physics

CHAPTER 1 : Mathematical Reminders

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1. Specific objectives

- Recall basic physical quantities and their units.
- Understand the concept of dimensional analysis and its importance in problem-solving.
- Apply dimensional analysis to convert units and solve problems

2. Introduction

The comprehension of physical phenomena remains incomplete without the acquisition of quantitative information, which essentially involves measuring physical quantities. In the exploration of a physical phenomenon, a thorough investigation of significant variables is imperative. The mathematical interrelation among these variables gives rise to a physical law. While such relationships can be established in certain instances, there are situations where a modeling approach becomes essential, such as in the case of dimensional analysis. This method proves invaluable when dealing with complex or less understood phenomena, enabling a systematic and insightful exploration of their underlying principles.

3. Physical Quantities and Dimensional Analysis

3.1. Physical Quantities

Physical quantities are classified according to two categories: base quantities and derived quantities

- Base quantities: they are self-defined quantities such as length, mass, time etc,
- Derived quantities: they are quantities that are derived from basic quantities and are

known by their meanings, such as speed, acceleration, force, and pressure etc.

3.2. The International System of Units (SI system)

Specific and uniform standards must be used across the world; quantities determine dimensions and dimensions are estimated in units. The international system of units SI or MKSA system consists of 7 base units adapted to 7 physical quantities, as shown in the following table:

Table I.1: The international system of units (SI system)

Physical quantities	**Unit of measurement (SI)**	**Dimensional symbol**
Length	Meter (m)	L
Mass	Kilogram (Kg)	M
Time	Second (s)	T
Temperature	Kelvin (K)	θ
Electric current intensity	Ampere (A)	I
Light intensity	Candela (cd)	J
Quantity of matter	Mole (mol)	N

3.3. Dimensional Analysis

It is a theoretical tool for interpreting problems based on the dimensions of the physical quantities involved: length, time, mass, etc. Dimensional analysis makes it possible to:

- Check the validity of equations with dimensions
- Research into the nature of physical quantities
- Search for the homogeneity of physical laws
- Determine the unit of a physical quantity based on the essential units (meter, second,kilogram, etc.)

The dimensional equation is represented by the following writing:

$[X]=M\ L\ T\ I\ \theta\ N\ J$

Where

M : Mass (Kg)

L : Length (m)

T :Time (s)

I : Electric current intensity (A)

θ :Temperature (K)

N :Mole (mol)

J : Light intensity (cd)

Where

[π]=1 , [*nombre*]=1 , [t]=T , [m] =M , [l]=L , [i]=I

Some quantities have no dimensions

Example

The dimensional equation of

Linear speed:

$$V = \frac{d}{t} \Rightarrow [V] = \left[\frac{d}{t}\right] = \frac{[d]}{[t]} = \frac{L}{T} = L.T^{-1}$$

Acceleration:

$$a = \frac{V}{d} \Rightarrow [\gamma] = \left[\frac{v}{t}\right] = \frac{[v]}{[t]} = \frac{L.T^{-1}}{T} = L.T^{-2}$$

Force:

$$F = m.a \Longrightarrow \boldsymbol{[F] = [m.a] = [m].[a] = M.L.T^{-2}}$$

Work :

$$W = F.d \Longrightarrow [W] = [F.d] = [F].[d] = M.L.T^{-2}.L$$

$$[W] = M.L^2.T^{-2}$$

Pressure :

$$P = \frac{F}{s} \Rightarrow [P] = \left[\frac{F}{S}\right] = \frac{[F]}{[S]} = \frac{M.L.T^{-2}}{L^2} = M.L^{-1}.T^{-2}$$

3.4. Dimensional Uniformity

Dimensional analysis helps to confirm the validity of physical laws by matching the dimensions between the two sides of the law. It also helps formulate the final picture of the mathematical relationship based on the principle of dimensional matching as a condition for the validity of the relationship, as the unit of the right side of the equation must equal the unit of the left side of the equation, otherwise the equation is incorrect.

- To prove the validity of any equation, the dimensions of the left side must be equal to the dimensions of the right side

Example

Verify the homogeneity of the following equation:

$$x = \frac{1}{2} a t^2 + v_0 t$$

$$[x] = [\frac{1}{2}. a.t^2 + v_0 t] \Rightarrow [x] = [\frac{1}{2}. a.t^2] + [v_0 t] \Rightarrow [x] = [\frac{1}{2}].[a].[t^2] + [v_0].[t]$$

$$\Rightarrow L = 1. LT^{-2}. T^2 + LT^{-1}. T^1$$

$\Rightarrow L = L$ So the equation is homogeneous.

4. Calculation of Error

In experimental science, there is no exact measurement. Measurements are subject to more or less significant errors depending on the quality of the instruments and the skill of the experimenter.

• A discrepancy exists between the obtained value and the exact value, which remains unknown.

• This discrepancy is referred to as "measurement error."

• The true value remaining unknown.

• The measurement error will remain undetermined.

4.1. The absolute error and the relative error:

In practice, errors can only be estimated there are two types of errors:

4.1.1. The absolute error

The absolute error represents the mathematical quantity that measures the difference between the measured or observed value of a quantity and its true or theoretical value. It is calculated by taking the absolute value of the difference between these two values. The general formula for absolute error (Eabs) is as follows:

$$E_{\text{abs}} = |\text{Measured value} - \text{True value}|$$

This measure allows the assessment of the overall discrepancy between the experimental measurement and the expected value, irrespective of the direction of this difference. The absolute error is a crucial tool in the analysis of experimental results and helps quantify the accuracy of the measurements taken.

a) the relative error

I provided the general formula for relative error in the previous response. If you have specific values for the approximate value and true value, you can plug them into the formula to calculate the relative error.

To reiterate:

$$RE = \frac{|\text{Approximate Value} - \text{True Value}|}{|\text{True Value}|}$$

And if you want the result as a percentage:

$$\text{Relative Error (\%)} = \left(\frac{|\text{Approximate Value} - \text{True Value}|}{|\text{True Value}|}\right) \times 100\%$$

4.2. Uncertainty calculations

For a quantity g=f(x,y,z), its total differential is expressed as:

$$dg = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy + \frac{\partial g}{\partial z}dz$$

The absolute uncertainty on the variable g is obtained by considering the variations in the variables that compose it, namely:

$$\Delta g = \left|\frac{\partial g}{\partial x}\right| \Delta x + \left|\frac{\partial g}{\partial y}\right| \Delta y + \left|\frac{\partial g}{\partial z}\right| \Delta z$$

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