Exercices

Exercice .1. ***(Linear Regression)***
Consider the following data points:

$$(1,3),(2,5),(3,7),(4,9),(5,11)$$

1. *Fit a linear model $y = mx + b$ using the least squares method.*
2. *Find the values of m and b that minimize the sum of squared differences between the observed and predicted values of y.*

Exercice .2. ***(Polynomial Regression)***
Given the data points:

$$(1,2),(2,5),(3,10),(4,17),(5,26)$$

1. *Fit a polynomial model $y = a_0 + a_1x + a_2x^2$ using the least squares method.*
2. *Determine the coefficients a_0, a_1, and a_2 that minimize the sum of squared differences.*

Exercice .3. ***(Exponential Regression)***
Given the data points:

$$(1,2),(2,4),(3,8),(4,16),(5,32)$$

1. *Fit an exponential model $y = ae^{bx}$ using the least squares method.*
2. *Find the values of a and b that minimize the sum of squared differences.*

Exercice .4. ***(Multiple Linear Regression)***
Consider the dataset with three variables:

$$(1,2,5),(2,4,8),(3,6,10),(4,8,12),(5,10,14)$$

1. *Fit a multiple linear regression model $y = \beta_0 + \beta_1x_1 + \beta_2x_2$ using the least squares method.*
2. *Find the coefficients β_0, β_1, and β_2 that minimize the sum of squared differences.*

Exercice .5. ***(Bias, Mean Squared Error, and Convergence)***

1. *Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with unknown mean μ and variance σ^2. Define an estimator $\hat{\mu}$ for μ. Calculate the bias of $\hat{\mu}$ and determine if it's an unbiased estimator.*
2. *Given a random variable X with known mean μ and variance σ^2, define the estimator $\hat{\sigma}^2$ for σ^2. Calculate the mean squared error of $\hat{\sigma}^2$.*
3. *Consider a sequence of estimators $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_n$ for a parameter θ. Show that $\hat{\theta}_n$ converges in probability to θ as n approaches infinity.*

Exercice .6. ***(Fisher Information, Cramer-Rao Bound, Efficiency)***

1. *Given a random sample $X_1, X_2, \ldots, X_n$ from a distribution with probability density function $f(x;\theta)$, derive the Fisher information $I(\theta)$ for the parameter θ.*

2. *For a given parameter θ, compute the Cramer-Rao Lower Bound (CRLB) for the variance of any unbiased estimator of θ, based on the Fisher information obtained in the previous exercise.*

3. *Suppose you have two unbiased estimators, $\hat{\theta}_1$ and $\hat{\theta}_2$, for the same parameter θ. Calculate the efficiency of $\hat{\theta}_1$ with respect to $\hat{\theta}_2$, and determine which estimator is more efficient.*

Exercice .7. ***(Efficiency)*** *Suppose you have two unbiased estimators, $\hat{\theta}_1$ and $\hat{\theta}_2$, for the same parameter θ. Calculate the efficiency of $\hat{\theta}_1$ with respect to $\hat{\theta}_2$, and determine which estimator is more efficient.*

Exercice .8. ***(Completeness)***

1. *Show that a random sample $X_1, X_2, \ldots, X_n$ from a distribution with a parameter θ has a complete sufficient statistic.*

2. *Given a continuous distribution with an unknown parameter θ, find a statistic that is minimal sufficient but not complete.*