Notions of Transport Phenomena

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Objectives

At the end of this course, the student will be able to:

- **Define** the conduction rate equation ;
    - **Illustrate** the thermal properties of matter ;
- **Develop** the general heat equation.

Introduction

Conduction is the transport of energy in a medium due to a temperature gradient, governed by Fourier's law. Understanding temperature distribution is crucial for calculating heat flux. This law is applicable to transient, multidimensional conduction in complex geometries, and it is limited to simplified conditions like one-dimensional, steady-state conduction in a plane wall.

To approach this subject, students must have acquired sufficient knowledge in ***differential equations*** (*as pre-requisites*).

Quiz : Pre-test: differential equations

Use direct integration to solve the differential equation:

$$\frac{dy}{dx} = x^2$$

II Chapter 2: Heat Transfer

1. Introduction

Heat transfer occurs when thermal energy moves from one place to another. Atoms and molecules inherently have kinetic and thermal energy, so all matter participates in heat transfer. There are three main types of heat transfer that move energy from **high temperature** to **low temperature**.

2. Conduction Heat Transfer

Conduction heat transfer is **transfer of thermal energy** from **more energetic** to **less energetic** particles due to their interaction.

Example

A one side steel plate is exposed to Bunsen burner; molecules **get more energy** from that heat and **start to vibrate**. They vibrate, interact with their neighbors **exchanging energy** and this energy goes from the hot side to the cold side, by **conduction**.

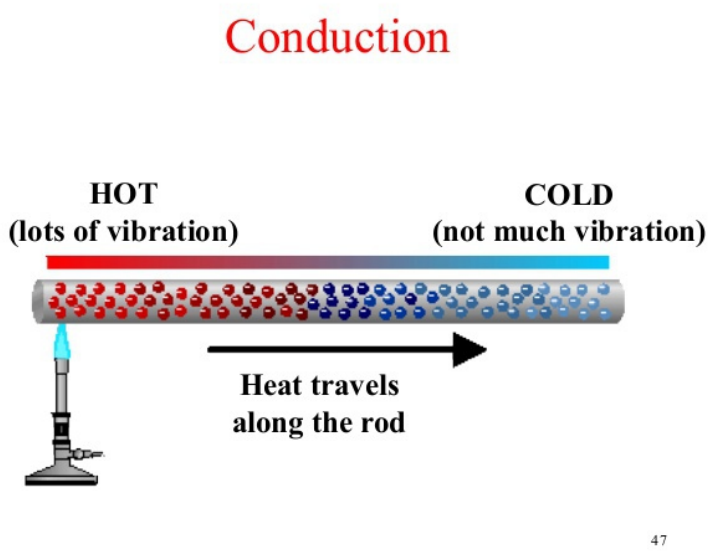


Figure 01 - Representation of Conduction

Heat transported through **a stationary medium** (solid or fluid without motion) by **vibrational energy of molecules** that increase with temperature.

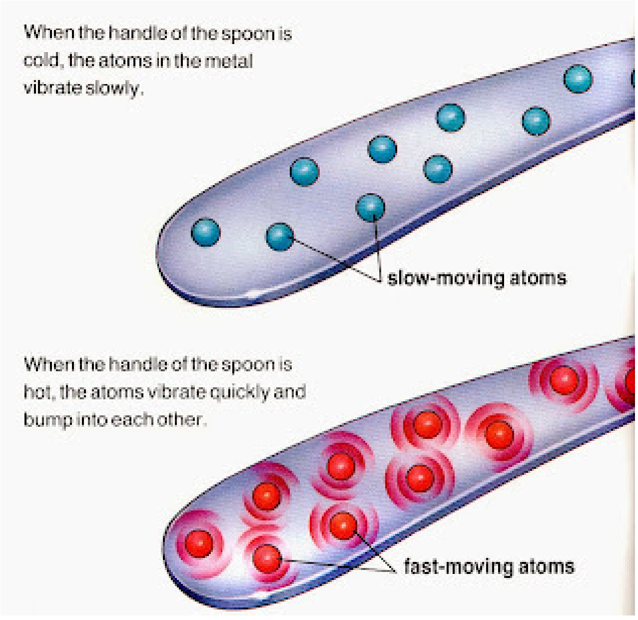


Figure 2 - Slow and fast motion of atoms

3. Fourier's law

To calculate this conduction, we will write down the **Fourier's Law**. Conductive heat flux is proportional to the temperature gradient. The constant of proportionality is a property of the material called: **thermal conductivity**. Thus:

$$q_x = -k.A.\frac{dT}{dx}$$

4. The thermal conductivity

Based on our life experiences, we know that some materials (like metals) conduct heat at a much faster rate than other materials (like glass). Thermal conductivity of a material is a measure of **its intrinsic ability to conduct heat**. It could be written as :

$$k = -\left(\frac{q_x}{{dT}/{dx}}\right)$$

Note

Metals are typically **good conductors** of heat, and hence have high thermal conductivity. *Gases* generally have low thermal conductivity and are **bad conductors** of heat (or, in other words, are good insulators).

5. The heat diffusion equation

A major objective in a conduction analysis is to determine the **temperature field** in a medium resulting from conditions imposed on its boundaries. That is, we wish to know the **temperature distribution**, which represents *how temperature varies with position in the medium*.

Once this distribution is known, the conduction heat flux at any point in the medium or on its surface may be computed from *Fourier's law*.

Note

Consider a homogeneous medium within which there is no bulk motion (advection) and the *temperature distribution* **T(x, y, z)** is expressed in **Cartesian coordinates**. The *medium is* assumed to be **incompressible**, that is, its density can be treated as constant. Following the four-step methodology of applying conservation of energy, we first define an **infinitesimally small** (*differential*) **control volume**, **dx·dy·dz**, as shown in Figure 3.

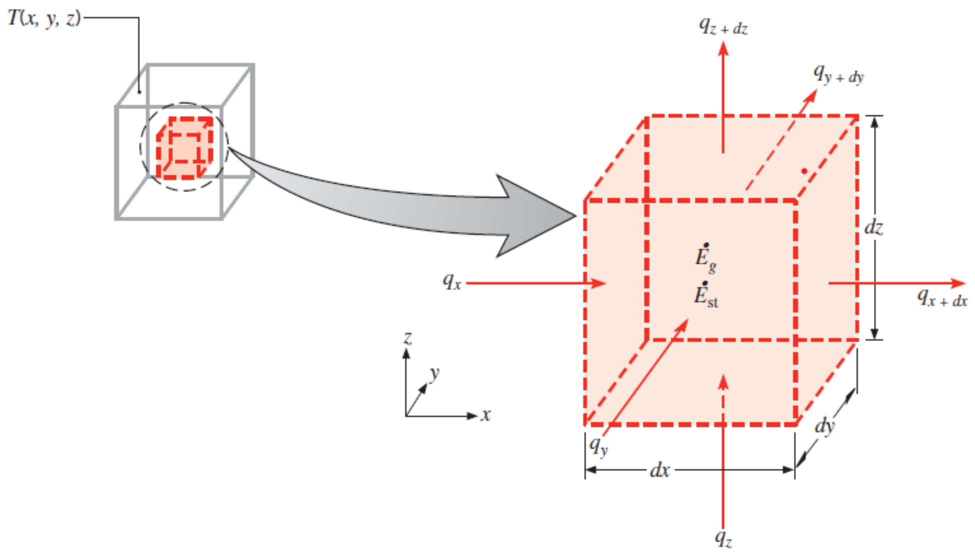


Figure 3 - Differential control volume, dx dy dz, for conduction analysis in Cartesian coordinates.

The express of **conservation of energy** using the foregoing rate equations. On a *rate basis*, the general form of the conservation of energy requirement is

$$\dot{E_i} + \dot{E_g} - \dot{E_{out}} = \dot{E_{st}}$$

The general form, in *Cartesian coordinates*, of the **heat diffusion equation**.

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \dot{q} = \rho.c_p.\frac{\partial T}{\partial t}$$

This equation, often referred to as the heat equation, provides the basic tool for heat conduction analysis. From its solution, we can obtain the **temperature distribution T(x, y, z)** as a function of time.

Advice

It is often possible to work with simplified versions. For example, if the **thermal conductivity** is constant, the heat equation is

$$(\frac{\partial^2 T}{\partial x^2}) + (\frac{\partial^2 T}{\partial y^2}) + (\frac{\partial^2 T}{\partial z^2}) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where

$$\alpha = \frac{k}{\rho c_p}$$

the thermal diffusivity

Note

Additional *simplifications* of the general form of the heat equation are often possible. For example, **under steady-state conditions**, there can be **no change in the amount of energy storage**; hence equation reduces to

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \dot{q} = 0$$

Moreover, if the heat transfer is **one-dimensional** (e.g., in the x-direction) and there is **no energy generation**, equation reduces to

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) = 0$$

Fundamental

The important implication of this result is that, **under steady-state**, **one-dimensional conditions** with **no energy generation**, the *heat flux* is *a constant* in the direction of transfer

$$\frac{d\ddot{q}_x}{dx} = 0$$

Quiz : Assessment end of chapter

The transfer of thermal energy occurs from the less energetic to the mor energetic particles.

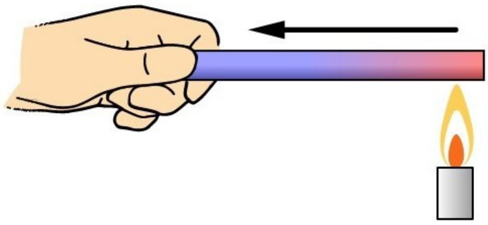
- **A** True
- **B** False

Quiz : Assessment end chapter

How does conduction heat transfer occurs?

Conclusion

As conclusion, conduction heat transfer is the transfer of thermal energy between two objects by direct contact.



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