Lecture 8

The Electric Current and the resistance

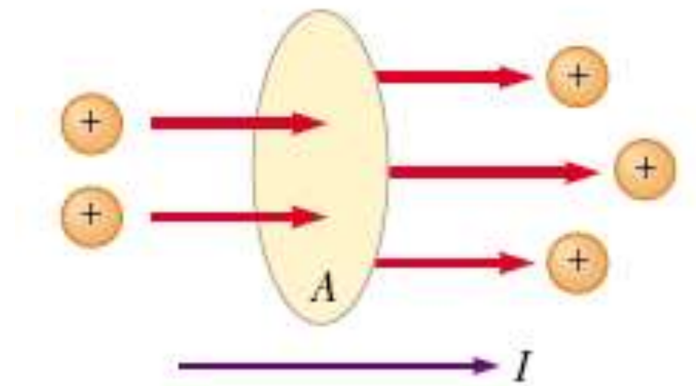
- **Electric current and Ohm’s law**
- **The Electromotive Force and Internal Resistance**
- **Electrical energy and thermal energy**
- **Resistors in series.**
- **Resistors in parallel.**
- **Kirchhoff’s Laws and its applications.**
- **Charging and Discharging Processes in RC**

Electric current and Ohm's law

The **current is defined as the flow of the charge**.

- The current is the rate at which charge flows through a surface of area *A*,
- If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through *A* per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$



The SI unit of current is the ampere (A):That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

The current density

- Consider a conductor of cross-sectional area ***A*** carrying a current ***I***. The current density ***J*** in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

The current density $$J \equiv \frac{I}{A} = nqv_d$$

where J has SI units of A/m^2.

A current density ***J*** and an electric field ***E*** are established in a conductor whenever a potential difference is maintained across the conductor, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

σ is called the conductivity of the conductor.

Ohm's Law

Ohm's law states that:

for many materials the ratio of the current density to the electric field is a constant **σ** that is independent of the electric field producing the current.

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

The quantity $R = \ell/\sigma A$ is called the **resistance** of the conductor.

$$R \equiv \frac{\Delta V}{I}$$

Ohm's Law

In order for a current ***I*** to flow there must be a potential difference, or voltage V , across the conducting material. We define the resistance, ***R*** , of a material to be:

$$R = \frac{V}{I}$$

The unit of resistance is Ohms (W): 1 W = 1 V/A

For many materials, ***R*** is constant, the material is said to be ohmic, and we write Ohm's Law:

$$V = IR$$

$$R = \frac{V}{I}$$

The inverse of conductivity is resistivity **ρ.** Where ρ has the units (Ω.m). Because

$$R = l / \sigma . A = \rho\, l / A$$

$$R = \rho \frac{\ell}{A}$$

Table 5.1 Values of Resistivity of Materials

Material	Resistivity ($\Omega \cdot m$)
Metals:	
Silver	1.47×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.63×10^{-8}
Tungsten	5.51×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Mercury	95×10^{-8}
Semiconductors:	
Pure carbon	3.5×10^{-5}
Pure germanium	0.60
Pure silicon	2300
Insulators:	
Amber	5×10^{14}
Mica	10^{11}–10^{15}
Teflon	10^{16}
Quartz	7.5×10^{17}

Electromotive Force

- When the current in the circuit is constant in magnitude and direction and is called **direct current DC**.

- A battery is called a *source of electromotive force* or, *emf*.
- **The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.**
- When an electric potential difference exists between two points, the source moves charges from the lower potential to the higher.

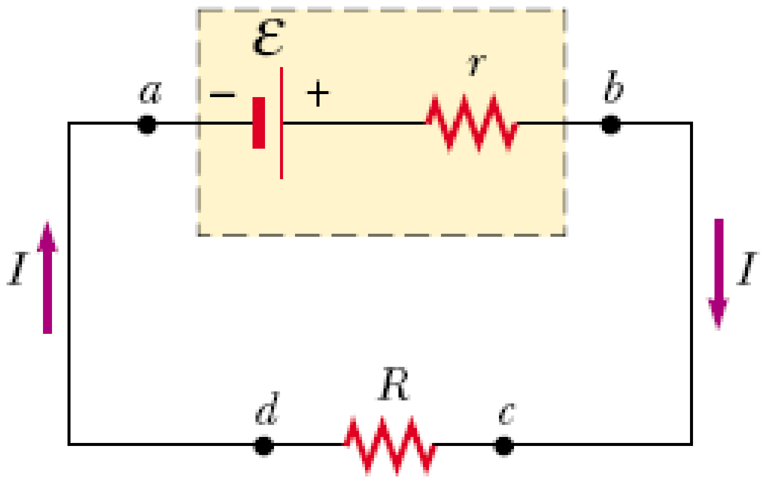
- What makes charges flow in circuits?
  - Potential difference ΔV
  - Source of charges

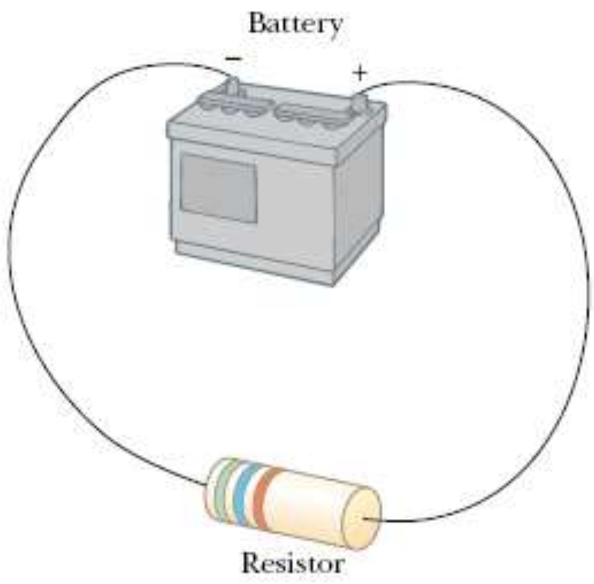
- This is what the EMF provides
  - NB: EMF=Electromotive force but it's not a force!!!

The internal resistance

- The resistance of the battery is called **internal resistance *r***.
- ***I*** is the current in the circuit, ***I_r*** is the current through the resistor, emf is ε
- The terminal voltage of the battery

$\Delta V = V_b - V_a$ is $\Delta V = \varepsilon - I_r$





Electrical energy and thermal energy.

- The resistor represents a *load* on the battery because the battery must supply energy to operate the device.
- The potential difference across the load resistance is
- **Δ*V* = IR** and

$$I = \frac{\varepsilon}{R + r}$$

- The total power output $I\varepsilon$ of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r.

$$I\varepsilon = I^2R + I^2r$$

Electrical Power and Electrical Work

All electrical circuits have three parts in common.

1. A voltage source.
2. An electrical device
3. Conducting wires.

- **The work done (W) by a voltage source is equal to the work done by the electrical field in an electrical device**,

1. **Work = Power x Time.**

The **electrical potential** is measured in **joules/coulomb** and a quantity of **charge is measured in coulombs**, so the **electrical work is measure in joules**.

A **joule/second** is a unit of power called the **watt**.

2. **Power = current x potential**

Or, $P = I\,V = I^2 R$

- **Energy = Power / Time**

Example 28.1 Terminal Voltage of a Battery

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

Solution Equation 28.3 gives us the current:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

and from Equation 28.1, we find the terminal voltage:

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

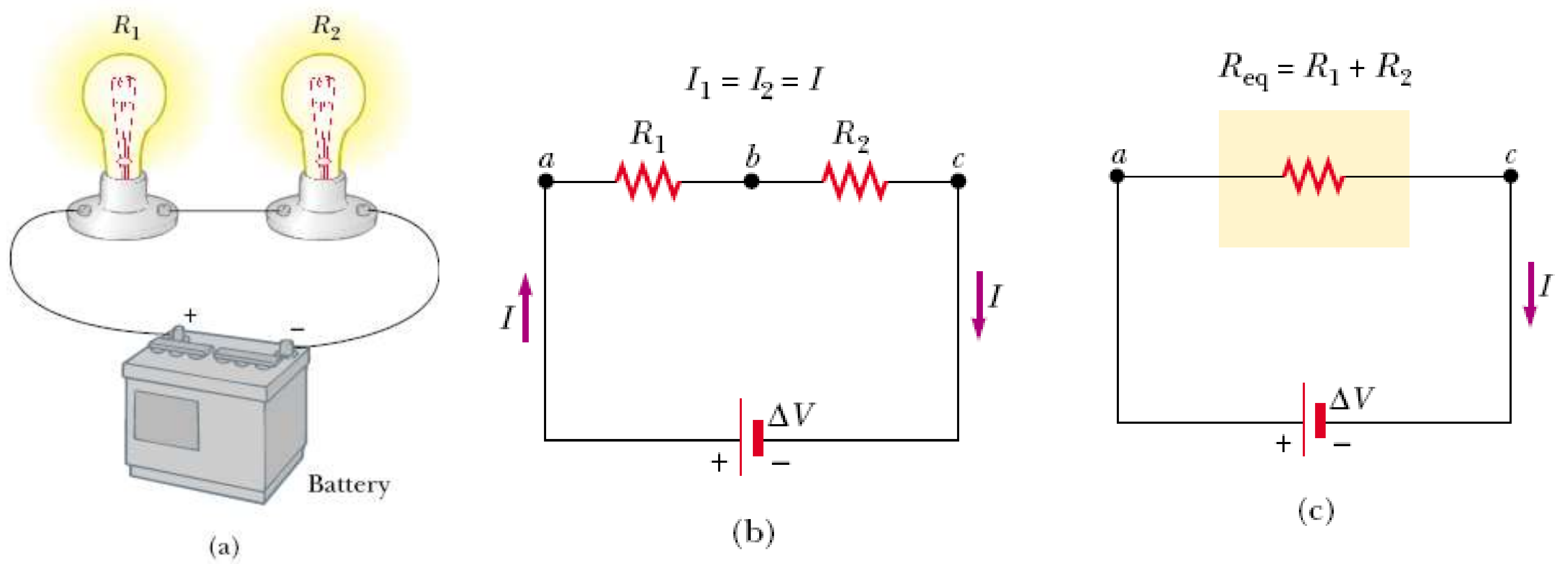
- Calculate the power delivered to the load resistor of 3 ohm when the current in the circuit is 3.93 A.

Solution The power delivered to the load resistor is

$$\mathcal{P}_R = I^2R = (3.93\ \text{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

Resistors in Series

- for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.



The potential difference across the battery is applied to the **equivalent resistance** $\boldsymbol{R_{eq}}$:

$$\Delta V = IR_{eq}$$

Resistors in Series

$\Delta V = IR_{eq}$

$$\Delta V = IR_{eq} = I(R_1 + R_2) \longrightarrow R_{eq} = R_1 + R_2$$

The equivalent resistance of three or more resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Problem 5.8. Consider the series portion of a circuit shown in Fig. 5-4. The current in the circuit flows from a to b and is 2.3 A.

(*a*) What is the equivalent resistance?

(*b*) What is the voltage across the entire circuit? Which point, a or b, is at the higher potential?

(*c*) What is the voltage across each resistor?

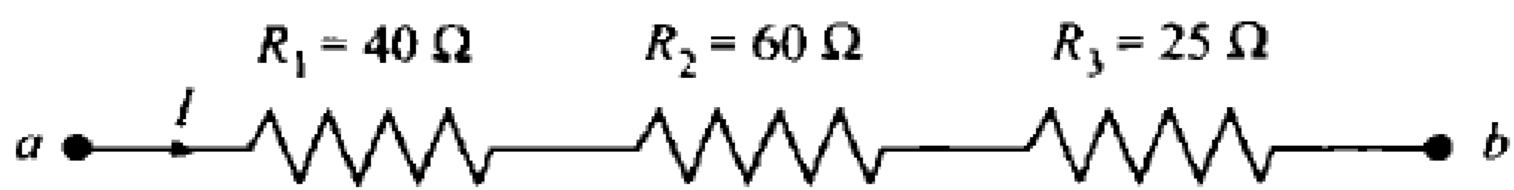


Fig. 5-4

Problem 5.8. Consider the series portion of a circuit shown in Fig. 5-4. The current in the circuit flows from *a* to *b* and is 2.3 A.

(*a*) What is the equivalent resistance?

(*b*) What is the voltage across the entire circuit? Which point, *a* or *b*, is at the higher potential?

(*c*) What is the voltage across each resistor?

Solution

(*a*) The equivalent resistance is the sum of all the resistances, or $R_{eq} = 40 + 60 + 25 = 125\ \Omega$.

(*b*) The voltage across the entire circuit is $V_{total} = IR_{eq} = (2.3\ \text{A})(125\ \Omega) = 288\ \text{V}$. Since the current flows from *a* to *b*, and the electric field does positive work in pushing charges through the resistors, energy is lost as the charges move through. Thus the potential at *a* is higher (by 288 V) than the potential at *b*.

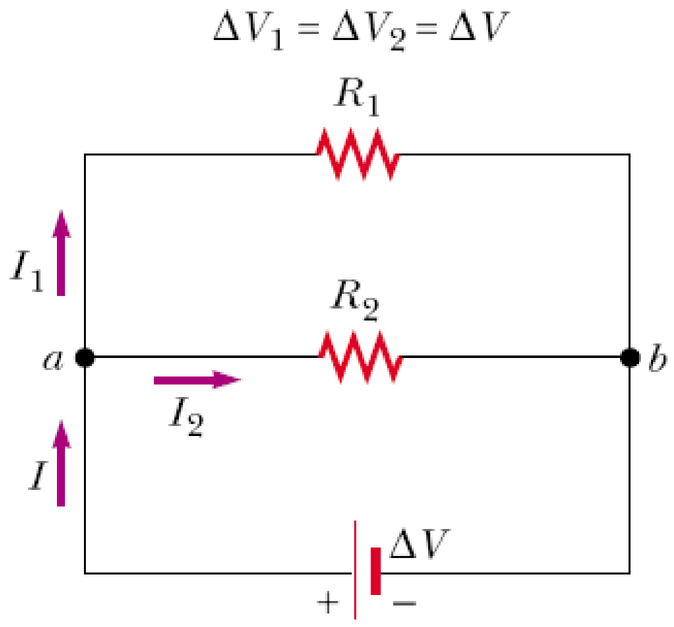
(*c*) The voltage across each resistor is IR_i. Thus $V_1 = (2.3\ \text{A})(40\ \Omega) = 92\ \text{V}$, $V_2 = (2.3\ \text{A})(60\ \Omega) = 138\ \text{V}$ and $V_3 = (2.3\ \text{A})(25\ \Omega) = 58\ \text{V}$.

> ***Note.*** Adding the voltages gives $92 + 138 + 58 = 288\ \text{V}$, which is the voltage we calculated in part(*b*).

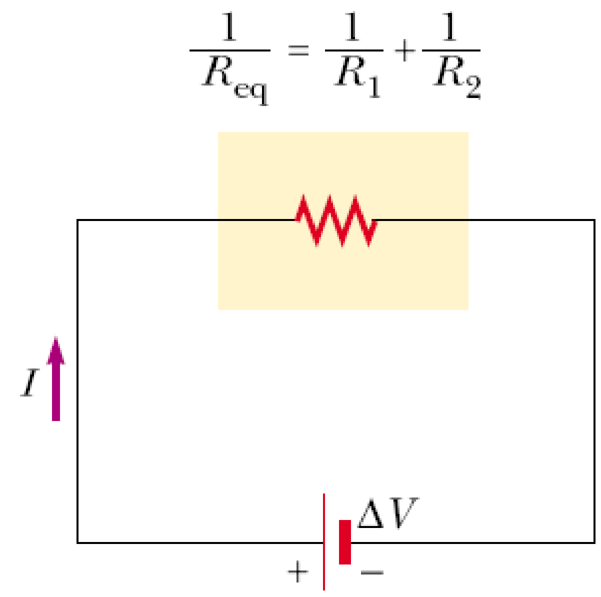
Resistors in Parallel

- The current I that enters point a must equal the total current leaving that point:

$$I = I_1 + I_2$$



(b)



(c)

When resistors are connected in parallel, the potential differences across the resistors is the same.

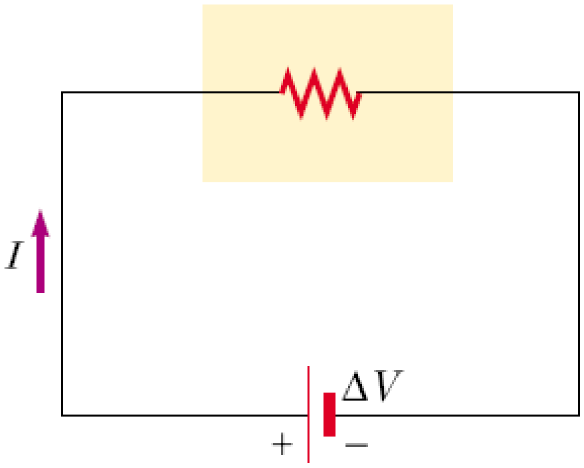
Resistors in Parallel

- Because the potential differences across the resistors are the same, the expression
- $\Delta V = IR$ gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



(c)

the equivalent resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Problem 5.9. Three resistors are connected in parallel, as in Fig. 5-5. The potential difference between *a* and *b* is 75 V.

(*a*) What is the equivalent resistance of this circuit?

(*b*) What is the current flowing from point *a*?

(*c*) What is the current in each resistor?

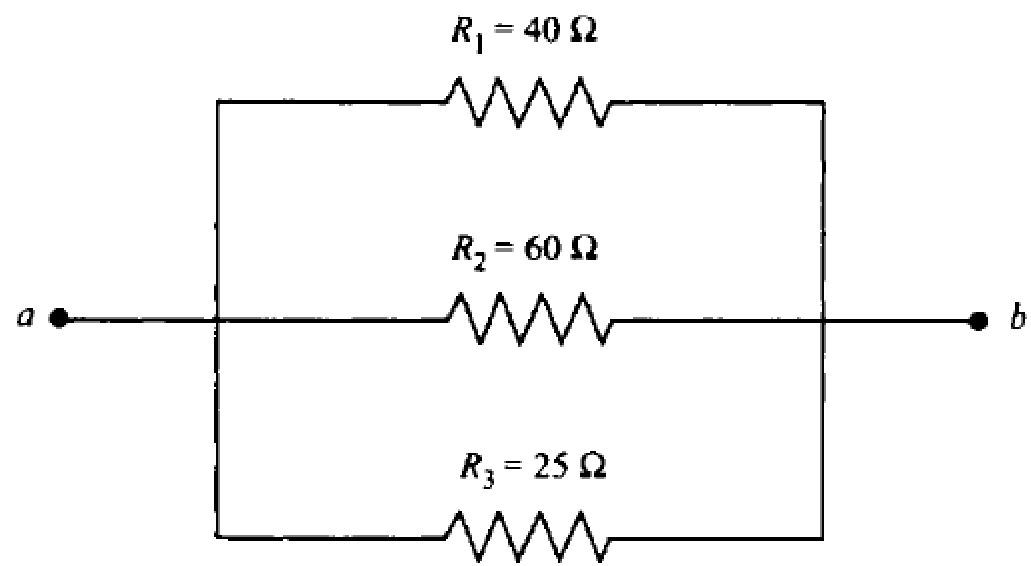


Fig. 5-5

Problem 5.9. Three resistors are connected in parallel, as in Fig. 5-5. The potential difference between a and b is 75 V.

(*a*) What is the equivalent resistance of this circuit?

(*b*) What is the current flowing from point a?

(*c*) What is the current in each resistor?

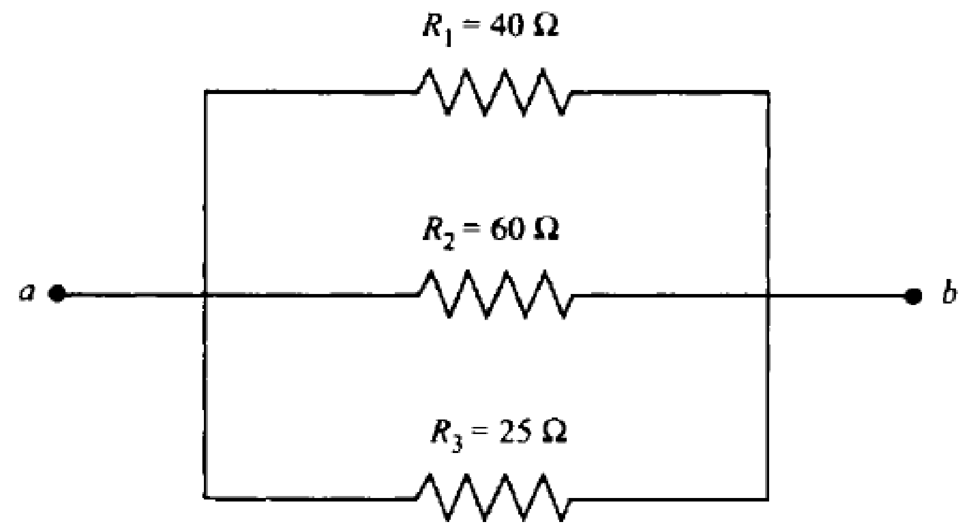


Fig. 5-5

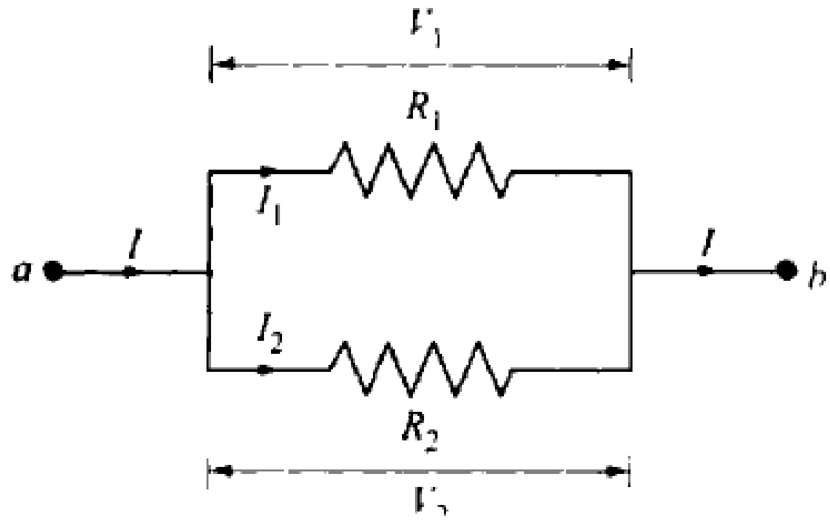
Solution

(*a*) The equivalent resistance is given by $1/R_{eq} = \Sigma(1/R_i) = 1/40 + 1/60 + 1/25 = 0.817$, or $R_{eq} = 12.2\ \Omega$.

(*b*) The total current is $I_{tot} = V/R_{eq}$. Thus $I_{tot} = 6.13$ A.

(*c*) The current in each resistor is $I_i = V/R_i$. Thus $I_1 = (75\ \text{V})/(40\ \Omega) = 1.88$ A, $I_2 = (75\ \text{V})/(60\ \Omega) = 1.25$ A, $I_3 = (75\ \text{V})/(25\ \Omega) = 3.0$ A. [The total current is $1.88 + 1.25 + 3.0 = 6.13$ A, as in part(b).]

Problem 5.20. In the circuit segment of Fig. 5-2(*b*), the voltage across the circuit is 55V. If $R_1 = 25\ \Omega$ and $R_2 = 35\ \Omega$, calculate (*a*) the current in each resistor; (*b*) the power dissipated in each resistor and (*c*) the power dissipated in the equivalent resistance. Compare the answer to this with the sum of the answers to (*b*).



(*b*)

Solution

(*a*) The voltage across each resistor is 55 V. Thus the current for each resistor is $I = V/R$. Then $I_1 = (55\ \text{V})/(25\ \Omega) = 2.2\ \text{A}$, and $I_2 = (55\ \text{V})/(35\ \Omega) = 1.57\ \text{A}$.

Problem 5.20. In the circuit segment of Fig. 5-2(*b*), the voltage across the circuit is 55V. If $R_1 = 25\ \Omega$ and $R_2 = 35\ \Omega$, calculate (*a*) the current in each resistor; (*b*) the power dissipated in each resistor and (*c*) the power dissipated in the equivalent resistance. Compare the answer to this with the sum of the answers to (*b*).

Solution

(*a*) The voltage across each resistor is 55 V. Thus the current for each resistor is $I = V/R$. Then $I_1 = (55\text{ V})/(25\ \Omega) = 2.2$ A, and $I_2 = (55\text{ V})/(35\ \Omega) = 1.57$ A.

(*b*) Since the voltage across each resistor is the same, the power in each resistor can be calculated using $P = V^2/R$. Thus $P_1 = (55\text{ V})^2/(25\ \Omega) = 121$ W, and $P_2 = (55\text{ V})^2/(35\ \Omega) = 86.4$ W. Alternatively, we could have used $P = I^2R$, using the current appropriate to each resistor. Then $P_1 = (2.2\text{ A})^2(25\ \Omega) = 121$ W, and $P_2 = (1.57\text{ A})^2(35\ \Omega) = 86.4$ W.

(c) The equivalent resistance is $R_{eq} = (25)(35)/(25 + 35) = 14.6\ \Omega$. The total power is therefore $P_{tot} = (55)^2/14.6 = 207.4$ W. This equals the sum of $P_1 + P_2 = 121 + 86.4$.

Kirchhoff's Rules

- The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff 's rules:

- **1. Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

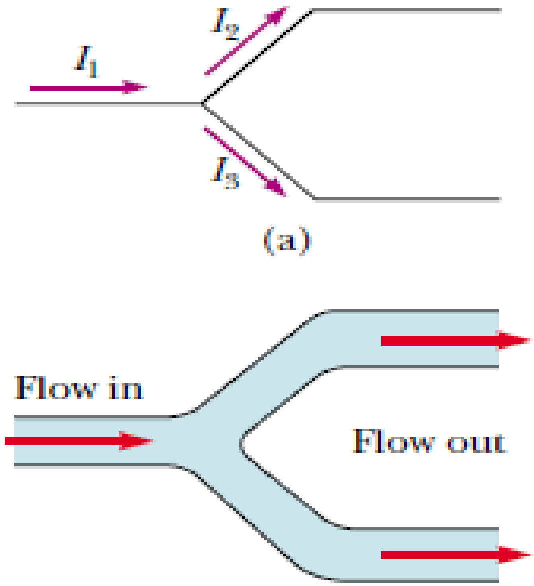
$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

- **2. Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0$$

Kirchhoff's Rules

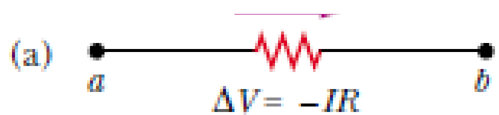
- **Kirchhoff's first rule is a statement of conservation of electric charge.**
- All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point.
- If we apply this rule to the junction shown in Figure, we obtain



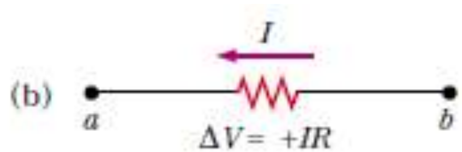
$$I_1 = I_2 + I_3$$

When applying Kirchhoff's second rule in practice, we imagine *traveling* around the loop and consider changes in *electric potential*, rather than the changes in *potential energy* described in the preceding paragraph. You should note the following sign conventions when using the second rule:

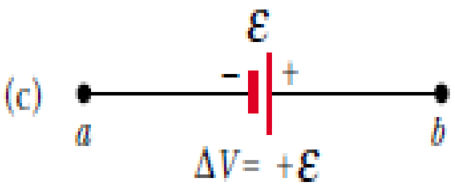
•Because charges move from the high-potential end of a resistor toward the low potential end, if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$.



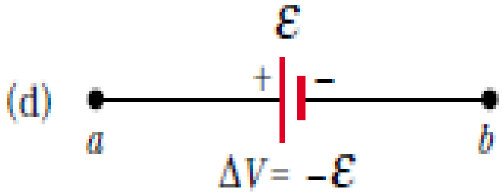
•If a resistor is traversed in the direction *opposite* the current, the potential difference ΔV across the resistor is $+IR$.



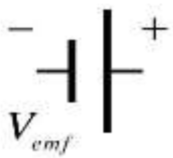
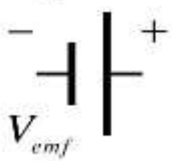
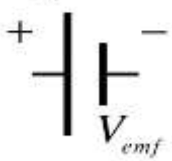
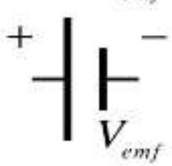
•If a source of emf (assumed to have zero internal resistance) is traversed in the DIRECTION of the emf (from - to +), the potential difference ΔV is +E. The emf of the battery increases the electric potential as we move through it in this direction.



•If a source of emf (assumed to have zero internal resistance) is traversed in the direction OPPOSITE the emf (from + to -), the potential difference ΔV is -E. In the case of the emf of the battery reduces the electric potential as we move through it.



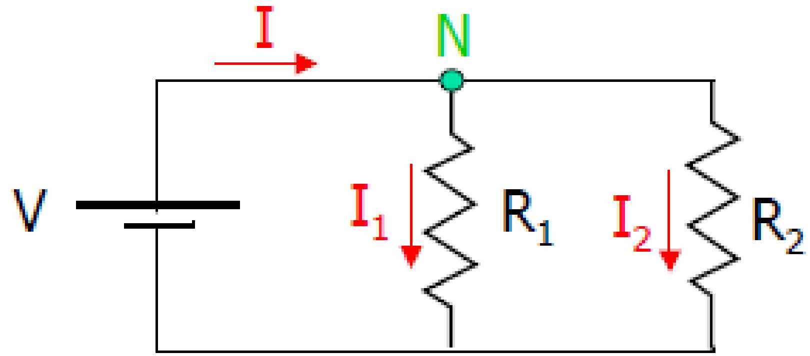
Circuit Analysis Conventions

Element	Analysis Direction	Current Direction	Voltage Drop
	$\rightarrow$	$\rightarrow$	$-iR$
	$\leftarrow$	$\rightarrow$	$+iR$
	$\rightarrow$	$\leftarrow$	$+iR$
	$\leftarrow$	$\leftarrow$	$-iR$
	$\rightarrow$		$+V_{emf}$
	$\leftarrow$		$-V_{emf}$
	$\rightarrow$		$-V_{emf}$
	$\leftarrow$		$+V_{emf}$

- Solve the circuit:

$$V = I_1 R_1 \quad \Rightarrow \quad I_1 = \frac{V}{R_1}$$

$$V = I_2 R_2 \quad \Rightarrow \quad I_2 = \frac{V}{R_2}$$



- Apply Kirchhoff's first law: $I = I_1 + I_2$

$$I = I_1 + I_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

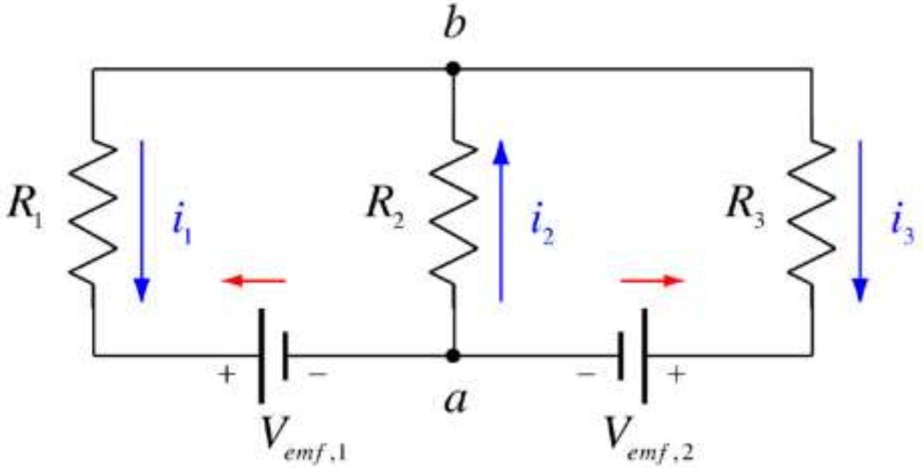
Example - Kirchhoff's Laws

- At junction ***b*** the incoming current must equal the outgoing current

$$i_2 = i_1 + i_3$$

- At junction ***a*** we again equate the incoming current and the outgoing current

$$i_1 + i_3 = i_2$$



- But this equation gives us the same information as the previous equation!
- We need more information to determine the three currents – 2 more independent equations

- We now have three equations

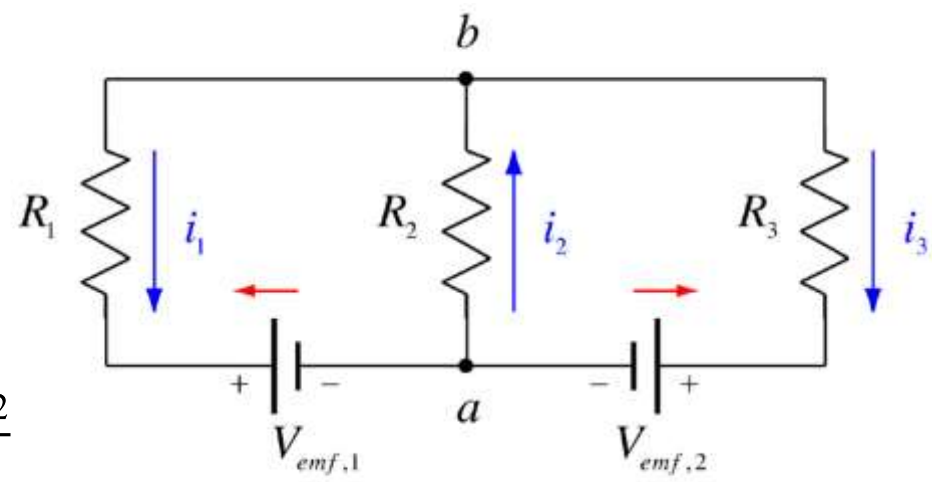
$$i_1 + i_3 = i_2 \qquad i_1 R_1 + V_{emf,1} + i_2 R_2 = 0 \qquad i_3 R_3 + V_{emf,2} + i_2 R_2 = 0$$

- And we have three unknowns i_1, i_2, and i_3
- We can solve these three equations in a variety of ways

$$i_1 = -\frac{(R_2 + R_3)V_{emf,1} - R_2 V_{emf,2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$i_2 = -\frac{R_3 V_{emf,1} + R_1 V_{emf,2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$i_3 = -\frac{-R_2 V_{emf,1} + (R_1 + R_2)V_{emf,2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



- Going around the left loop counterclockwise starting at point *b* we get

- Going around the right loop clockwise starting at point *b* we get

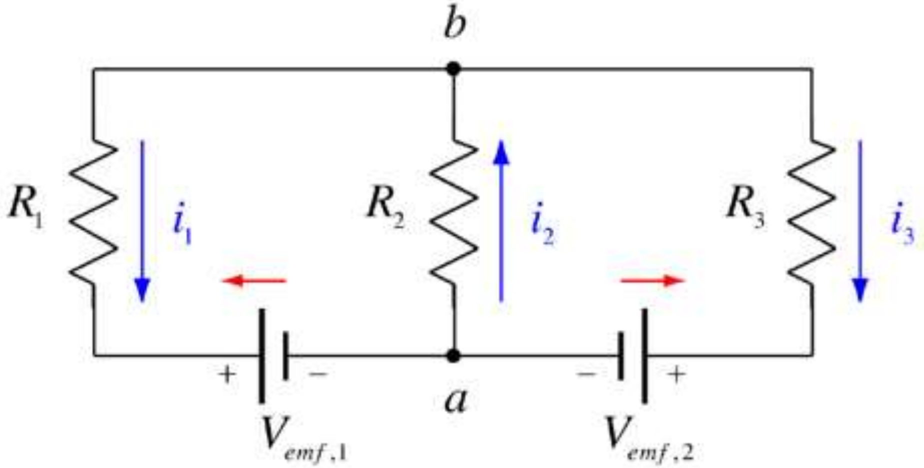
$$-i_1R_1 - V_{emf,1} - i_2R_2 = 0 \quad \Rightarrow \quad i_1R_1 + V_{emf,1} + i_2R_2 = 0$$

- Going around the outer loop clockwise starting at point *b* we get

$$-i_3R_3 - V_{emf,2} - i_2R_2 = 0 \quad \Rightarrow \quad i_3R_3 + V_{emf,2} + i_2R_2 = 0$$

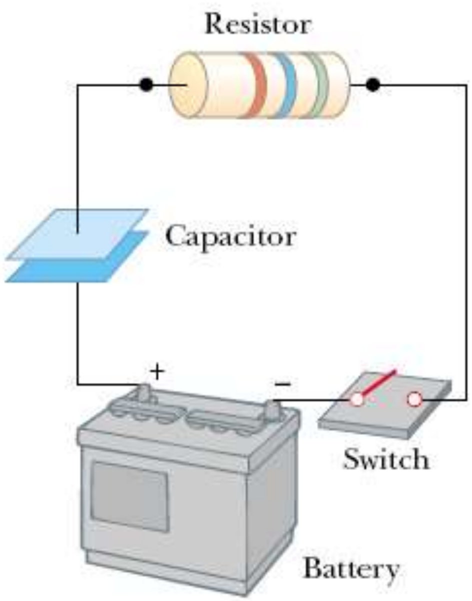
- But this equation gives us no new information!

$$-i_3R_3 - V_{emf,2} + V_{emf,1} + i_1R_1 = 0$$

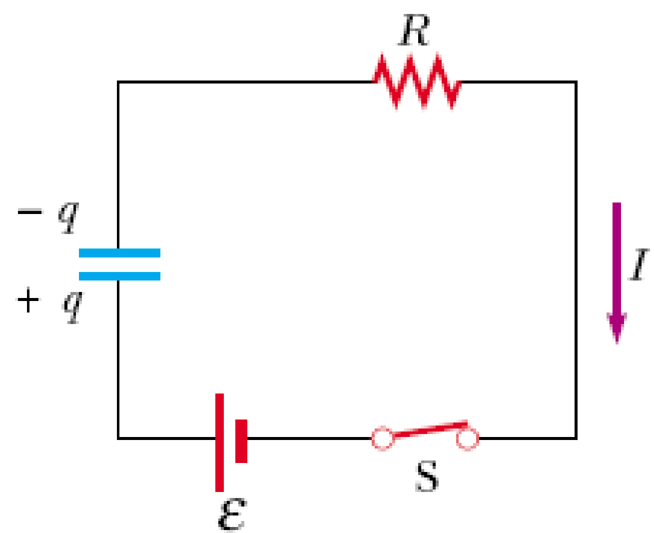


Charging and Discharging Processes in RC Circuits

- A circuit containing a series combination of a resistor and a capacitor is called An **RC Circuits**



(a)



(c) $t > 0$

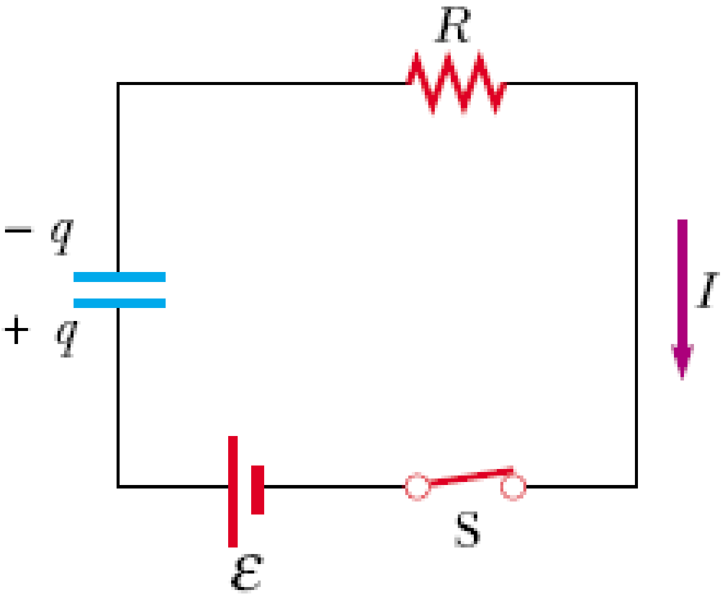
Circuit diagram

A capacitor in series with a resistor, switch, and battery.

RC Circuits

- To analyze this circuit quantitatively, let us apply Kirchhoff's loop rule to the circuit after the switch is closed. Traversing the loop in Fig. c clockwise gives

$$\mathcal{E} - \frac{q}{C} - IR = 0$$



(c) $t > 0$

where q/C is the potential difference across the capacitor and IR is the potential difference across the resistor.

RC Circuits

- **Charging a Capacitor**

Charge as a function of time
for a capacitor being charged

$$q(t) = C\mathcal{E}(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

The charging current is

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$

The quantity *RC*, which appears in the exponents of Equations, is called the time constant - of the circuit.
It represents the time interval during which the current decreases to 1/*e* of its initial value.

RC Circuits

- Discharging a Capacitor

Charge as a function of time for a discharging capacitor

$$q(t) = Qe^{-t/RC}$$

Current as a function of time for a discharging capacitor:

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}e^{-t/RC}$$

where $Q/RC = I_0$ is the initial current.
The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged.