Electric Field

- Definition of the electric field:

- An **electric field** is said to exist in the region of space around a charged object
  - This charged object is the **source charge**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it.
- The *electric field vector* $\vec{E}$ at a point in space is the electric force $\vec{F}_e$ acting on a positive test charge q_0 placed at that point divided by the test charge.

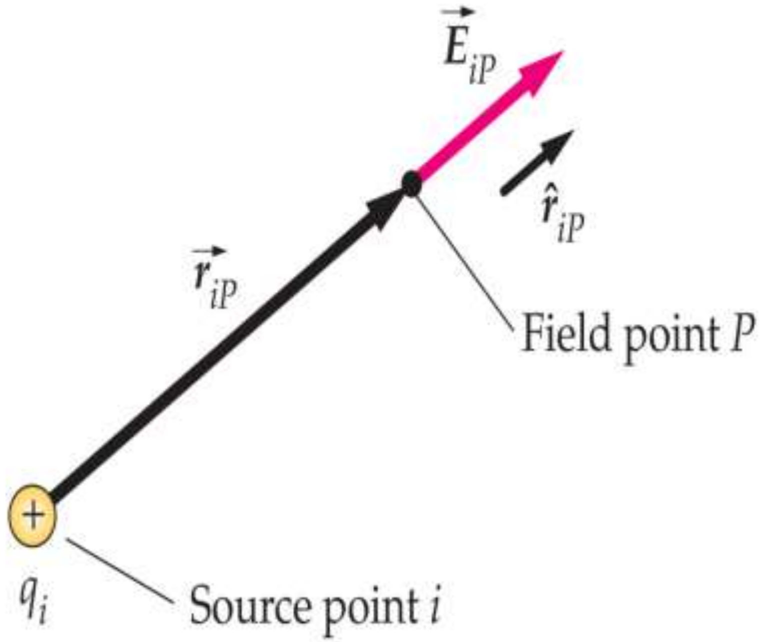
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_0}.$$

- the units in SI: Newton per Coulomb (N/C=V/m)

Electric Field, Vector Form

- Remember Coulomb's law, between the source charge q_i and test charge q_0, can be expressed as:

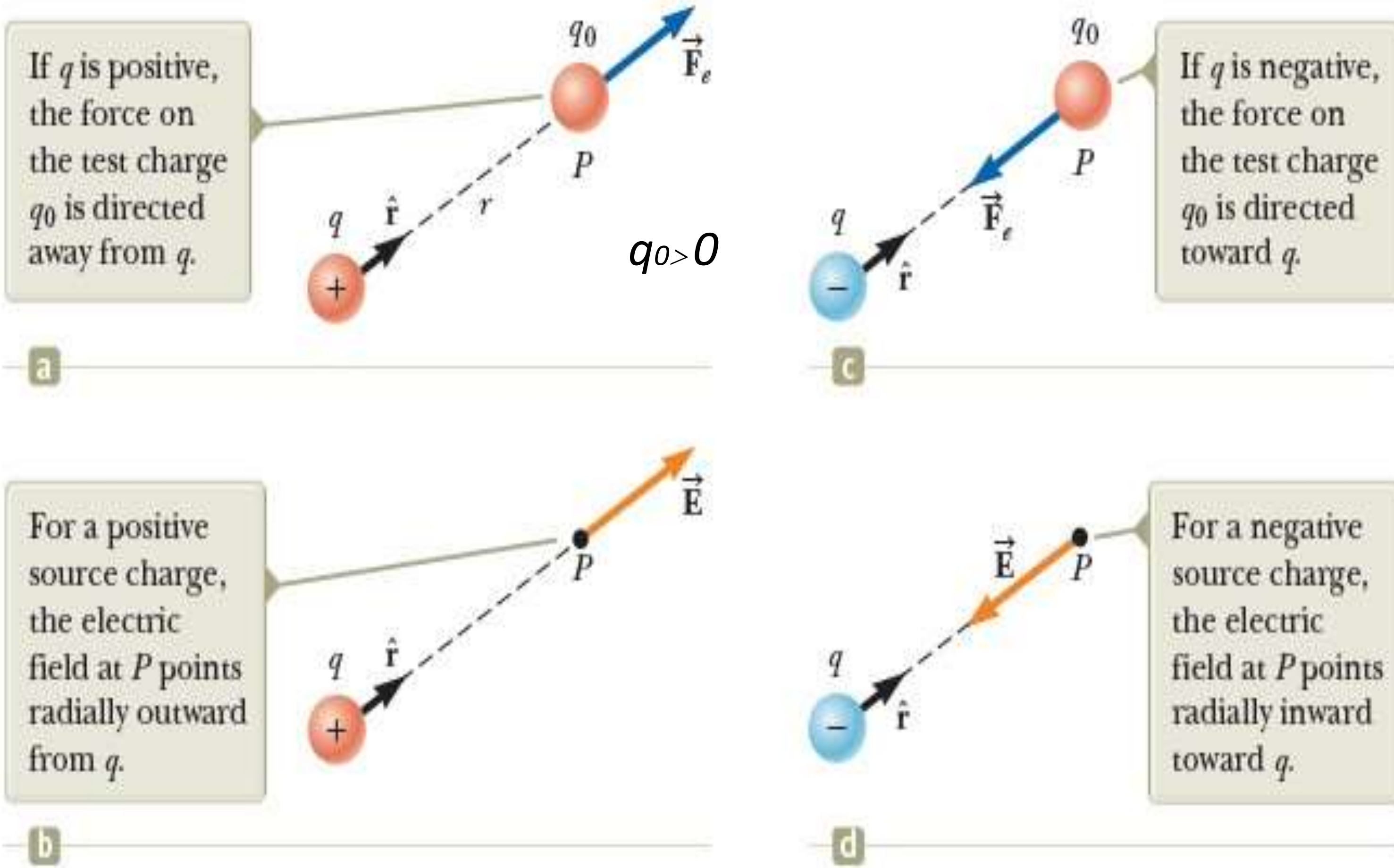
$$\vec{\mathbf{F}}_{i0} = k_e \frac{q_i q_o}{r_{ip}^2} \hat{\mathbf{r}}_{ip}$$



- Then, the electric field will be

$$\vec{\mathbf{E}}_p = \frac{\vec{\mathbf{F}}_{i0}}{q_o} = k_e \frac{q_i}{r_{ip}^2} \hat{\mathbf{r}}_{ip}$$

More About Electric Field Direction



- If q is positive, the force and the field are in the same direction
- If q is negative, the force and the field are in opposite directions

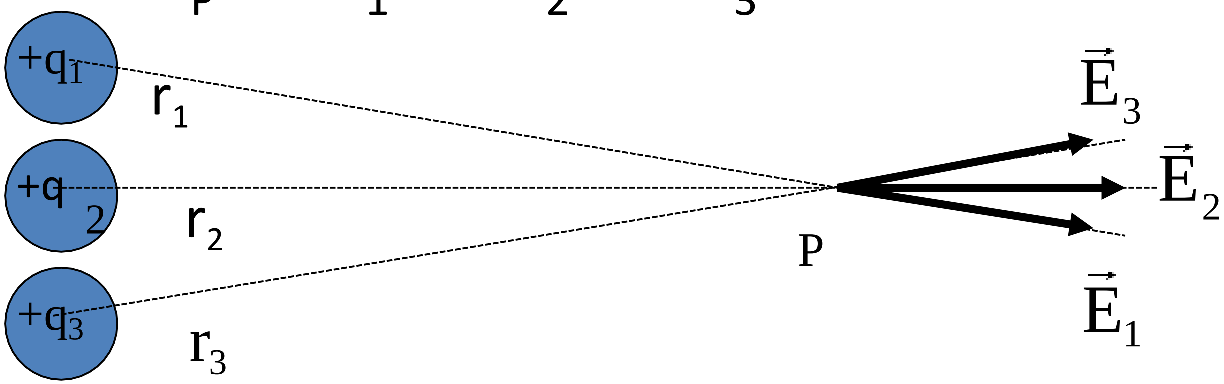
Superposition with Electric Fields

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Superposition of Fields

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots.$$



$$\vec{E}_P = \frac{kq_1}{r_1^2}\hat{r}_{10} + \frac{kq_2}{r_2^2}\hat{r}_2 + \frac{kq_3}{r_3^2}\hat{r}_3 + \ldots.$$

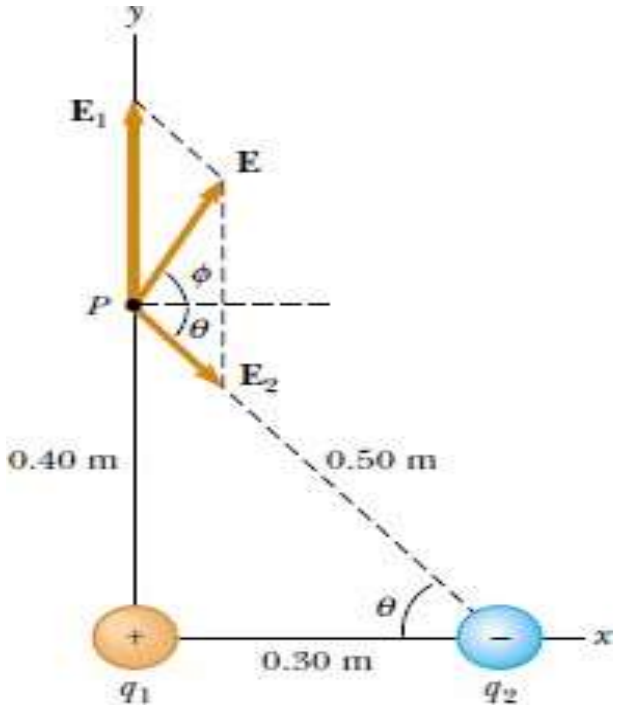
$$\vec{E}_P = k\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_{20} + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots.\right) = k\sum_i^N \frac{q_i}{r^2}\hat{r}_i$$

Example 3. Electric Field Due to Two Charges

- Find the electric field at the point ***P***, which has coordinates (0,0,40)m.

  q_1= 7μC, q_2 = -5 μC.
  - Remember, the fields add as vectors
  - The direction of the individual fields is the direction of the force on a positive test.

- Find the electric field due to q_1, $\vec{E}_1$
- Find the electric field due to q_2, $\vec{E}_2$
- The total electric field due to two charges q_1,q_2.
- $\vec{E} = \vec{E}_1 + \vec{E}_2$



Electric Potential

A charge q moving in a constant electric field E experiences a force $F = q.E$ from that field.

Also, as we know from our study of work and energy**, the work done on the charge by the field as it moves from point r_1 to r_2 is;**

$$\mathbf{W}= \int_{r1}^{r2} \boldsymbol{F}.\, dr = q \int_{r1}^{r2} \mathbf{E}.dr$$

The electric force is conservative and it allows us to calculate an **electric potential energy**, which as usual we will denote by ***U*** and **the change in potential energy** is the negative of the work done by the electric force:

$$\Delta \mathbf{U}_{\mathbf{elec}}\mathbf{= -W}= -q \int_{r1}^{r2} \mathbf{E}.dr$$

We usually want to discuss the potential energy of a charge at ***a particular point***, that is, we would like a function $U(r)$.

Usually we will make the choice that the potential energy is zero when the charge is infinitely far away: $U(\infty) = 0$.

Potential Difference ΔV

- Potential energy difference per unit charge:

$$\Delta V = \frac{\Delta U}{q_0} = -\int_{r1}^{r2} E \cdot dr$$

- Potential of a Point Charge and Groups of Points Charges

The potential due to a point charge *q* is :

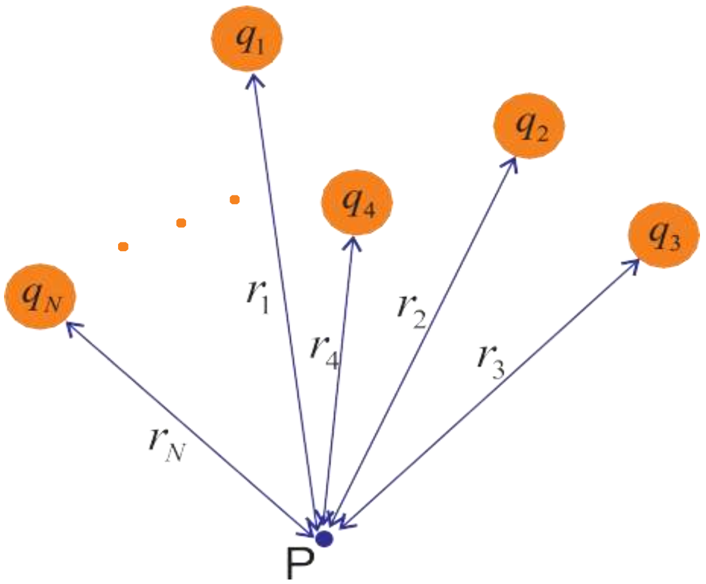
$$V(r) = K \frac{q}{r} = \frac{1}{4\pi\varepsilon 0} \cdot \frac{q}{r}$$

•Similarly, we take $V(r = \infty) = 0$.

$$V(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

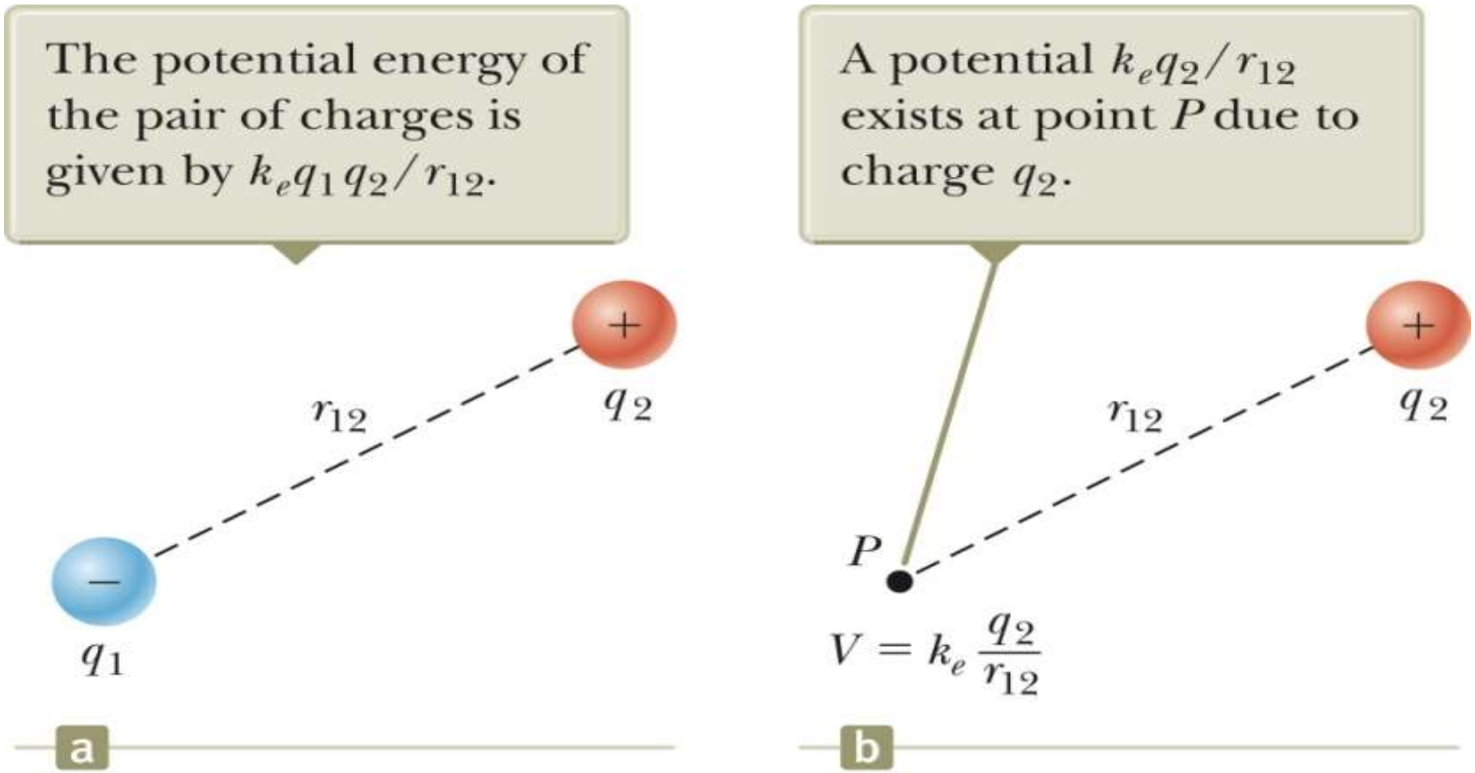
- Total potential at point ***P*** due to ***N*** charges: The sum is the algebraic sum
- V=0, $r = \infty$
- $V = V_1 + V_2 + \cdots + V_N$ (The superposition principle)
- $V = q_1/4\pi\epsilon_0 r_1 + q_2/4\pi\epsilon_0 r_2 + \ldots + q_N/4\pi\epsilon_0 r_N$

$$V(r) = \sum_{i=1}^{i=N} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$$



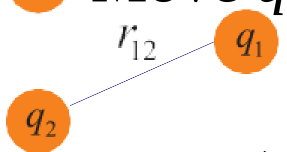
Note: It is the difference in potential energy that is important.

- Reference point: $U(r = \infty) = 0$
- If q_1, q_2 same sign, then U (r) > 0 for all r, work must be done to bring the charges together.
- If q_1, q_2 opposite sign, then U (r) < 0 for all r, work is done to keep the charges apart.
- $U(r) = q_1 q_2 / 4\pi\epsilon_0 r_{12}$



• Potential Energy of A System of Charges

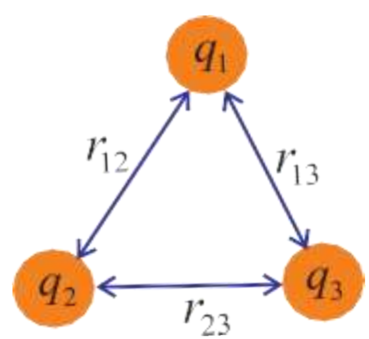
- **Example 04:** P.E. of 3 charges q_1, q_2, q_3
- **Start:** q_1, q_2, q_3 all at $r = \infty$, $U = 0$
- **Step1:** 

Move q_1 from ∞ to its position $\Rightarrow U = 0$
- **Step2:** 

Move q_2 from ∞ to new position

  $\Rightarrow U = q_1 q_2 / 4\pi\epsilon_0 r_{12}$
- **Step3:** Move q_3 from ∞ to new position $\Rightarrow$ Total P.E.

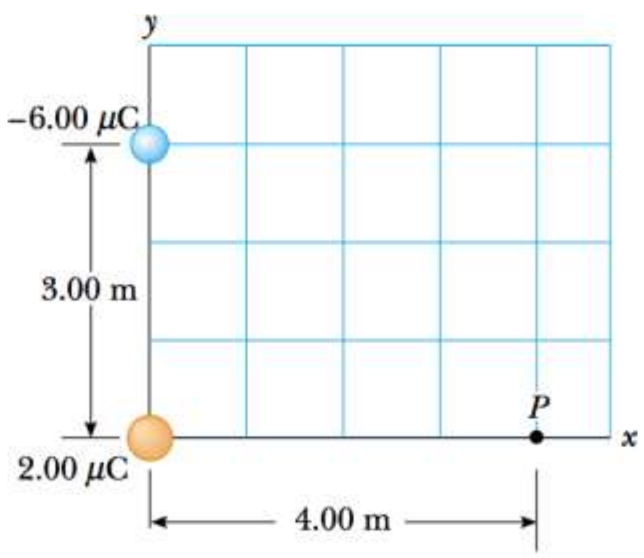
$$U = q_1 q_2 / 4\pi\epsilon_0 r_{12} + q_1 q_3 / 4\pi\epsilon_0 r_{13} + q_2 q_3 / 4\pi\epsilon_0 r_{23}$$



Example 5: The Electric Potential Due to Two Point Charges

A charge q_1 =2μC is located at the origin, and a charge q_2 = -6 μC is located at (0,3.00) m, as shown in Figure.

➢ **Find** the total electric potenial due to these charges at the point ***P***, whose coordinates are (4.00, 0) m.



(a)

Example 6: The Electric Potential Due to Two Point Charges

B) Find the change in potential energy of the system pf two charges plus

a charge q_3= 2μC as the latter charge moves from infinity to point ***P***.