Institute of Science and Technologies 2nd year: Engineering of Proceeding

Numerical methods 2023/2024

Series N°1

Exercise 1

Consider the equation $x^3 + x - 4 = 0$

1- Show that this equation has a root within the interval [1, 2].
2- Is this root unique?
3- Find an approximation of this root by the bisection method ($\varepsilon = 10^{-2}$).

Exercise 2

Consider the following equation, $cos\ x - x = 0$.

1- Show that this equation has a root within the interval [0, 1].
2- Find the function g that allows the fixed-point method to converge.
3- Find an approximation of this root with a precision equal to 10^{-2} . Take $x_0 = 0.5$

Exercise 3

Consider the equation$f(x) = x^2 + lnx$. with $x > 0$.

1- Show that $f(x)$ admits a root r within$\left[\frac{1}{4}, 1\right]$.
2- Show that $f(x) = 0 \Leftrightarrow x = g(x)\ where\ g(x) = \exp(-x^2)$.
3- Show that g(x) verifies the conditions of the fixed point theorem within $\left[\frac{1}{4}, 1\right]$.
4- Calculate the first four iterations using this method.

Exercise 4

Find the approximate value of $\sqrt[3]{25}$ using the Newton-Raphson method.

Exercise 5 :

Consider the equation $f(x) = 2x^2 + 5 - e^x = 0$

1- Show that this equation has a root within the interval [3, 4].
2- What will be this root in the fourth iteration if we use the
    a. The bisection method.
    b. The Newton-Raphson method.