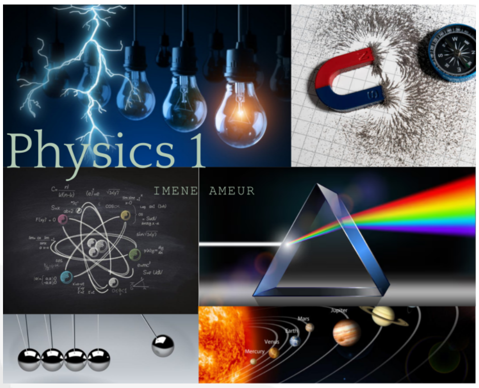
Course Physics1



Course Physics 1
CUMILA

Dr Imene Ameur

Abdelhafid Boussouf University Center- Mila

Faculty of Natural and Life Sciences

Department of Earth and Universe Sciences

E-mail: *ameur.i@centre-univ-mila.dz*

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1. Introduction

Newtonian mechanics is one of the most successful early theories. It provides a comprehensive mathematical framework allowing the movement of a point to be described in a predictive and precise manner. Due to the vectorial nature of the formalism.

We will review a set of basic mathematical prerequisites necessary for the introduction of point mechanics, both in the ***kinematics*** and ***dynamics*** aspects and relating to coordinates, the vectors as well as the different operations to which they are subjected.

2. Kinematics

[cf. kinematics]

The kinematics of the material point is the study of motion of point as a function of time (the position, the velocity, the acceleration, etc.) without reference to the forces that caused the motion.

2.1. Position, Velocity and Acceleration Vectors

2.1.1. Position Vector

The position of a mobile at a time t is determined with respect to a reference frame by a vector $\overrightarrow{OM}$ which is called the position vector. Its origin is the center of the frame O and its end is the mobile M

$$\overrightarrow{OM}=\overrightarrow{OM}(t)$$

2.1.2. Displacement Vector

We define the vector $\overrightarrow{M_1M_2}$ the displacement vector

$$\overrightarrow{M_1M_2}=\overrightarrow{OM_2}-\overrightarrow{OM_1}$$

2.1.3. Velocity Vector

- ***Average speed***

The average speed is defined as follows

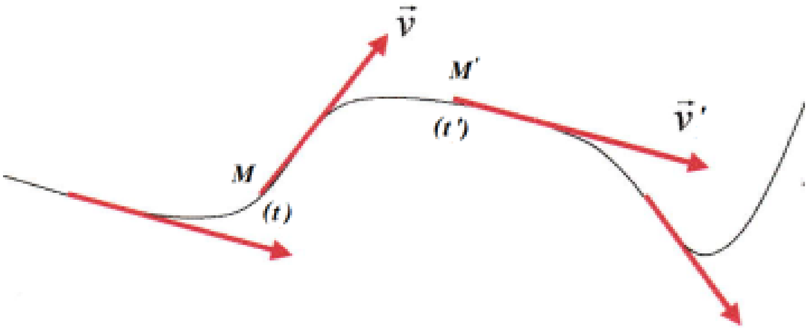
$$\vec{V}_a=\frac{\overrightarrow{OM_2}-\overrightarrow{OM_1}}{t_2-t_1}=\frac{\overrightarrow{M_1M_2}}{\Delta t}$$

- ***Instant velocity***

It is the velocity at a given time t and it is defined as follows

$$\vec{V}=\lim_{\Delta t\to 0}\vec{V}_a=\frac{d\overrightarrow{OM}}{dt}$$

The instantaneous velocity vector is tangent to the trajectory and its direction follows the direction of movement.



2.1.4. Acceleration Vector

- ***Average Acceleration***

Average acceleration is the change in speed between two positions with respect to time. The mobile undergoes an average acceleration such that

$$\vec{a}_a=\frac{\vec{V}_2-\vec{V}_1}{t_2-t_1}=\frac{\Delta\vec{V}}{\Delta t}$$

V_1 is the speed of the mobile at time t_1 and V_2 its speed at time t_2

- ***Instant Acceleration***

Instantaneous acceleration is the acceleration at a given time t

$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_a = \frac{d\vec{V}}{dt} = \frac{d^2\overrightarrow{OM}}{dt^2}$$

2.2. Study of Some Particular Movements: Rectilinear and in-Planar Movements

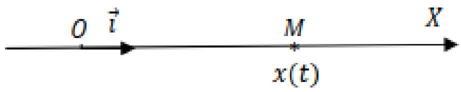
2.2.1. Rectilinear Movement

A movement is rectilinear if the trajectory followed by the mobile is a straight line.
The position M of the mobile is identified by the position vector

- ***Position Vector***

$$\overrightarrow{OM} = x(t)\vec{i}$$

If the movement is linear along Ox



- ***Instantaneous velocity vector***

$$\vec{V}(t) = \frac{d\,\overrightarrow{OM}}{dt} = \frac{dx}{dt}\vec{i}$$

- ***Instantaneous acceleration vector***

$$\vec{a}(t) = \frac{d\vec{V}}{dt} = \frac{d^2x}{dt^2}\vec{i}$$

Fundamental

We have two types of rectilinear motion: uniform rectilinear motion and uniformly varied rectilinear motion

Uniform rectilinear movement: uniform rectilinear motion is characterized by a constant speed and therefore the acceleration is zero.

$$\vec{a} = \frac{d\vec{V}}{dt} = 0$$

Uniformly varied rectilinear movement: uniformly varied rectilinear motion is characterized by constant acceleration.

$$\vec{a} = \frac{d\vec{V}}{dt} = cst$$

Note

If we have:

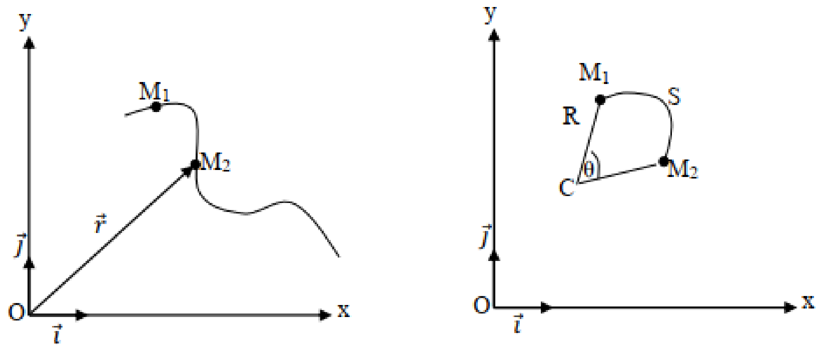
$\vec{a}.\vec{v} > 0 \Rightarrow$ the movement is uniformly accelerated

$\vec{a}.\vec{v} < 0 \Rightarrow$ the movement is uniformly delayed

2.2.2. Planar Movement

If the trajectory of the mobile is in planar, we study the movement following the Cartesian coordinates ($O, \vec{i}, \vec{j}$), the polar ($O, \vec{U}_\rho, \vec{U}_\theta$) or curvilinear coordinates $(\vec{U}_T, \vec{U}_N)$, in this section we choose the curvilinear coordinates.

Curvilinear movement is characterized by a curvilinear trajectory which requires knowledge of the radius of curvature R and the center C.



Definition

The curvilinear coordinates is a base connected to the mobile in curvilinear motion. It is defined by the orthonormal base $(\vec{U}_T, \vec{U}_N)$ such as

$\vec{U}_T$: is a unit vector tangential to the trajectory and in the direction of movement

$\vec{U}_N$: is perpendicular to the vector $\vec{U}_T$, and it is directed towards the center of the curvature of the trajectory (the concavity of the trajectory).

Method

- ***Position vector***

The position of the mobile is determined by the curvilinear abscissa S such that

$$S(t)=R\theta(t)$$

S is the length of the arc between the two points M(1) and M(2)

- ***Instantaneous velocity vector***

$$\vec{v}=v\vec{U}_T=\frac{dS}{dt}\vec{U}_T$$

- ***Instantaneous acceleration vector***

$$\vec{a}=\frac{dv}{dt}\vec{U}_T+v\dot{\theta}\vec{U}_N$$

$$\vec{a}=\frac{dv}{dt}\vec{U}_T+\frac{v^2}{R}\vec{U}_N=a_T\vec{U}_T+a_N\vec{U}_N$$

3. Dynamics

The dynamics of the point makes it possible to connect the movement to its causes. The causes of movement are modeled in mechanics by vector quantities called forces.

3.1. Interaction Forces

A material point is in motion because of the interactions between the particle and its environment which undergoes them; these interactions are called ***"forces"***

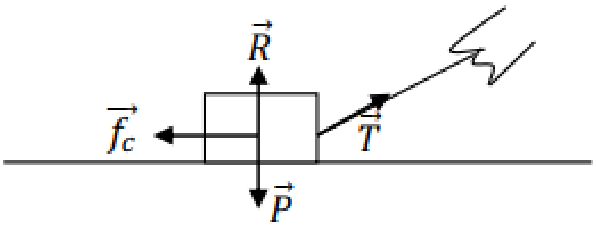
Definition

Force is defined as the amount of effort or pressure applied that causes an object to accelerate, as a result of which the direction and speed of the body changes and can cause its temporary or permanent deformation. In general, force causes a change in the motion of an object. The force is represented by a vector, such as:

- ***Its origin:*** is the point of contact (force/body)
- ***Its direction:*** is the direction of movement supported on the wire for the case for example of force thread tension
- ***Its module:*** is the value of the force in Newton (N)

Example

A body slides on a horizontal surface by a thread. The forces exerted on this body are: weight force, thread tension, reaction force and friction force.



Force exerted on a body

The purpose of dynamics is to predict the movements of bodies subjected to forces. However, these forces act on bodies at a distance, like the gravitational force, and in contact like the reaction force.

Force at a distance	**Force in contact**
Gravitational force	Support reaction
Electrostatic force	Friction force
Electromagnetic force	Elastic force
Nuclear force	Tension force

Forces types (force at a distance and in contact)

3.2. Fundamental Laws: Newton's laws of motion

Newton's laws of motion, three statements describing the relations between the forces acting on a body and the motion of the body.

See "Newton's laws of motion"

3.2.1. Principle of inertia - Newton's 1st law

In a Galilean frame of reference, if a body (material system) is isolated or pseudo-isolated its center of inertia (center of mass):

- It remains fixed if it is not moving.

- It remains in uniform rectilinear motion if it is in motion.

Then the sum of the external forces applied to the material point is zero

$$\sum \vec{F}_{ext} = \vec{0} \rightarrow \vec{V} = cst$$

3.2.2. Fundamental relationship of dynamics - Newton's 2nd law

Newton's second law represents the fundamental principle of dynamics. In a Galilean frame of reference, the sum of the external forces applied to a system is equal to the mass times the acceleration

$$\sum \vec{F}_{ext} = m\vec{a}$$

3.2.3. Principle of action and reaction - Newton's 3rd law

When two bodies interact, the force exerted by the first on the second is equal and opposite to the force exerted by the second on the first

Example

Let two material points (1) and (2) interact with each other, the action exerted by (1) on (2) $\vec{F}_{12}$ is equal and opposite to that exerted by (2) on (1) $\vec{F}_{21}$

$$\vec{F}_{12} = -\vec{F}_{12} \rightarrow ||\vec{F}_{12}|| = ||\vec{F}_{21}||$$

These two forces are of the same nature.

4. Gravitation

4.1. Gravitational Force

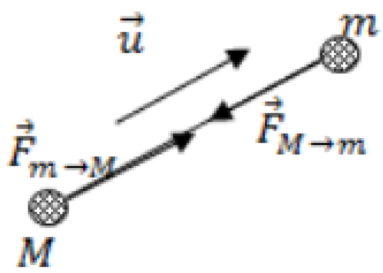
The gravitational force is the force exerted by a mass M on another mass m. This force follows Newton's law of gravitation or the law of universal attraction (Newton's 4th law), is defined by

$$\vec{F}_{M \to m} = G\frac{Mm}{r^2}\vec{u}$$

r: is the distance between the centers of mass of M and m.

G: is the universal gravitational constant $(G = 6.67384 . 10^{-11}\,Nm^2 Kg^{-2}(SI))$

$\vec{u}$: is a unit vector, it is directed from M to m.



Advice

For more information, watch the video by clicking *here*

4.2. The Acceleration Gravity

The acceleration gravity g is calculated by using the previous equation (the gravitational force). If M represents the mass of the earth and m located at a distance r from the center of the earth

$$\vec{F}_{M \to m} = G\frac{Mm}{r^2}\vec{u} = m\vec{g}$$

With

$$\vec{g} = G\frac{M}{r^2}\vec{u}$$

This vector represents the gravitational field of the earth it is always oriented towards the center of the earth.

$$\vec{F}_{M \to m} = m\vec{g}$$

Represents the weight of the body m, such that $\vec{P} = m\vec{g}$

On the surface of the earth (r = R) R is the ray of the earth (R = 6371 Km) and M is the mass of the earth $(M = 5.9742 . 10^{24}\,kg)$

$$g = G\frac{M}{r^2} = \frac{6.67384 . 10^{-11}\,5.9742 . 10^{24}}{(6.371 . 10^6)^2} = 9.80\,m . s^{-2}$$

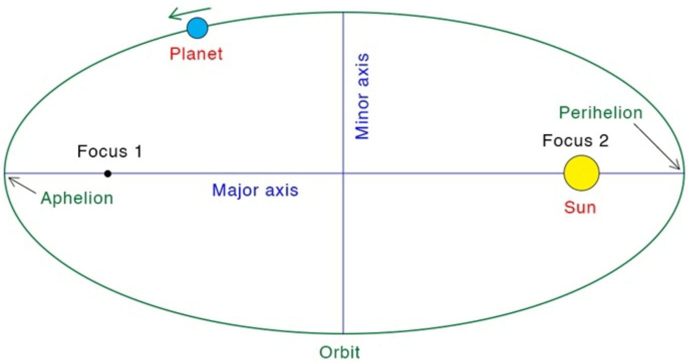
For a body on earth the constant g is $g=9.80\,m.s^{-2}$

4.3. Kepler's Laws

Kepler observed most of the planets in the solar system, taking advantage of his predecessors (Copernicus and Galilei, etc.), then he extracted the three laws which are famous in his name

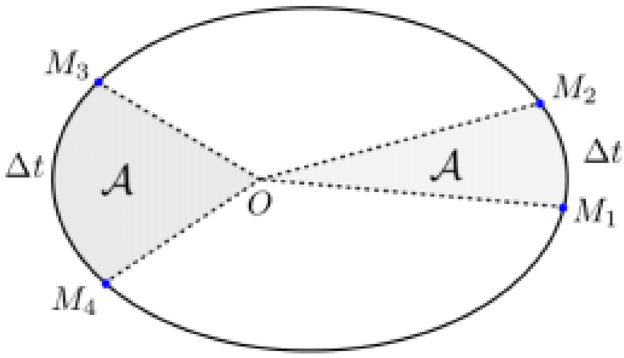
4.3.1. Kepler's 1st law: Law of orbits

The planets move around the sun in elliptical orbits with the sun at one of the foci.



4.3.2. Kepler's 2nd law: Law of areas

The straight line connecting the sun and a planet covers equal areas over equal time periods



4.3.3. Kepler's 3rd law: law of periods

For all planetary orbits (satellites) the ratio of the square of the periods of revolution (T in s) to the cube of the semi-major axis of the orbit (a in m) is constant.

$$\frac{T^2}{a^3}=k=cst$$

4.4. The Movement of Planets

By applying the Newton's 2nd law to a planet considered

$$\vec{F}_{S/P}=m\vec{a}$$

The force being radial

$$G\frac{m M_S}{r^2}=m a_n \rightarrow a_n=G\frac{M_S}{r^2}$$

The acceleration of the planet in its movement is only radial, directed towards the center of the sun.

Hence, the movement of the planets around the sun can be considered a uniform circular motion

Also The speed and period of a planet is defined as follow

$$v=\sqrt{\frac{GM_S}{r}}, T=\frac{2\pi r}{v}$$

$$\frac{T^2}{r^3}=\frac{4\pi^2}{GM_S}=cst$$

Note

From this equation $\frac{T^2}{r^3}=\frac{4\pi^2}{GM_S}=cst$, we can conclude that Kepler's 3rd law is verified

5. Work and Energy

[cf. Work and energy]

We saw in the previous part how to solve dynamic problems using the fundamental principle of dynamics (Newton's law). In addition to this method we can use theorems based on kinetic, potential and mechanical energy and even the work of forces.

5.1. Concept of Work

Definition

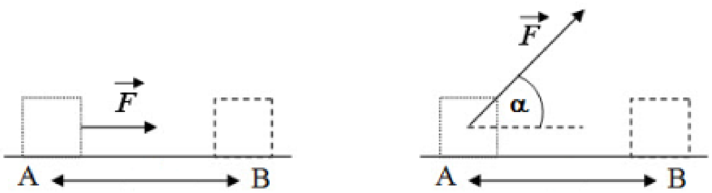
Mechanical work: in physics is the amount of energy required to move an object with force and distance, is a scalar quantity measured in Joule

5.1.1. Work of a constant force

The work of a constant force $\vec{F}$ when a fixed point whose effect moves from A to point B is given by scalar

$$W_{A\rightarrow B}(\vec{F})=\vec{F}.\overrightarrow{AB}=||\vec{F}||.||\overrightarrow{AB}||.\cos(\alpha)$$

α is the angle between the force vector $\vec{F}$ and the vector $\overrightarrow{AB}$



✎ *Note*

The work of a constant force is not related to the trajectory followed by the point of impact from A to B, i.e. the work $W_{A\to B}(\vec{F})$ has the same value, whether the path is straight or curved.

5.1.2. Work of a non-constant force

The work of a non-constant force is finding by dividing the path into elementary transitions $d\vec{r}$ and we call the work of the force in these transitions elementary work dW. The total work of the force on the line AB is equal to the sum of the elementary work dW.

$$W_{A\to B}(\vec{F})=\int_A^B \vec{F}.d\vec{r}$$

5.2. Concept of Power

Power is defined as the derivative of work with respect to time

$$P=\frac{dW}{dt}=\frac{\vec{F}.d\vec{r}}{dt}=\vec{F}.\vec{v}$$

$\vec{F}$: It is the force that acts on the body.

$\vec{v}$: The speed of the body in motion

The power unit is the Watt 1W = 1J/s = 1Nm /s

5.3. Concept of Energy

Definition

Energy: in physics energy is defined as the ability to do work. For example, increasing the speed of a car or lifting a stone requires work. Energy and work are measured in the same units (J). There' re three types of energy: kinetic, potential and mechanical.

5.3.1. Kinetic Energy

Kinetic energy: Kinetic energy is a type of energy that an object has due to its motion. We define the linear kinetic energy E_k of a material point of mass m which moves with a speed v in a Galilean frame of reference by $E_k=\frac{1}{2}mv^2$

5.3.2. Potential Energy

Potential energy: Potential energy is the energy that exists in an object due to its position or state, it represents the work actually done and is sometimes called stored energy. The work of conservative forces is not about the path followed, it is about the starting point and the ending point. The work of these forces can be expressed by a function called potential energy E_p , for example: the gravitational potential energy is equal to $E_p(z)=mgz$

5.3.3. Mechanical Energy

Mechanical energy: Mechanical energy is the energy resulting from movement, that is, due to the effect of force on objects.

Consider a system moving between two points A and B under the influence of conservative and non-conservative forces. The mechanical energy (total) is

$$E_M(B)-E_M(A)=\sum W_{A\rightarrow B}\vec{F}_{NC}\rightarrow\Delta E_M=\sum W_{A\rightarrow B}\vec{F}_{NC}$$

With

$$E_M=E_k+E_p$$

5.3.4. Principle of Conservation of Energy

The principle of conservation of energy is divided in two cases:

- Case of conservative forces

$$E_M=E_k+E_p=cst\rightarrow\Delta E_M=0$$

This means that the change in kinetic energy is equal to the change in potential energy

$$\Delta E_k=-\Delta E_p$$

- Case of non-conservative forces

$$E_M(B)-E_M(A)=\int W_{A\rightarrow B}(\vec{F}_{NC})\rightarrow\Delta E_M=\int W_{A\rightarrow B}(\vec{F}_{NC})$$

✎ *Note*

We notice from here that the total energy is not constant and its variation is not non-existent, but it is equal to the work of the non-conservative forces, which represents the energy loss.

Reminder

- ***Conservative forces:***

Forces are said to be ***conservative*** when their work does not depend on the path followed but on the starting point and the arrival point, it is symbolized by $\vec{F}_C$. For example: Force of gravity, force of weight, spring return force.

- ***Non-conservative forces:***

The forces are said to be ***non-conservative*** or active forces when their work depends on the path followed, it is symbolized by $\vec{F}_{NC}$. It leads to the dissipation of kinetic energy (waste or unnecessary conversion) into heat which is lost to the surrounding environment, and is considered the main consumer of the energy used. For example: Friction force.