

Tutorial exercises set 2: Analysis 2

Exercise 01:

Give the following integrals:

$$\begin{array}{lll} 1) \int x^5 dx & 2) \int (x + \sqrt{x}) dx & 3) \int \left(\frac{3}{\sqrt{x} - \frac{x\sqrt{x}}{4}} \right) dx \\ 4) \int \left(\frac{x^2}{\sqrt{x}} \right) dx & 5) \int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx & 6) \int \left(x^2 + \frac{1}{x^{\frac{1}{3}}} \right)^2 dx \end{array}$$

Exercise 02: (Integration by variable substitution)

$$\begin{array}{lll} 1) \int e^{5x} dx & 2) \int \cos(5x) dx & 3) \int \sin(ax) dx \\ 4) \int \frac{\ln x}{x} dx & 5) \int \frac{dx}{\sin^2 3x} & 6) \int \frac{dx}{\cos^2(7x)} dx \\ 7) \int \frac{dx}{3x-7} dx & 8) \int \frac{dx}{1-x} & 9) \int \frac{dx}{5-2x} \\ 10) \int \tan(2x) & 11) \int \cotan(5x-7) dx & 12) \int \frac{dy}{\cotan 3y} dy \\ 13) \int \cotan\left(\frac{x}{3}\right) & 15) \int \tan \varphi \sec^2 \varphi d\varphi & 16) \int (\cotan e^x) e^x dx \\ 17) (\tan 4S - \cotan \frac{S}{4}) dS & 18) \int \sin^2(x) \cos(x) dx & 19) \int \cos^3(x) \sin(x) dx \\ 20) \int \sqrt{x^2+1} dx & 21) \int \frac{x^2}{\sqrt{x^3+1}} dx & 22) \int \frac{\cos(x) dx}{\sin^2(x)} \\ 23) \int \frac{\sin(x)}{\cos^3(x)} dx & 24) \int \frac{\tan(x)}{\cos^2(x)} dx & \end{array}$$

Exorcise 03: (Integration of rational fractions)

$$1) I = \int \frac{2x-1}{(x-1)(x-2)} dx \quad 2) I = \int \frac{xdx}{(x+1)(x+3)(x+5)} \quad 3) I = \int \frac{x^5+x^4-8}{x^3-4x} dx$$

Exercise 04:(Integration by parts)

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|----------------------------|---|--|
| 1) $\int xe^x dx$ | 2) $\int x \ln(x) dx$ | 3) $\int x \sin(x) dx$ |
| 4) $\int \ln(x) dx$ | 5) $\int \arcsin(x) dx$ | 6) $\int \ln(-x + 1) dx$ |
| 7) $\int x^n \ln x dx$ | 8) $\int x \arctan(x) dx$ | 9) $\int x \arcsin(x) dx$ |
| 10) $\int \ln(x^2 + 1) dx$ | 11) $\int \arctan(\sqrt{x}) dx$ | 12) $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}} dx$ |
| 12') $\int x \cos^2(x) dx$ | 13) $I = \int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$ | 14) $\int \frac{\arcsin(x)}{x^2} dx$ |

Exercise 05:(تمرين إضافي)

Prove that:(برهن أن)

- 1) $\int \frac{x}{x^2 - 3x + 2} dx = 2 \ln(x - 2) - \ln(x - 1) + C$
- 2) $\int \frac{dx}{x^2 + x - 2} = \frac{1}{3} \ln \left(\frac{x - 1}{x + 2} \right) + C$
- 3) $\int \frac{dx}{(x - 1)^2(x - 2)} = \frac{3}{x - 2} + \ln \frac{(x - 2)}{(x - 1)} + C$
- 4) $\int \frac{x - 8}{x^3 - 4x^2 + 4x} = \frac{3}{x - 2} + \ln \frac{(x - 2)^2}{x^2} + C$
- 5) $\int \frac{x^5}{x^3 - 1} dx = \frac{1}{3} [x^3 + \ln(x^3 - 1)] + C$
- 6) $\int \frac{\arcsin x}{x^2} dx = \ln \left| \frac{1 - \sqrt{1 - x^2}}{x} \right| + \frac{1}{x} \arcsin(x) + C$