to decide between*

$H_0 : \mu = \mu_0 = 2$,

$H_1 : \mu \neq 2$.

Assuming $\alpha = 0.1$, do you accept H_0 or H_1? If we require significance level α, find β as a function of μ and α.

Exercice 3. *Let $X_1, X_2, \ldots, X_{100}$ be a random sample from an unknown distribution. After observing this sample, the sample mean and the sample variance are calculated to be*

$\bar{X} = 21.32, S^2 = 27.6$.

Design a level 0.05 test to choose between

$H_0 : \mu = 20$,

$H_1 : \mu > 20$.

Do you accept or reject H_0?

Exercice 4. *Let X_1, X_2, X_3, X_4 be a random sample from a $N(\mu, \sigma^2)$ distribution, where μ and σ are unknown. Suppose that we have observed the following values $3.58, 10.03, 4.77, 14.66$. We would like to decide between*

$H_0 : \mu \geq 10$,

$H_1 : \mu < 10$.

Assuming $\alpha = 0.05$, do you accept H_0 or H_1?

Exercice 5. *Let $X_1, X_2, \ldots, X_{81}$ be a random sample from an unknown distribution. After observing this sample, the sample mean and the sample variance are calculated to be $\bar{X} = 8.25, S^2 = 14.6$.*

Design a test to decide between

$H_0 : \mu = 9$,

$H_1 : \mu < 9$,

and calculate the P-value for the observed data.