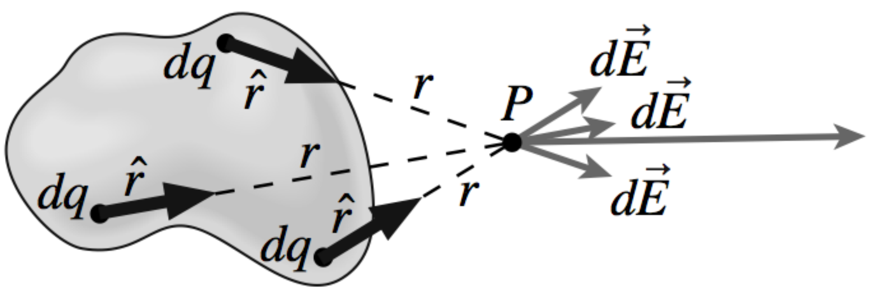
Continuous charge distributions

- Charge ultimately resides on individual particles, but it’s often convenient to consider it distributed continuously on a line, over an area, or throughout space.
- The electric field of a charge distribution follows by summing that is, integrating the fields of individual charge elements ***dq***, each treated as a point charge:



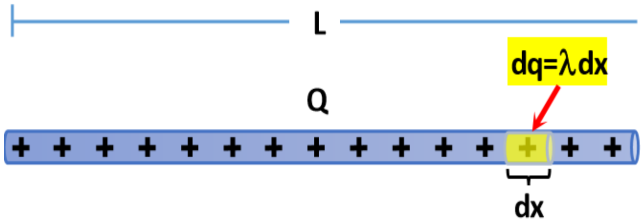
$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\,\hat{r}$$

Continuous charge distributions

General charge distributions:

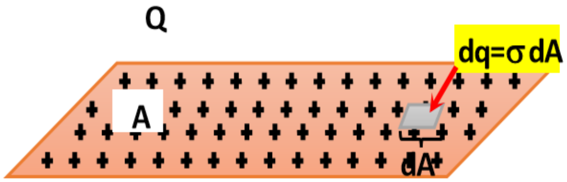
1- Linear charge distribution: $\lambda = \frac{Q}{L}$

- **Linear charge density:** λ (charge per unit length).
- **Differential charge element:** $dq = \lambda dx$ (where dx is a differential element of length as shown) .



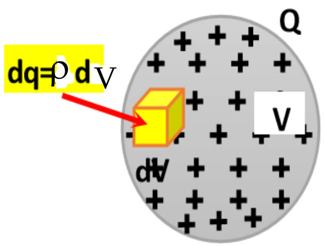
2- Surface charge distribution: $\sigma = \frac{Q}{A}$

- **Surface charge density: σ** (charge per unit surface).
- **Differential charge element:** $dq = \sigma dA$ (where dA is a differential element of surface area as shown).

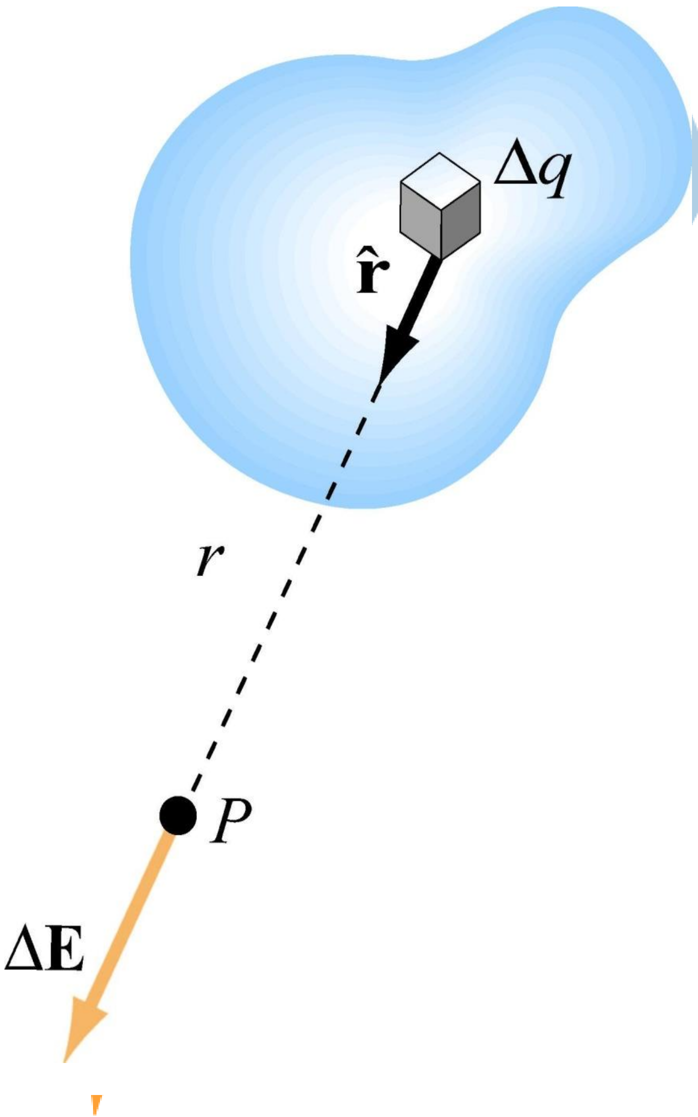


3- Volume charge distribution: $\rho = \frac{Q}{V}$

- **Volume charge density: ρ** (charge per unit Volume)
- **Differential charge element:** $dq = \rho dV$ (where dV is a differential element of volume as shown).



Continuous Charge Distributions



Break distribution into parts:

$$Q = \sum_i \Delta q_i \rightarrow \iiint_V dq$$

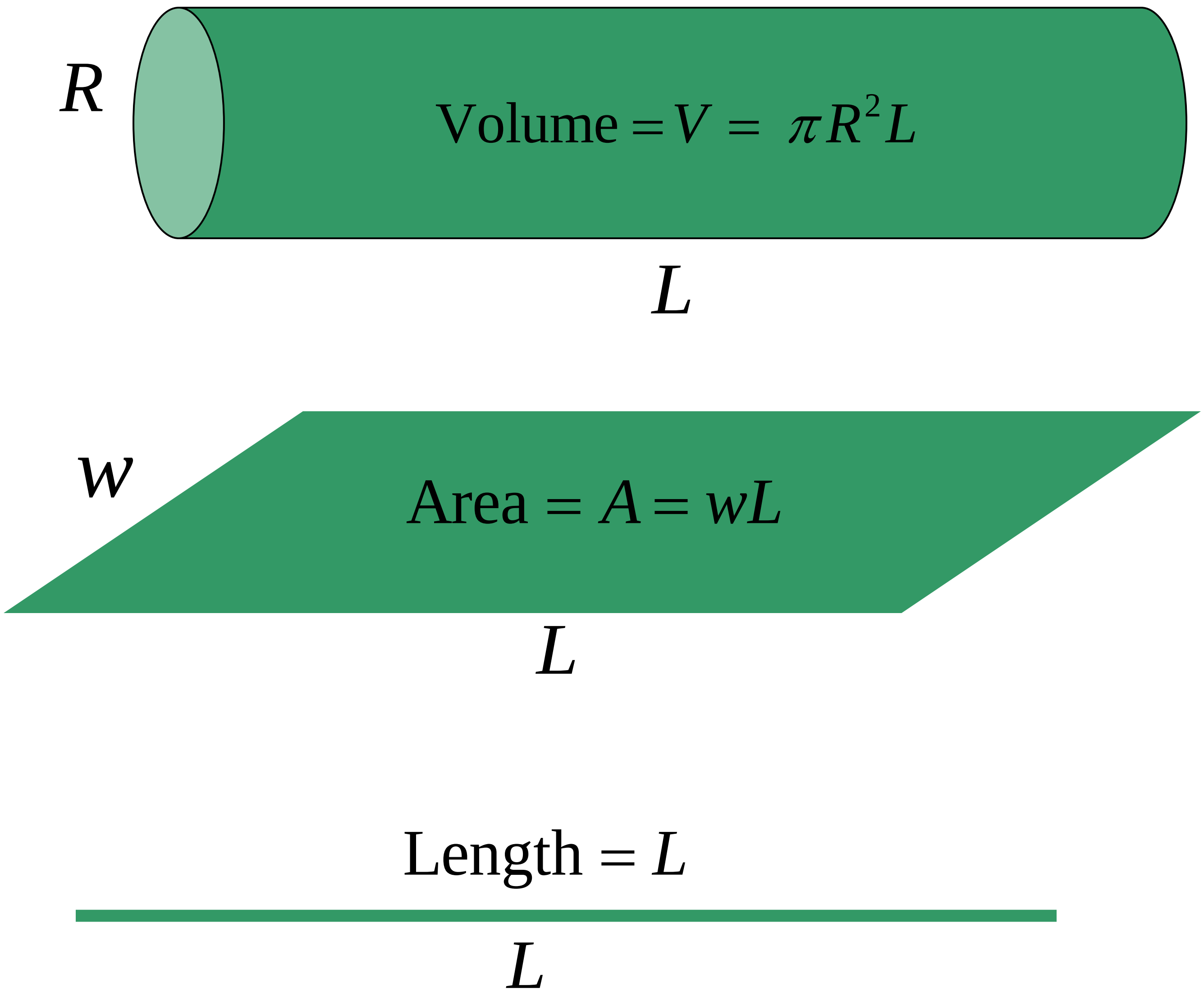
E field at *P* due to *dq*

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}} \rightarrow d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}}$$

Superposition:

$$\boxed{\vec{\mathbf{E}} = \sum \Delta\vec{\mathbf{E}} \rightarrow \int d\vec{\mathbf{E}}}$$

Continuous Sources: Charge Density



$$dQ = \rho\, dV$$

$$\rho = \frac{Q}{V}$$

$$dQ = \sigma\, dA$$

$$\sigma = \frac{Q}{A}$$

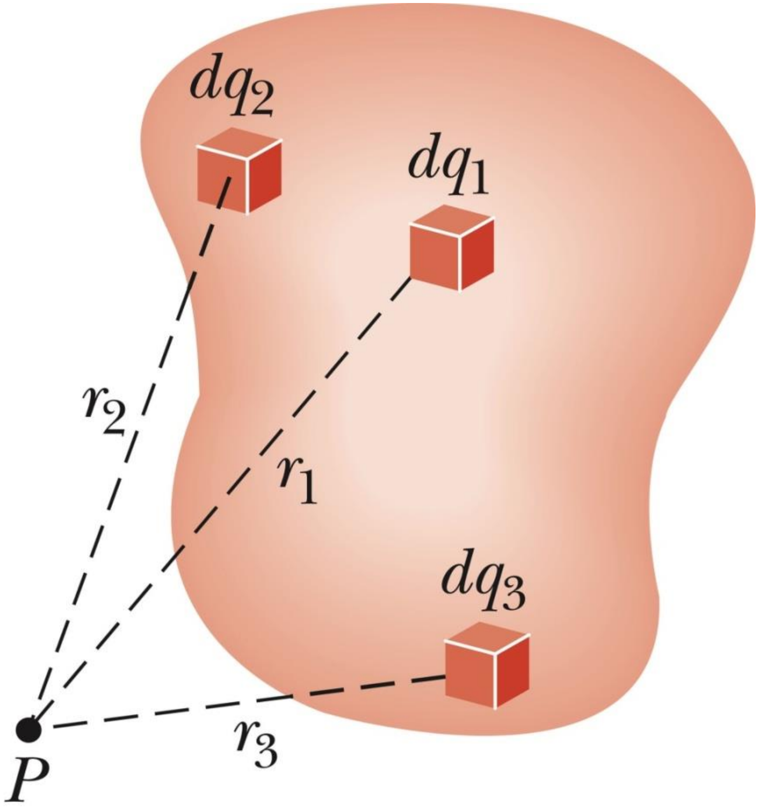
$$dQ = \lambda\, dL$$

$$\lambda = \frac{Q}{L}$$

Electric Potential for a Continuous Charge Distribution

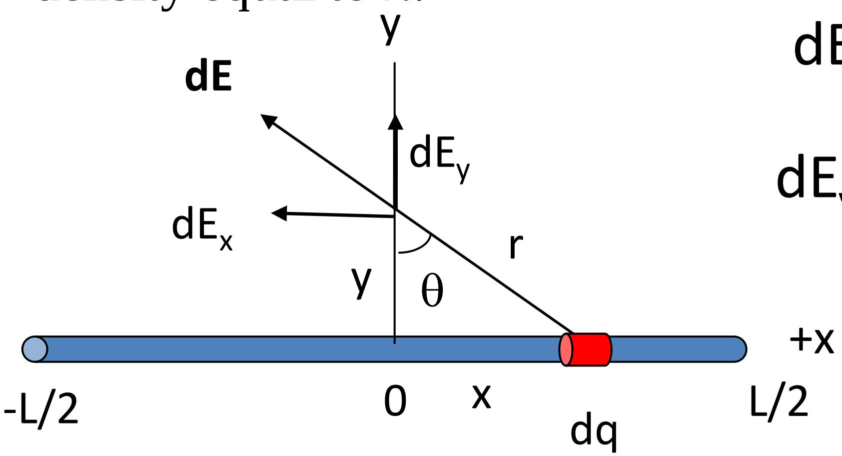
- **Method 1**: The charge distribution is known.
- Consider a small charge element *dq*
- Treat it as a point charge.
- The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$



Example 5:

Find electric field due to a line of uniform charge of length *L* with linear charge density equal to λ.



$$dE = k\,dq/r^2$$

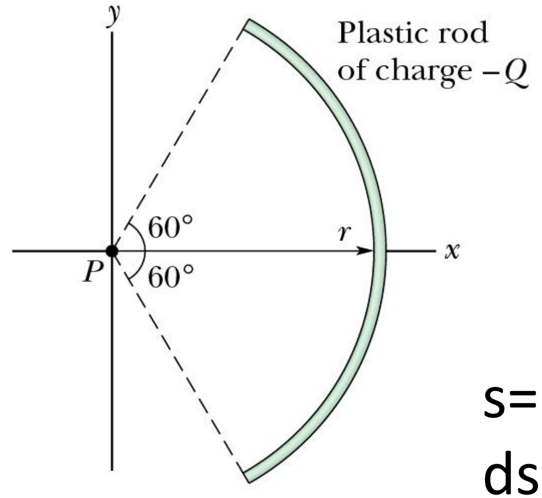
$$dE_y = dE\cos\theta$$

$$dq = \lambda dx$$

Field due to arc of charge

Example 6:

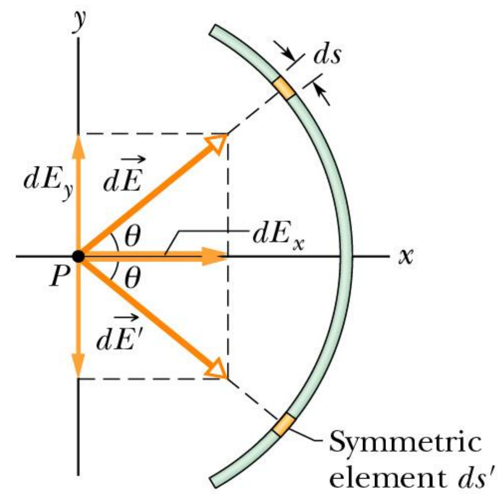
Find electric field due to a arc of uniform charge of length ***s*** with linear charge density equal to λ.



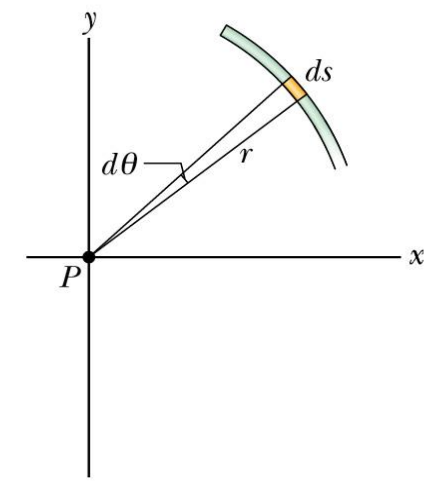
(a)

s=r θ

ds=r dθ



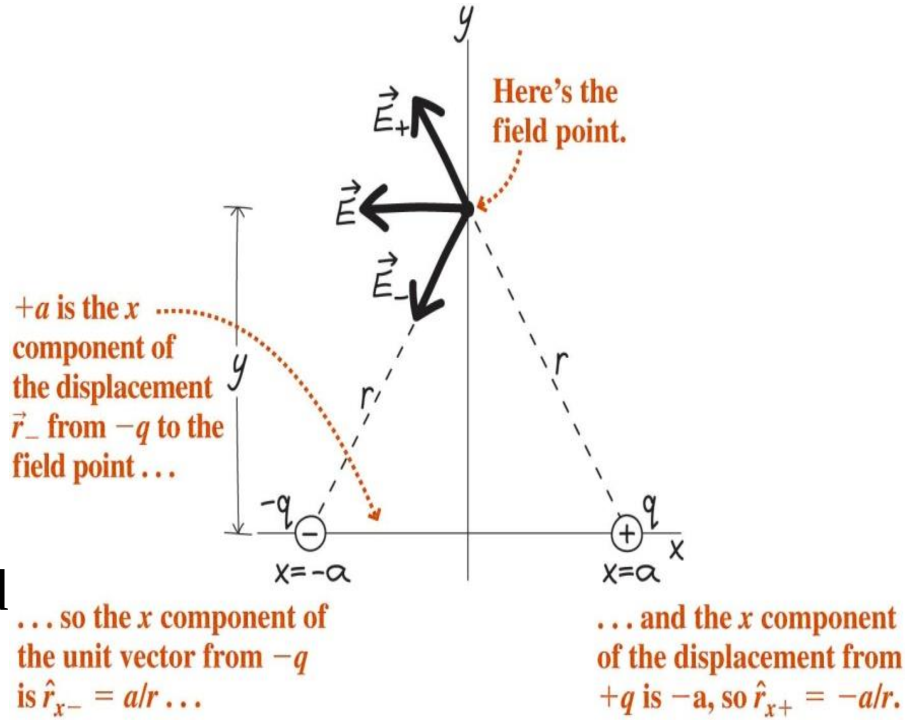
(b)

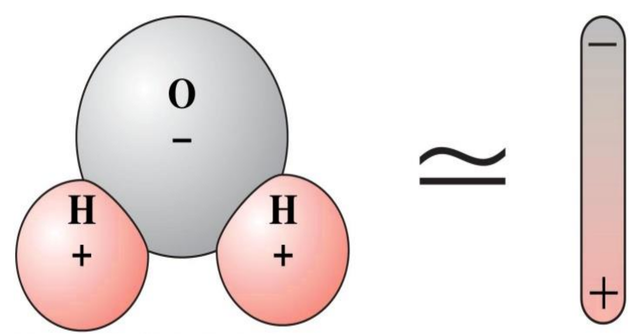


(c)

The dipole: an important charge distribution

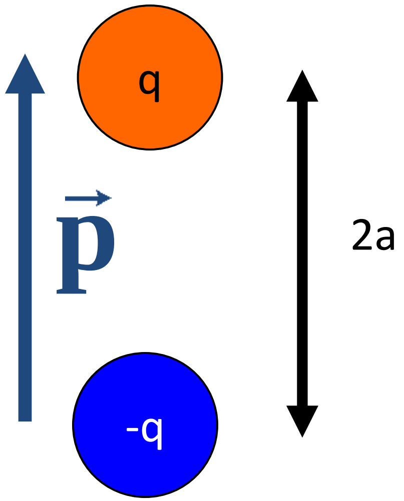
- The dipole is electrically neutral, but the separation of its charges results in an electric field.
- Many charge distributions, especially molecules, behave like electric dipoles.
- The product of the charge and separation is the dipole moment: $p = qd$.
- Far from the dipole, its electric field falls off as the inverse cube of the distance.





Electric Dipole

Two equal but opposite charges **+*q*** and **–*q***, separated by a distance **2a**



<u>Dipole Moment</u>

$$\vec{\mathbf{p}} \equiv \text{charge} \times \text{displacement}$$

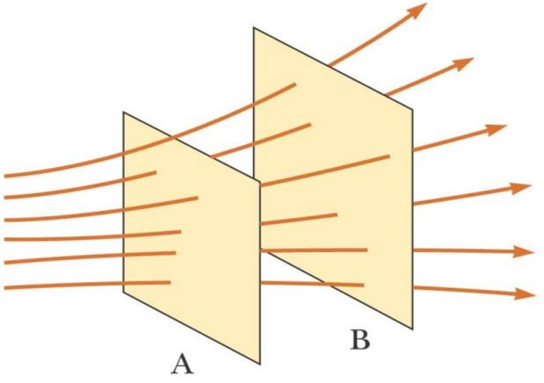
$$= q \times 2a\hat{\mathbf{j}} = 2qa\hat{\mathbf{j}}$$

$\vec{\mathbf{p}}$ points from negative to positive charge

Electric Field Lines

- The field lines are related to the field by:

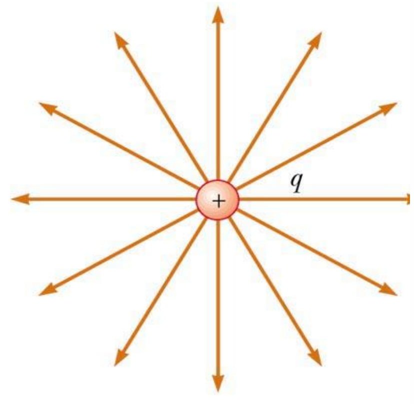
The electric field vector, *E* , is tangent to the electric field lines at each point. A direction is indicated by an arrow head

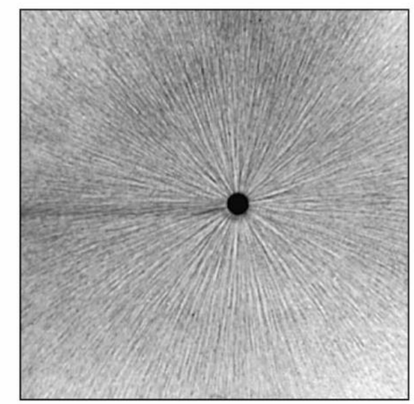


The number of lines per unit area through a surface perpendicular to the lines is proportional to the strength of the electric field in a given region

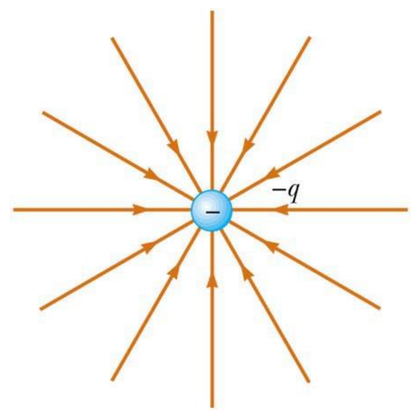
Electric Field Line Patterns

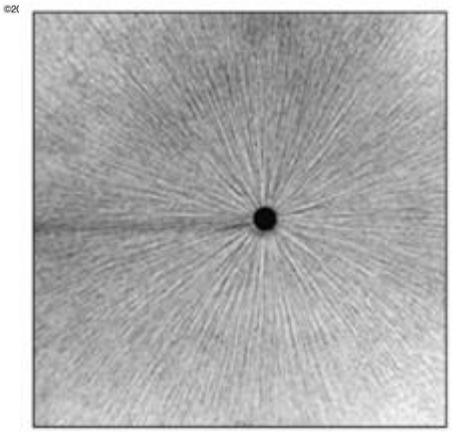
- Point charge
- The lines radiate equally in all directions
- For a ***positive source*** charge, the lines will radiate ***outward***
- For a ***negative source*** charge, the lines will radiate ***inward***





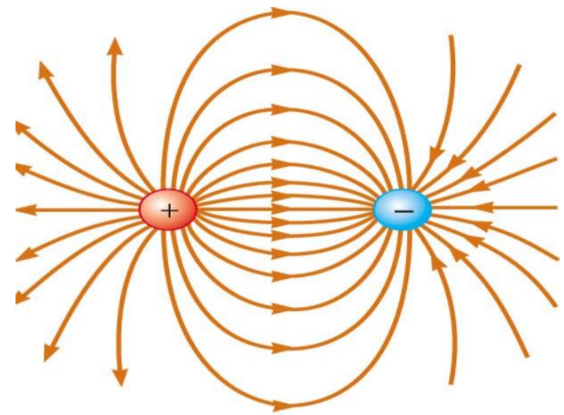
(c)



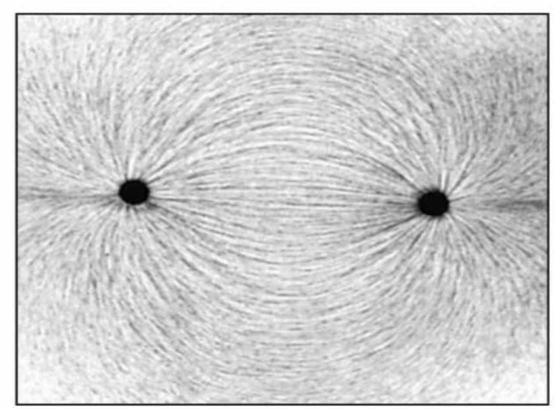


Electric Field Line Patterns

- An electric *dipole* consists of two equal and opposite charges
- The charges are equal and opposite
- The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge



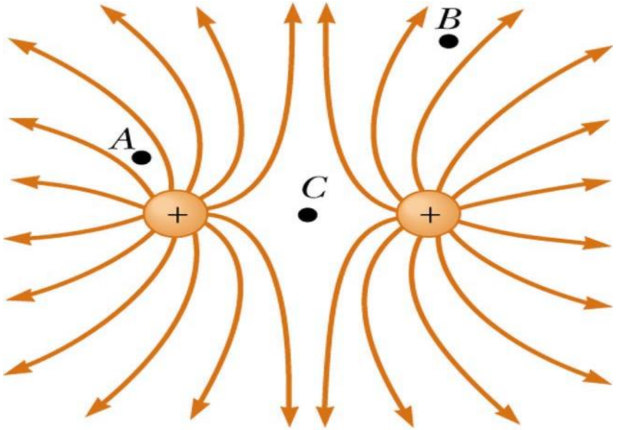
(a)

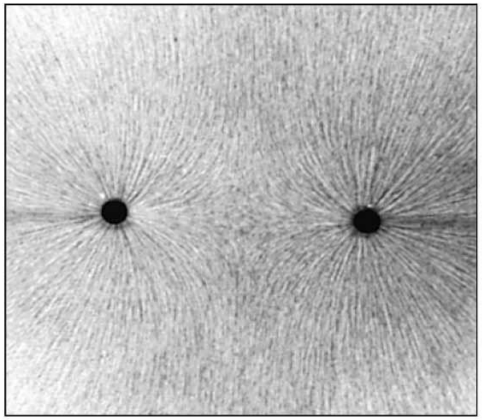

(b)

Electric Field Line Patterns

- Two equal positive point charges

- At a great distance from the charges, the field would be approximately that of a single charge of 2q
- The bulging out of the field lines between the charges indicates the repulsion between the charges
- The low field lines between the charges indicates a weak field in this region

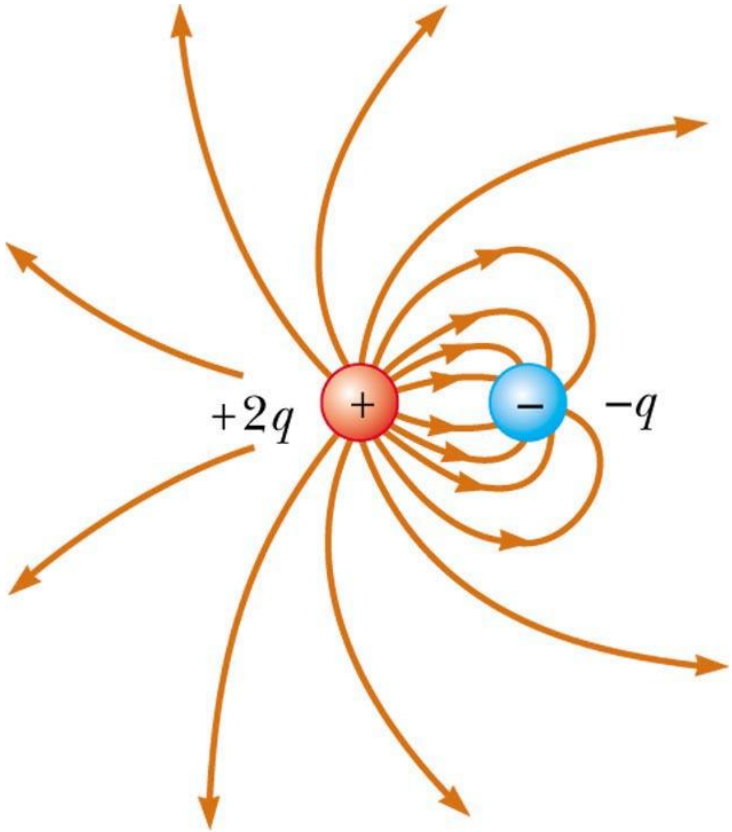




(b)

Electric Field Patterns

- Unequal and unlike point charges
- Note that two lines leave the +2q charge for each line that terminates on -q



Rules for Drawing Electric Field Lines

- The lines must begin on positive charges and terminate on negative charges
- In the case of an excess of charge, some lines will begin or end infinitely far away
- The number of lines drawn leaving a positive charge or ending on a negative charge is proportional to the magnitude of the charge
- No two field lines can cross each other

Electric Field Lines

- A convenient aid for visualizing electric field patterns is to draw lines pointing in the direction of the field vector at any point

- These are called *electric field lines* and were introduced by Michael Faraday