Exercises on Determining Interval Estimators

Correction exercice 1. *The* 95% *confidence interval for* μ *is given by:*

$$\bar{X} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}}$$

where $\bar{X}$ *is the sample mean,* σ *is the known population standard deviation, and* n *is the sample size. In this case,* 1.96 *corresponds to the* 97.5*th percentile of the standard normal distribution.*

Correction exercice 2. *The* 99% *confidence interval for* μ *is given by:*

$$\bar{X} \pm 2.576 \cdot \frac{\sigma}{\sqrt{n}}$$

where $\bar{X}$ *is the sample mean,* σ *is the known population standard deviation, and* n *is the sample size. In this case,* 2.576 *corresponds to the* 99.5*th percentile of the standard normal distribution.*

Correction exercice 3. *The* 90% *confidence interval for* μ *is given by:*

$$\bar{X} \pm z \cdot \frac{S}{\sqrt{n}}$$

where $\bar{X}$ *is the sample mean,* S *is the sample standard deviation,* n *is the sample size, and* z *is the critical value from the t-distribution with* $n - 1$ *degrees of freedom that corresponds to the desired confidence level.*

The general formula for the confidence interval is:

$$\bar{X} \pm z \cdot \frac{S}{\sqrt{n}}$$

Correction exercice 4. *The* 98% *confidence interval for* σ^2 *is given by:*

$$\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right)$$

where S^2 *is the sample variance,* n *is the sample size,* $\alpha = 0.02$*, and* $\chi^2_{\alpha/2}$ *and* $\chi^2_{1-\alpha/2}$ *are the* $\alpha/2$ *and* $1-\alpha/2$ *percentiles of the chi-squared distribution with* $n-1$ *degrees of freedom, respectively.*

Correction exercice 5. *The* 95% *confidence interval for the ratio* $\frac{\sigma_1^2}{\sigma_2^2}$ *is given by:*

$$\left(\frac{S_1^2}{S_2^2} \cdot \frac{1}{F_{\alpha/2}}, \frac{S_1^2}{S_2^2} \cdot F_{\alpha/2}\right)$$

where S_1^2 *and* S_2^2 *are the sample variances,* n_1 *and* n_2 *are the sample sizes,* $\alpha = 0.05$*, and* $F_{\alpha/2}$ *and* $F_{1-\alpha/2}$ *are the* $\alpha/2$ *and* $1-\alpha/2$ *percentiles of the F-distribution with* $n_1 - 1$ *and* $n_2 - 1$ *degrees of freedom, respectively.*

Correction exercice 6. *The* 90% *confidence interval for* μ *is given by:*

$$\bar{X} \pm 1.645 \cdot \frac{\sigma}{\sqrt{n}}$$

where $\bar{X}$ *is the sample mean,* σ *is the known population standard deviation,* n *is the sample size, and* 1.645 *corresponds to the* 95*th percentile of the standard normal distribution.*

In this context, the confidence interval represents a range of values within which we are 90% *confident that the true mean time* μ *falls. This means that if we were to take many random samples and construct confidence intervals from them, about* 90% *of those intervals would contain the true mean time.*