University center of Mila
Department of Mathematics

2023/2024
M2-S03

Algebra And Coding
Work Sheet 01

Exercice 1. Let R be a ring and $(I_\lambda)_{\lambda \in L}$ family of ideals of R.
Proof that $I = \bigcap\limits_{\lambda \in L} I_\lambda$ is an ideal of R.

Exercice 2. Let $m, n \in \mathbb{N}$ such that $(m, n) = 1$. Show that it exists non-trivial $\overline{x} \in \mathbb{Z}/_{m \cdot n\mathbb{Z}}$ such that

$$\overline{x}^2 = \overline{x}.$$

Indication : use Bezout theorem.

Exercice 3. Let R be a commutative ring and, I,J,K are ideals of R. Proof that

(a) $I \cdot (J + K) = I \cdot J + I \cdot K$.

(b) $I + J = A$ so $I \cdot J = I \cap J$.

Exercice 4. Let A,B,C be three rings and, $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are homomorphisms of rings, we have

(a) $g \circ f$ is an homomorphism.

(b) If f is an isomorphism, so f^{-1} is an isomorphism.

Exercice 5. let R be an integral domain of characteristic p. So $p = 0$ or p is a prime number.

Exercice 6. let R be a principal domain and $a_1, \cdots, a_n \in R$. The following properties are equivalents.

(a) gcd$(a_1, \cdots, a_n) = 1$.

(b) There are elements $u_1, \cdots, u_n \in R$ such that

$$a_1u_1 + \cdots +, a_nu_n = 1.$$

Exercice 7. Let $a \in \mathbb{Z}$ and $\mathbb{Z}/_{n\mathbb{Z}}, n \in \mathbb{N}$ is the quotient ring, we have

(a) $\overline{a}$ is invertible iff gcd$(a, n) = 1$.

(b) $\mathbb{Z}/_{n\mathbb{Z}}$ is a field iff n is prime.

Exercice 8. Let M be an ideal of R. M is maximal iff $R/_M$ is a field.

Exercice 9. Let K $\neq \{0\}$ be a commutative ring. K is a field, iff the only ideals of K are $\{0\}$ and K.