Electric Flux

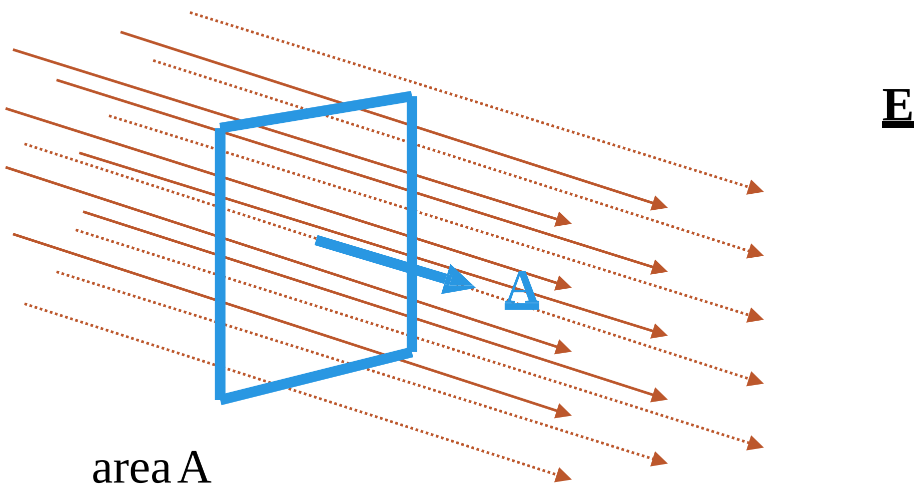
- ***Flux :*** The rate of flow through an **area** or **volume**. It can also be viewed as the product of an area and the vector field across the area.

- ***Electric Flux:*** The rate of flow of an electric field through an area or volume represented by the number of E field lines penetrating a surface.

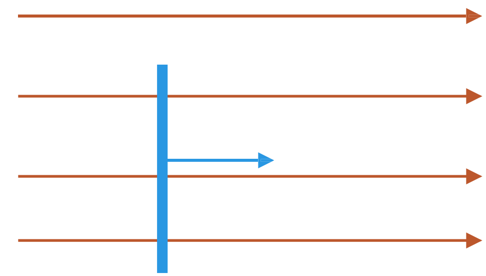
- ***Electric Flux:*** is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field.
- $\Phi_E = E.A$
- Units: $N \cdot m^2 / C$

Electric Flux

We define the electric flux Φ, of the electric field $\underline{E}$, through the surface A, as:



$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$



Normal to surface,
magnitude A

$\underline{\mathbf{A}} \equiv \longrightarrow$

Where:

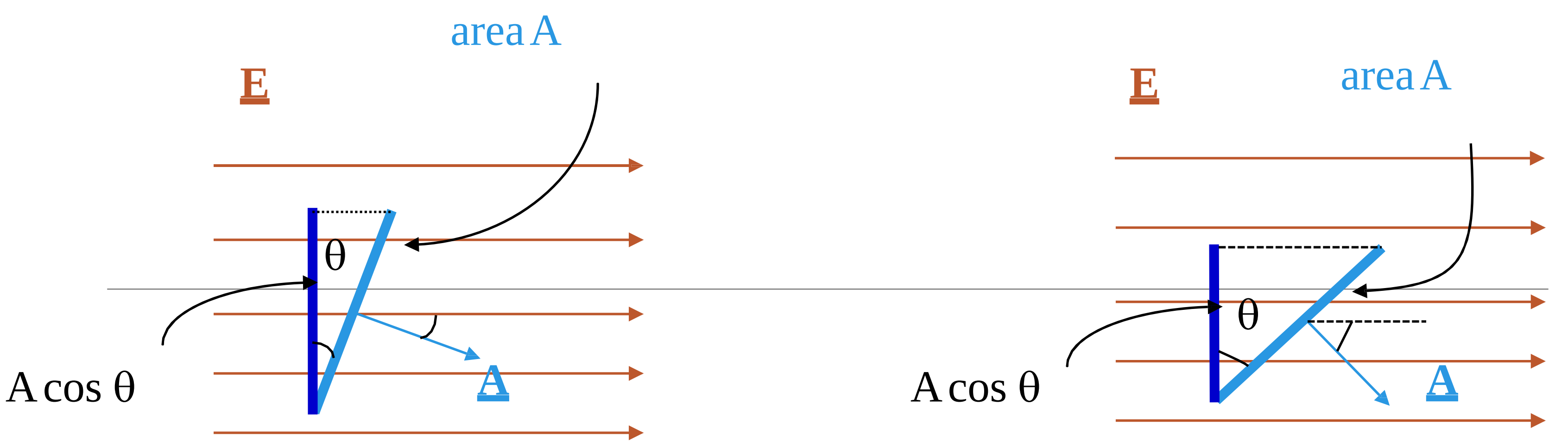
$\underline{A}$ is a vector normal to the surface
(magnitude A, and direction normal to the surface).

Electric Flux

The flux also depends on orientation

$$\Phi = \underline{E} \cdot \underline{A} = E\,A\cos\theta$$

θ is the angle between $\underline{E}$ and $\underline{A}$



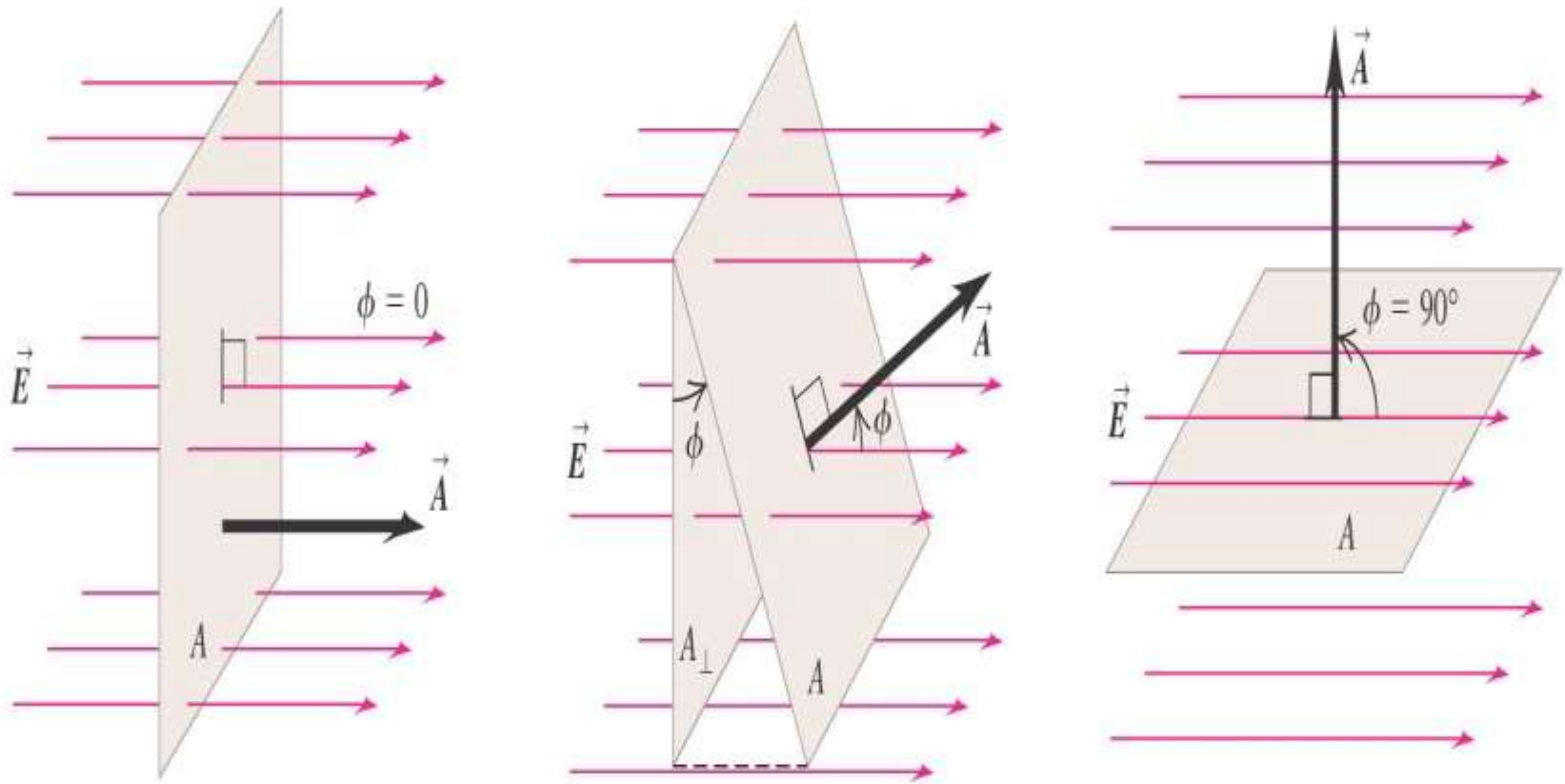
The number of field lines through the tilted surface equals the number through its projection. Hence, the flux through the tilted surface is simply given by the flux through its projection: E (A cos θ).

Flux of a Uniform Electric Field

(a) Surface is face-on to electric field:
- $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.

(b) Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{E}$ and $\vec{A}$ is ϕ.
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.

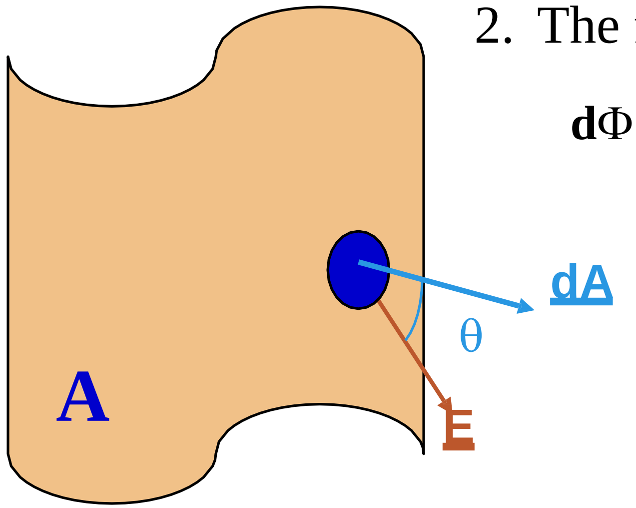
(c) Surface is edge-on to electric field:
- $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.



What if the surface is curved, or the field varies with position ??

$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$

1. We divide the surface into small regions with area dA



2. The flux through dA is $d\Phi = E\, dA \cos\theta$

$d\Phi = \underline{\mathbf{E}} \cdot d\underline{\mathbf{A}}$

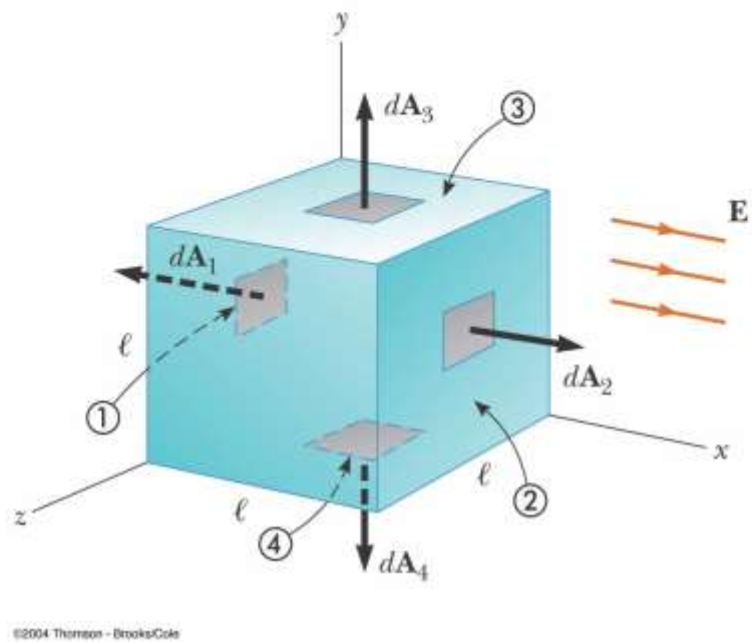
3. To obtain the total flux we need to integrate over the surface A

$$\Phi = \int d\Phi = \int \underline{\mathbf{E}} \cdot d\underline{\mathbf{A}}$$

Electric flux has SI units of volt metres (V m), or, equivalently, newton metres squared per coulomb ($N\ m^2\ C^{-1}$). Thus, the SI base units of electric flux are $kg \cdot m^3 \cdot s^{-3} \cdot A^{-1}$.

Flux Through a Cube

- Uniform electric field **E** = $(E, 0,0)$



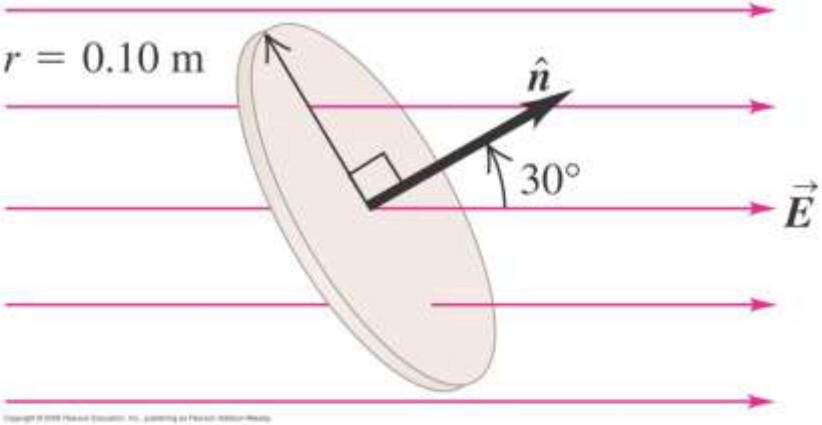
- The flux through the surfaces 3, 4, 5, and 6 equal zero =>
  Side 1: $\Phi = -E\,l^2$
  Side 2: $\Phi = E\,l^2$
  For the other sides, $\Phi = 0$
  Therefore, $\Phi_{total} = 0$

- The net flux over all six faces is

$$\Phi_E = -El^2 + El^2 + 0 + 0 + 0 + 0 = 0$$

Few examples on calculating the electric flux

-Find electric flux

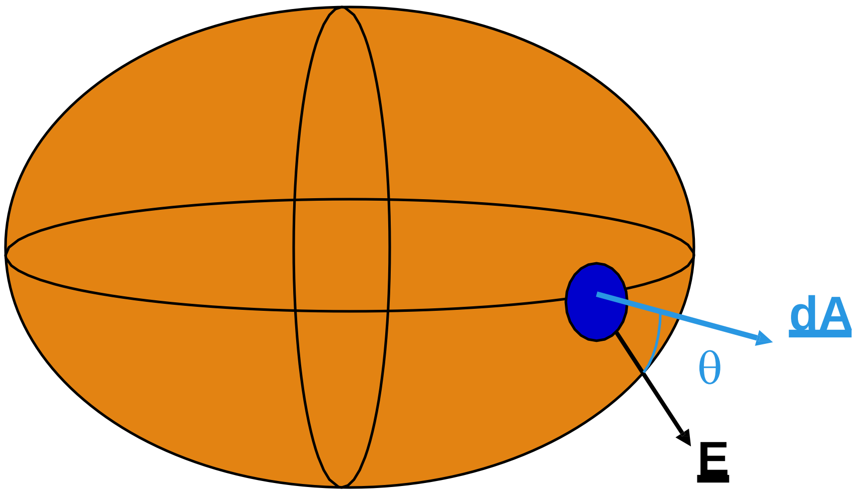


$$E = 2 \cdot 10^3 [N/C]$$

In the case of a closed surface

$$\Phi = \oint d\Phi = \oint \underline{E} \bullet \underline{dA} = \frac{\sum_{inside} q}{\varepsilon_0}$$

The loop means the integral is over a closed surface.



Definitions

- ***Symmetry***:

  The balanced structure of an object, the halves of which are alike

- ***Closed surface:***

  A surface that divides space into an inside and outside region, so one can't move from one region to another without crossing the surface

- ***Gaussian surface:***

  A hypothetical closed surface that has the same symmetry as the problem we are working on note this is not a real surface it is just an mathematical one

Gauss' Law

Carl Friedrich Gauss
1777 – 1855

- Gauss' Law: electric flux through any closed surface is proportional to the net charge $\boldsymbol{Q}$ inside the surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$1/\varepsilon_0 = 4\pi k_e$$

- $\varepsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$: permittivity of free space
- The area in Φ is an imaginary Gaussian surface (does not have to coincide with the surface of a physical object)

Gauss's Law

The total flux within a closed surface …

… is proportional to the enclosed charge.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Gauss's Law is always true, but is only useful for certain very simple problems with great symmetry.

Gauss' Law

- *Gauss' Law* depends on the enclosed charge only

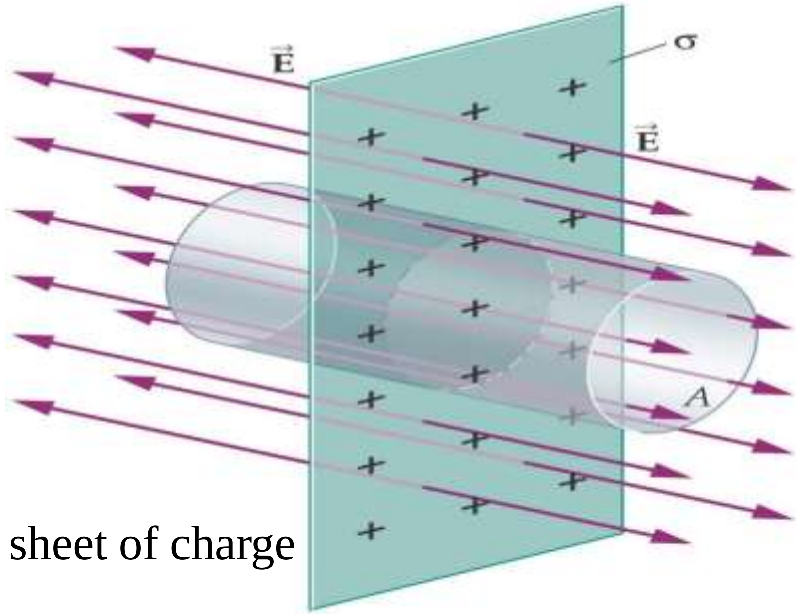
$$\Phi = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

1. If there is a positive net flux there is a net positive charge enclosed
2. If there is a negative net flux there is a net negative charge enclosed
3. If there is a zero net flux there is no net charge enclosed

- Gauss' Law works in cases of symmetry

Applying Gauss's Law: Field Due to a Plane of Charge

Gauss's law is useful only when the electric field is constant on a given surface



Infinite sheet of charge

- The uniform field must be perpendicular to the sheet and directed either toward or away from the sheet
- Use a cylindrical Gaussian surface
- The flux through the ends is EA and there is no field through the curved part of the surface
- Surface charge density $\sigma = Q / A$

1.Select Gauss surface In this case a cylindrical

2.Calculate the flux of the electric field through the Gauss surface
$\Phi = 2EA$

3.Equate $\Phi = q_{encl}/\varepsilon_0$
$2EA = q_{encl}/\varepsilon_0$

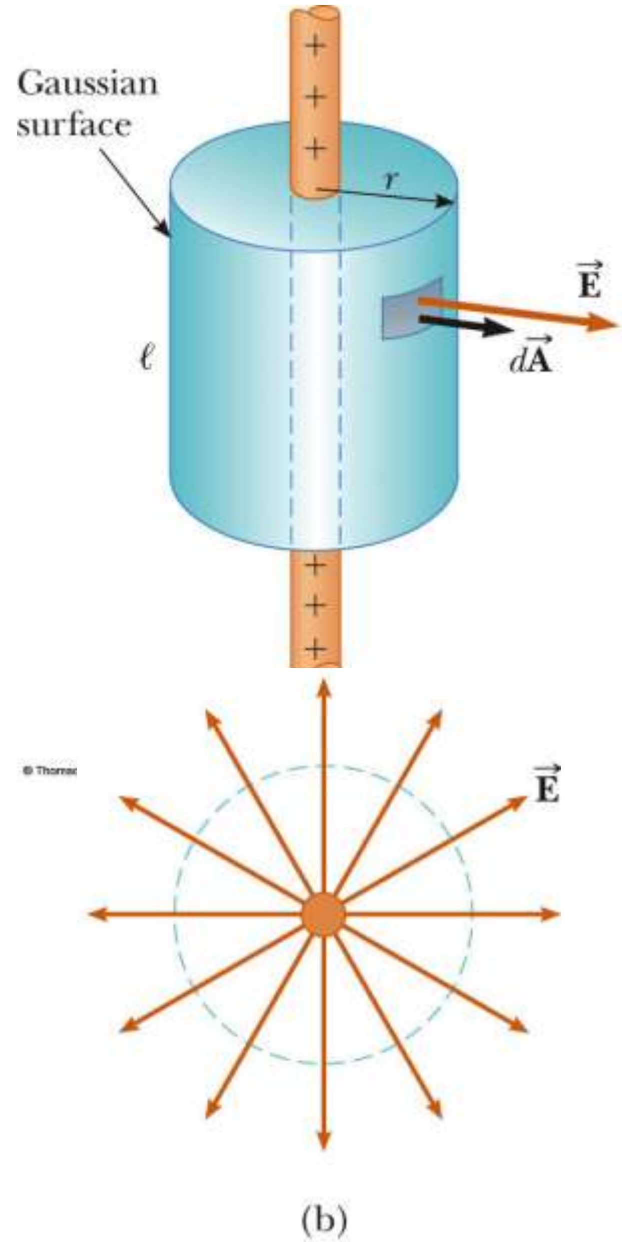
4.Solve for E
$E = q_{encl} / 2A\varepsilon_0 = \sigma / 2\varepsilon_0$
with $\sigma = (q_{encl} / A)$

Field Due to a Line of Charge

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{Q}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = 2k\frac{\lambda}{r}$$

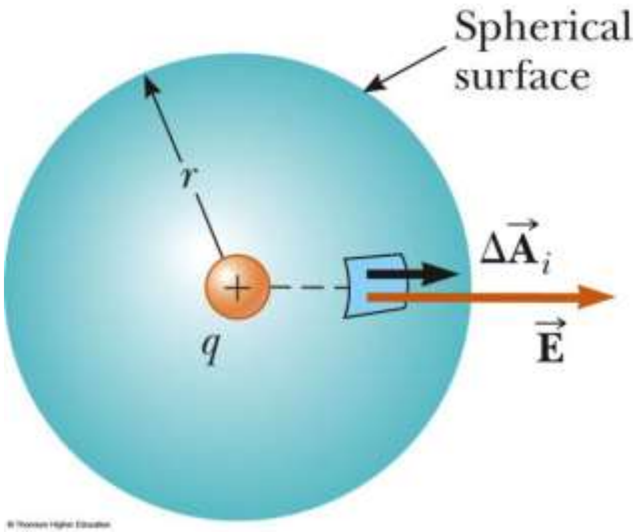


Electric field produced by a point charge

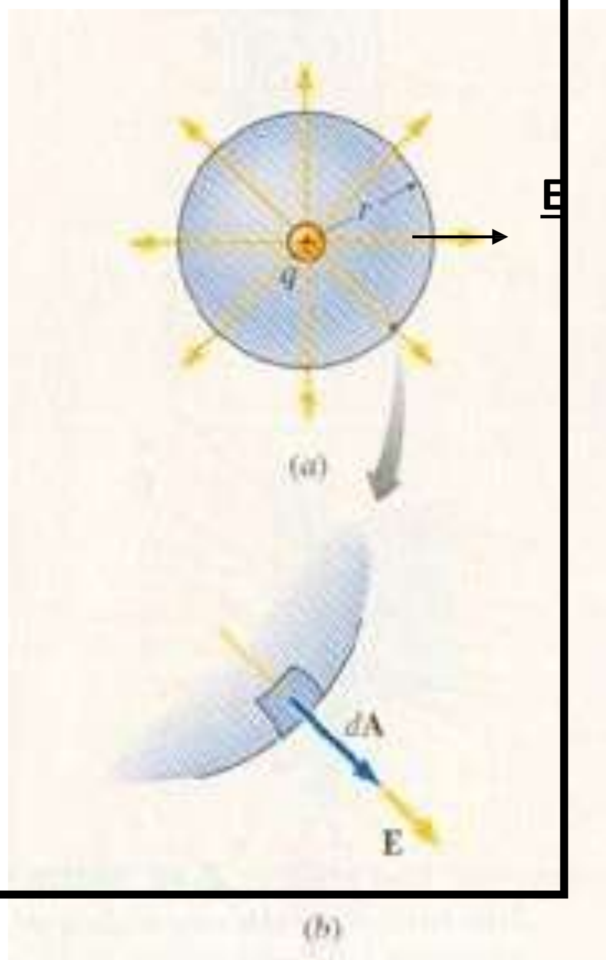
- A positive point charge q is located at the center of a sphere of radius r
- The magnitude of the electric field everywhere on the surface of the sphere is $E = k_e q / r^2$
- $A_{\text{sphere}} = 4\pi r^2$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA$$

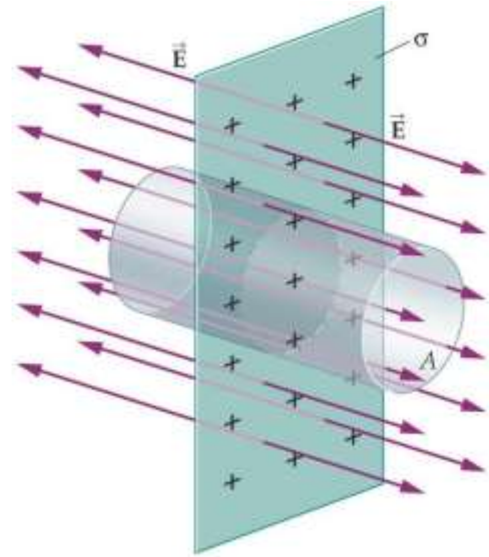
$$= k_e \frac{q}{r^2} \cdot 4\pi r^2 = 4\pi k_e q = \frac{q}{\varepsilon_0}$$



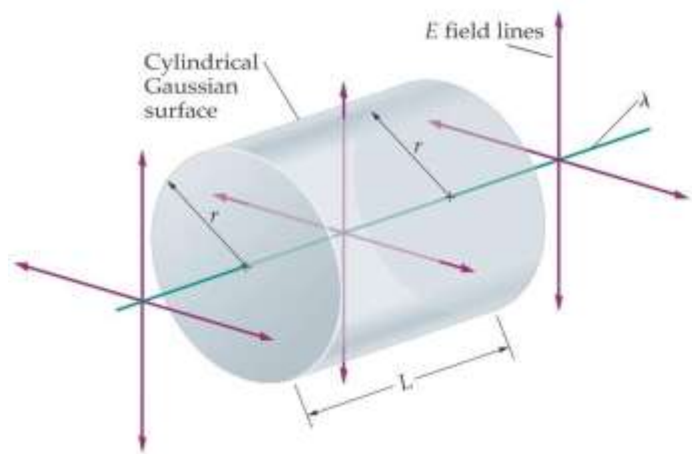
Spherical geometry



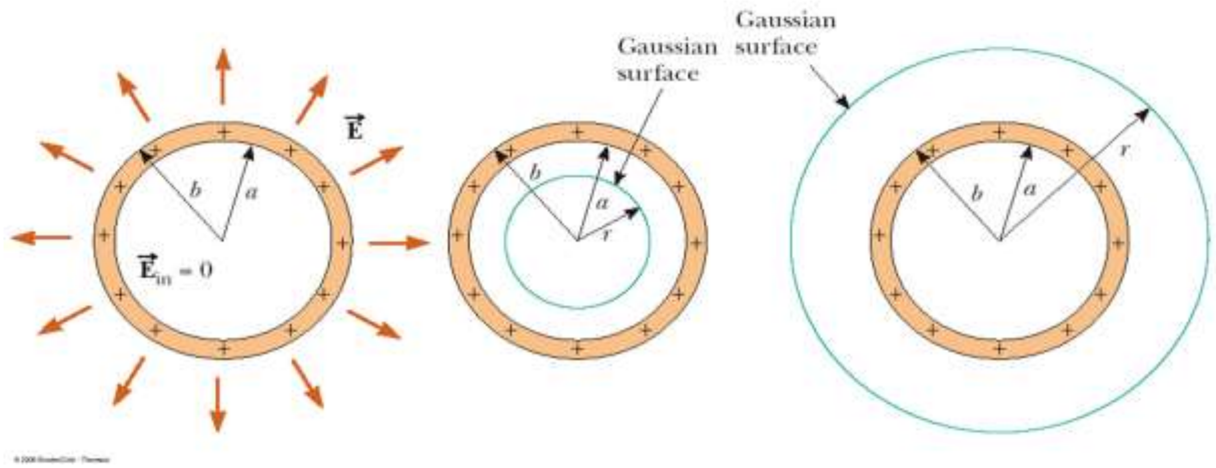
Planar geometry



Cylindrical geometry



Electric Field of a Charged Thin Spherical Shell



- The electric field **inside** the shell is ***zero***
- The calculation of the field **outside** the shell is identical to that of a point charge

$$E = \frac{Q}{4\pi r^2 \varepsilon_o} = k_e \frac{Q}{r^2}$$

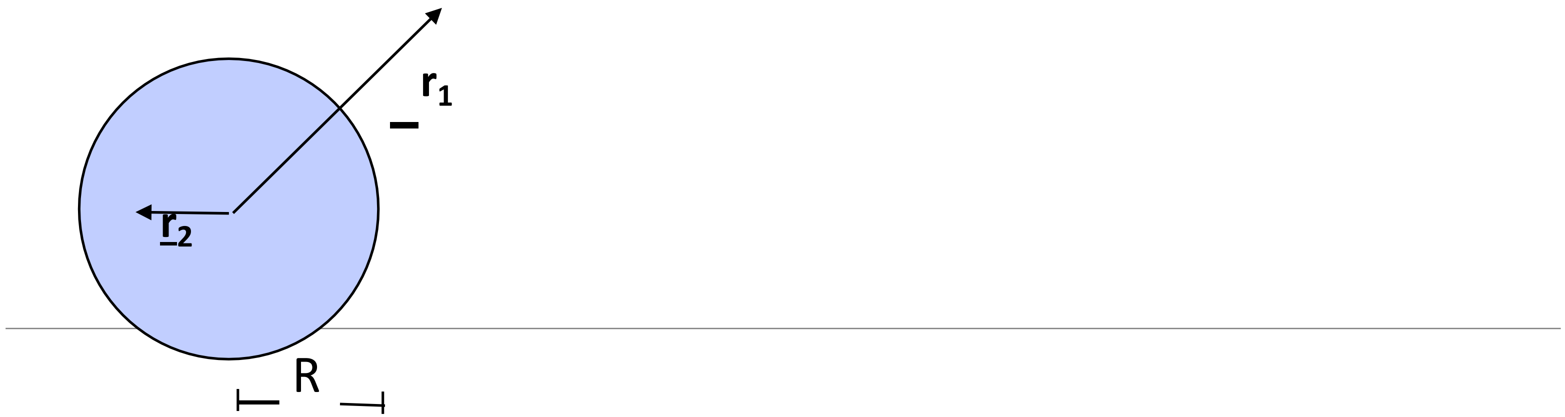
Example 08

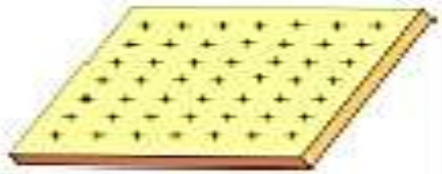
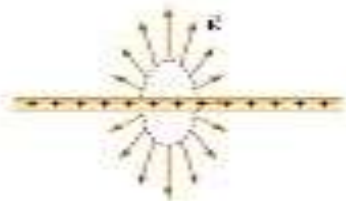
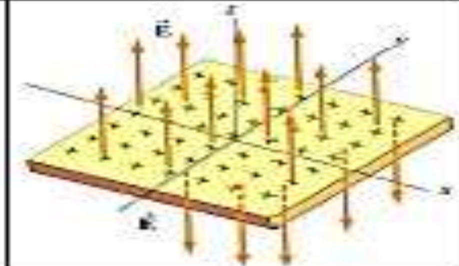
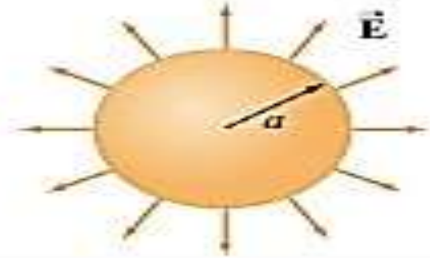
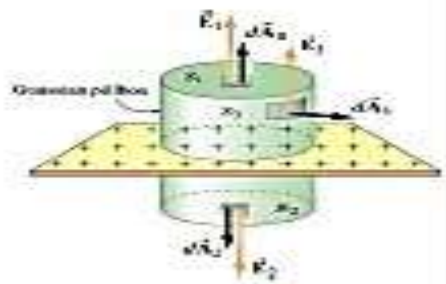
- A sphere with centre ***O*** and radius ***R*** is charged in volume with charge density $\rho = \rho_0 . r / R$ (ρ_0 is constant).

1) What is the total charge inside the sphere ?
2) Apply Gauss's theorem to determine the electric field at any point ***M*** in space.
3) Deduce the expression for the potential ***V(r)*** at any point in space.

Problem: Sphere of Charge Q

A charge Q is uniformly distributed through a sphere of radius R. What is the electric field as a function of **r** ?. Find **E** at $\mathbf{r}_1$ and $\mathbf{r}_2$.



System	Infinite line of charge	Infinite plane of charge	Uniformly charged solid sphere
Figure			
Identify the symmetry	Cylindrical	Planar	Spherical
Determine the direction of $\vec{E}$			
Divide the space into different regions	$r > 0$	$z > 0$ and $z < 0$	$r \le a$ and $r \ge a$
Choose Gaussian surface	 Coaxial cylinder	 Gaussian pillbox	 Concentric sphere
Calculate electric flux	$\Phi_E = E(2\pi r l)$	$\Phi_E = EA + EA = 2EA$	$\Phi_E = E(4\pi r^2)$
Calculate enclosed charge q_{in}	$q_{enc} = \lambda l$	$q_{enc} = \sigma A$	$q_{enc} = \begin{cases} Q(r/a)^3 & r \le a \\ Q & r \ge a \end{cases}$
Apply Gauss's law $\Phi_E = q_{in}/\varepsilon_0$ to find E	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$	$E = \dfrac{\sigma}{2\varepsilon_0}$	$E = \begin{cases} \dfrac{Qr}{4\pi\varepsilon_0 a^3}, & r \le a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}, & r \ge a \end{cases}$