Democratic and Popular Republic of Algeria

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Analysis 1

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Exercises Serie N° 4

Exercise 1

Study the differentiability of the function f at the point x_0 in the following cases:

1. $f(x) = \begin{cases} x^2 \cos \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}, \qquad x_0 = 0.$

2. $f(x) = \begin{cases} \sin x \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}, \qquad x_0 = 0.$

3. $f(x) = \begin{cases} \exp(\frac{1}{x^2-a^2}), & |x| < a \\ 0, & |x| \geq a \end{cases}, \qquad |x_0| = a, \quad a \in \mathbb{R}_+$

Exercise 2

Let the function f be defined on $\mathbb{R}_+$ by:

$$f(x) = \begin{cases} ax^2 + bx + 1, & 0 \leq x < 1 \\ \sqrt{x}, & x \geq 1 \end{cases}$$

Determine the real numbers a and b so that f is differentiable on $\mathbb{R}_+$. Calculate $f'(x)$.

Exercise 3

1. *Calculate the derivatives of the following functions:*
   (a) $y_1(x) = \sqrt{\ln x + 1} + \ln(\sqrt{x} + 1)$.
   (b) $y_2(x) = \dfrac{\sqrt{\cos x}}{1 - e^x}$.
   (c) $y_3(x) = e^{\cos \sqrt{x}}$.
2. *Calculate the $n - th$ derivatives of the following functions:*
   (a) $y_1(x) = \ln(1 + x)$.
   (b) $y_2(x) = \dfrac{1 + x}{1 - x}$.
   (c) $y_3(x) = (x + 1)^3 e^{-x}$.
   (d) $y_4(x) = x^2 \sin 3x$.

Exercise 4

Determine the extrema of the following functions:

1. $f(x) = \sin x^2$, *on* $[0, \pi]$.
2. $g(x) = x^4 - x^3 + 1$, *on* $\mathbb{R}$.

Exercise 5

1. *Can we apply Rolle's theorem to the following functions?*
   (a) $f(x) = \sin^2 x,$ *on* $[0, \pi]$.
   (b) $g(x) = \dfrac{\sin x}{2x},$ *on* $[-\dfrac{\pi}{2}, \dfrac{\pi}{2}]$.
2. *Show that* $\forall x, y \in \mathbb{R}_+^*,\ 0 < x < y : \qquad x < \dfrac{y - x}{\ln y - \ln x} < y$

Exercise 6

Using l'Hopital's theorem, calculate the following limits:

1. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{e^x - 1}$.
2. $\lim\limits_{x \to \pi} \dfrac{\sin x}{x^2 - \pi^2}$.
3. $\lim\limits_{x \to 1} \dfrac{e^{x^2+x} - e^{2x}}{\cos(\frac{\pi}{2}x)}$.