

TABLE 7 Laplace transforms of causal functions

| no. | $f(t)$            | $F(s) = (\mathcal{L}f)(s)$<br>$= \int_0^\infty f(t)e^{-st}dt$ | half-plane of<br>convergence                  | condition(s)          |
|-----|-------------------|---|---|-----------------------|
| 1   | 1                 | $\frac{1}{s}$   | $\operatorname{Re} s > 0$                     |                       |
| 2   | $e^{at}$          | $\frac{1}{s-a}$   | $\operatorname{Re} s > \operatorname{Re} a$   | $a \in \mathbb{C}$    |
| 3   | $t^n$             | $\frac{n!}{s^{n+1}}$  | $\operatorname{Re} s > 0$                     |                       |
| 4   | $\sin at$         | $\frac{a}{s^2 + a^2}$   | $\operatorname{Re} s > 0$                     | $a \in \mathbb{R}$    |
| 5   | $\cos at$         | $\frac{s}{s^2 + a^2}$   | $\operatorname{Re} s > 0$                     | $a \in \mathbb{R}$    |
| 6   | $\sinh at$        | $\frac{a}{s^2 - a^2}$   | $\operatorname{Re} s > a$                     | $a \in \mathbb{R}$    |
| 7   | $\cosh at$        | $\frac{s}{s^2 - a^2}$   | $\operatorname{Re} s > a$                     | $a \in \mathbb{R}$    |
| 8   | $\sin(at + b)$    | $\frac{a \cos b + s \sin b}{s^2 + a^2}$                       | $\operatorname{Re} s > 0$                     | $a, b \in \mathbb{R}$ |
| 9   | $\cos(at + b)$    | $\frac{s \cos b - a \sin b}{s^2 + a^2}$                       | $\operatorname{Re} s > 0$                     | $a, b \in \mathbb{R}$ |
| 10  | $t^n e^{-at}$     | $\frac{n!}{(s+a)^{n+1}}$                                      | $\operatorname{Re} s > \operatorname{Re}(-a)$ | $a \in \mathbb{C}$    |
| 11  | $\epsilon(t - a)$ | $\frac{e^{-as}}{s}$   | $\operatorname{Re} s > 0$                     | $a > 0$               |

TABLE 8 Properties of the Laplace transform of causal functions

| no. | $f(t), g(t)$  | $F(s), G(s)$                                  | $condition(s)$        |
|-----|---|---|-----------------------|
| 1   | $af(t) + bg(t)$   | $aF(s) + bG(s)$                               | $a, b \in \mathbb{C}$ |
| 2   | $\epsilon(t-a)f(t-a)$   | $e^{-as}F(s)$                                 | $a \geq 0$            |
| 3   | $e^{at}f(t)$  | $F(s-a)$                                      | $a \in \mathbb{C}$    |
| 4   | $f(at)$   | $a^{-1}F(a^{-1}s)$                            | $a > 0$               |
| 5   | $f^{(n)}(t)$  | $s^n F(s)$                                    |                       |
| 6   | $(-1)^n t^n f(t)$   | $F^{(n)}(s)$                                  |                       |
| 7   | $\int_0^t f(\tau) d\tau$  | $\frac{F(s)}{s}$                              |                       |
| 8   | $(f * g)(t)$  | $F(s)G(s)$                                    |                       |
| 9   | $f(t)$  | $\frac{\int_0^T f(t)e^{-st} dt}{1 - e^{-sT}}$ | $f(t+T) = f(t)$       |
| 10  | $\lim_{s \rightarrow \infty} sF(s) = f(0+)$                       |   |                       |
| 11  | $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$ |   |                       |