Democratic and Popular Republic of Algeria

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Analysis 1

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Exercises Serie N° 3

Exercise 1

Determine the definition domains of the following functions:

1. $f(x) = \sqrt{\dfrac{x+1}{x-1}}$.
2. $g(x) = \sqrt{x^2+x-2}$.
3. $h(x) = \ln\left(\dfrac{2+x}{2-x}\right)$
4. $k(x) = \dfrac{\sin x - \cos x}{x-\pi}$.
5. $p(x) = (1+x)^{\frac{1}{x}}$.
6. $\phi(x) = \begin{cases} \dfrac{\sin x . \cos x}{x-\pi} & if \quad x \neq \pi \\ 1 & Otherwise \end{cases}$

Exercise 2

Let the function f be defined on $]-1,1[$ by: $f(x) = \dfrac{x}{1+|x|}$.
Show that f is strictly increasing.

Exercise 3

Calculate the following limits:

1. $\lim\limits_{x\to+\infty} e^{x-\sin x}$.
2. $\lim\limits_{x\to 0} \dfrac{(\tan x)^2}{\cos(2x)-1}$.
3. $\lim\limits_{x\to 0^+} \dfrac{x}{b}\left[\frac{c}{x}\right]$.
4. $\lim\limits_{x\to 0} \dfrac{\ln(1+x^2)}{\sin^2 x}$.
5. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$.
6. $\lim\limits_{x\to+\infty} \dfrac{x\ln x + 5}{x^2+4}$.

Exercise 4

Determine the values a and b so that the functions f, and g are continuous on $\mathbb{R}$

$$f(x) = \begin{cases} \dfrac{\sin(ax)}{x}, & x<0 \\ 1, & x=0 \\ 2be^x - x, & x>0 \end{cases} \qquad g(x) = \begin{cases} \sqrt{x} - \dfrac{1}{x}, & x \geq 4 \\ (x+a)^2, & x<4 \end{cases}$$

Exercise 5

Are the following functions continuous at the point $x_0 = 0$?

$$f(x) = \begin{cases} x + \dfrac{\sqrt{x^2}}{x} & : x \neq 0 \\ 0 & : x = 0 \end{cases} \qquad\qquad g(x) = \begin{cases} 1 + x\cos(\dfrac{1}{x}) & : x \neq 0 \\ 0 & : x = 0 \end{cases}.$$

Exercise 6

Show that the following functions are continuous over their defined domains:

$$f(x) = \frac{x^3 + 2x + 3}{x^3 + 1}, \qquad\qquad g(x) = \frac{(1+x)^n - 1}{x}.$$

Study the existence of extension by continuity over $\mathbb{R}$.

Exercise 7

1. *Show that any periodic and non-constant function does not admit a limit in $+\infty$.*
2. *Let $f : [0, +\infty[\longrightarrow \mathbb{R}$ be a function such that $f(0) > 0$. We assume that $\lim\limits_{x \to +\infty} \dfrac{f(x)}{x} = a < 1$. Show that there exists $x_0 \in [0, +\infty[$ such that $f(x_0) = x_0$.*