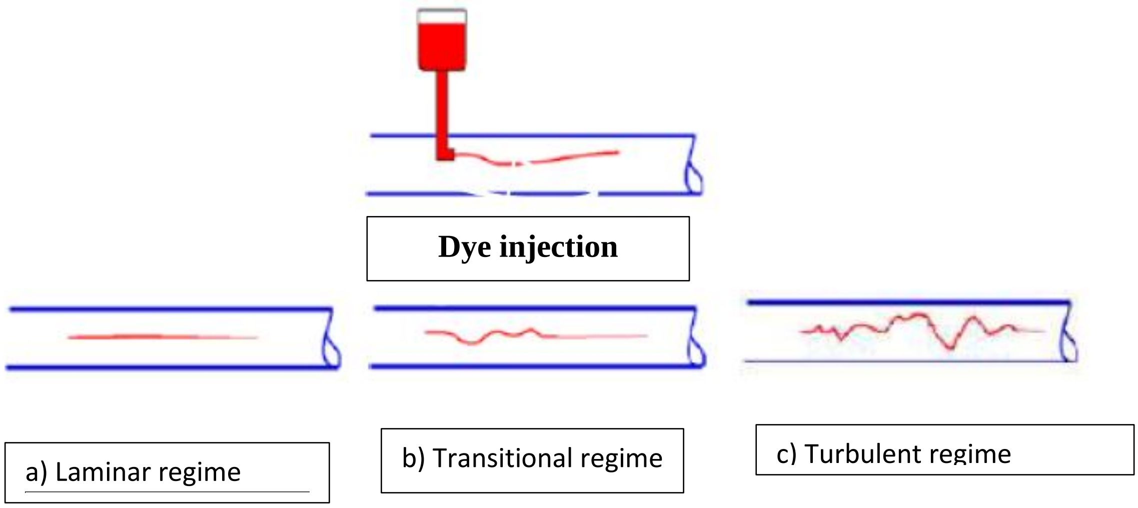
Chapter 3: Fluid dynamics: Stresses and strains in continuous media; Equation of motion of real fluids; Flow regime ,Applications of the Navier and Stockes equations (poiseuille flow, couette flow, free surface flow)

In the previous chapter we assumed that the fluid was perfect for apply the energy conservation equation. The flow of a real fluid is more complex than that of an ideal fluid. Indeed, there are frictional forces, due to the viscosity of the fluid, which acts between the fluid particles and the walls, as well as between the particles themselves. To solve a flow problem of a real fluid, we call on experimental results, in particular those of British engineer and physicist Osborne Reynolds. A simplified method for calculating pressure losses based on these experimental results is proposed. It is essential for the sizing of various installations hydraulics (pumping, turbines, hydraulic and thermal machines in which carries a real fluid…etc.)

3.1 Flow regimes, Reynolds experiment

Knowledge of the flow regime of a fluid is a key point in process engineering, because it has an influence on most phenomena, in particular the transfer of heat, material, load losses etc.

In 1883 Osborne Reynolds (1842-1912) professor of engineering at the University of Manchester carried out experiments during the flow of a fluid in a pipe rectilinear cylindrical. By injecting a dye onto the axis of the pipe, he noticed that at low speed, the dye remains concentrated very close to the axis, characteristic of a flow stable laminar (a), then at greater speed/flow, vortex structures appear form, more and more energetic, causing rapid diffusion due to turbulence (b and c) which largely takes over the barely observable molecular diffusion at low speed.



By using various fluids (different viscosity), varying the flow rate and diameter of the pipeline, Reynolds showed that the parameter which made it possible to determine whether flow is laminar or turbulent is a dimensionless number called the number of Reynolds . It expresses the relationship between the inertia force. $\rho\mu$ and the viscous force $\mu*\frac{u}{d}$

He is given by the following relation:

$$Re = \frac{\rho . u . d}{\mu} = \frac{u . d}{\vartheta}$$

U: Average flow speed in /d

d: Diameter of the pipe in

µ: Dynamic viscosity of the fluid in 1:_d

v: Kinematic viscosity of the fluid in _/d.

If Re < 2000: the flow is laminar.

If 2000 <Re <3000: the flow is transient.

If Re > 3000: the flow is turbulent.

These values may vary slightly from one work to another, but in practice, the values leave no ambiguity. They will be clearly higher or lower than these limits. In turbulent flows, the value of is very important and can reach 10^5 up to 10^8.

3.2 Stresses and strains in continuous media

Stress:

1. **Normal Stress (σ):** It's the force per unit area perpendicular to the material's section. Defined as the force F applied over an area A:

$$\sigma = F / A.$$

2. **Shear Stress (τ):** It's the force per unit area parallel to the material's section. It occurs in shear situations and can be defined as

$$\tau = F / A.$$

Strain:

1. **Normal Strain (ε):** the relative change in length per unit length in the direction of the applied force. For a linear elastic material, strain is directly proportional to the applied stress by Hooke's Law:

$$\varepsilon = \Delta L / L = \sigma / E$$

   where E is the Young's modulus.

2. **Shear Strain:** It occurs in shear situations and is defined as the relative angular change per unit length in the direction of the applied force.

Constitutive Laws:

1. Hooke's Law:For linear elastic materials, stress is proportional to strain. It's expressed as

$$\sigma = E \times \varepsilon,$$

   where E is the Young's modulus, a constant characterizing the material's stiffness.

2. **Nonlinear Laws:** Some materials exhibit nonlinear behaviors where the relationship between stress and strain is not linear. More complex laws can be used to model these behaviors, such as the plastic behavior law for plastic materials or the viscoelastic behavior law for viscoelastic materials.

Practical Applications:

These concepts are applied in various fields such as structural engineering, aerospace, material manufacturing, and more to design structures and materials that withstand stress while maintaining acceptable levels of deformation.

University courses in mechanical engineering, civil engineering, or materials science delve into these concepts using practical examples, laboratory experiments, and numerical simulations to illustrate basic principles and advanced applications in these fields.

3.3 **Navier -stoks Equations**

The Navier -Stokes equation describe the movement of viscous fluids and are given by :

1.Eqaution of conservation of momentum (or Navier -stoks Equations):

For an incompressible fluid , it is often written in vector form for each dimension (X,Y,Z) and time (t)

$$\rho \ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \ = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

Where

ρ:is the fluide density

v: is the fluid velocity field

p is the pressure

μis the dynamic viscosity of the fluid

f represents external forces per unit mass applied to the fluid

2. Mass conservation equation
   For an incompressible fluid , it's expressed by the continuity equation

$$\nabla \cdot \mathbf{v} = 0$$

   Where (nabla) is the divergence operator and v is the fluid velocity field

3.4: Internal flows

3.4.1 : Introduction :

Internal flows refer to the movement of fluids inside conduits, pipes or canals where the fluid is confined by solid walls. These flows are essential in many areas such as engineering, medicine and natural sciences. Here is an introduction to internal flows

3.4.2 Characteristics of internal flows:

Confainment by solid walls: unlike external flows, internal flows take place within conduits or canals, which distinguishes them by the presence of walls which define the geometry of the passage.

Effects of viscosity: In internal flows, the viscosity of the fluid plays a crucial role. It influences the way in which the fluid interacts with the walls, causing friction and shear effects.

Laminary and turbulent flows: depending on the conditions, the geometry of the conduit and the speed of the fluid, the internal flows can be laminar (ordered and regular) or turbulent (chaotic and unstable), thus influencing the distribution of the speed and the structure of the 'flow.

Fluid resistance: internal flows generate resistance to the movement of fluid due to friction along the walls. This resistance is often characterized by load losses.

3.4.3 Types of internal flows:

Slow in conduits: Like flow in pipes, industrial pipes or pipes in hydraulic systems.

Slows between flat surfaces: like the flow of duvet, where the fluid moves between two parallel plates.

Free surface flows: like flows in rivers, canals or flows in tanks and dams

3.4.4 Importance and applications:

Internal flows are fundamental for the design of fluid transport systems, industrial processes, blood circulation in the human body, climatology, meteorology and many other areas. Understanding these flows optimizes systems and predicting performance under various conditions, which is essential for many practical applications.

3.4.5 Poiseuille flow:

Description: Poisuille flow occurs in a cylindrical duct when the fluid flows in a laminar manner under the effect of a pressure difference. It is characterized by higher speeds in the center of the duct and lower speeds near the walls.

Relationship: Poiseuille equation for an incompressible stationary flow in a radius cylindrical duct R and length L is:

$$Q = \frac{\pi R^4}{8\mu} \frac{\Delta P}{L}$$

Where
Q is the volume flow,
μ is the viscosity of the fluid,
ΔP is the pressure difference applied,
R is the radius of the conduit,
L is the length of the duct
This relationship shows that flow ,Q is proportional to the fourth power of the radius ,R of the duct and with the difference in pressure, ΔP, but inversely proportional to the viscosity, μ and the length , L of the conduit.

3.4.6Couette flow:

Description: couette flow occurs between two parallel plates where one is fixed and the other moves at a constant speed compared to the first. This flow leads to a linear gradient of the speed of the fluid between the plates.

Relationship: The equation for the flow of duvet in a flat geometry is:

$$u(y) = \frac{U}{h} \cdot y$$

Where;
u (y) is the speed of fluid at a distance y of the fixed plate,

U is the movement speed of the mobile plate,

H is the distance between the two plates.

This relationship shows that the speed of the fluid varies linearly with the distance y from the fixed plate, with a zero speed in contact with the plate and a maximum speed to the mobile plate.

3.4.7 Free surface flow:

Description: Free surface flows occur when the fluid is in contact with air and its surface is not confined by solid walls. For example, rivers, canals or flows in dams.

Relationship: the equations to model free surface flows, such as the Saint-Venant equation for permanent flows in open channels, are more complex and often depend on specific geometry and flow conditions.

These equations integrate terms representing gravity, resistance to friction, the shape of the watercourse bed and other factors to model the movement of fluid in free surface situations. They are essential to predict water levels, flow speeds and characteristics of flows in these unconfined environments.