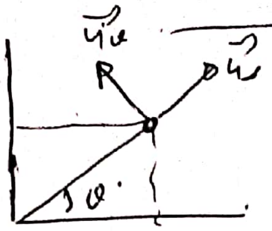
Exam ; 1st Year Common core.
2023/2024 C.U. Mila

Ex 1



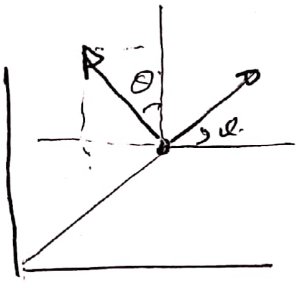
$x = \rho \cos(\theta)$
$y = \rho \sin(\theta)$ 0.5

$\vec{r} = \rho \vec{u_\rho}$

0.5 $\rho = \sqrt{x^2+y^2}$
$\theta = \tan(\frac{y}{x})$

$\vec{r} = \sqrt{x^2+y^2} \cdot \vec{u_\rho}$

(2)



$\vec{u_\rho} = \cos(\theta)\vec{i} + \sin(\theta)\vec{j}$
$\vec{u_\theta} = -\sin(\theta)\vec{i} + \cos(\theta)\vec{j}$ 0.5

$\frac{d\vec{u_\rho}}{dt} = -\dot{\theta}\sin(\theta)\vec{i} + \dot{\theta}\cos(\theta)\vec{j}$
$\frac{d\vec{u_\theta}}{dt} = -\dot{\theta}\cos(\theta)\vec{i} + \dot{\theta}\sin(\theta)\vec{j}$ 0.5

$\Rightarrow \dot{\vec{u_\rho}} = \dot{\theta}(-\sin(\theta)\vec{i} + \cos(\theta)\vec{j})$ 0.5
$\dot{\vec{u_\theta}} = -\dot{\theta}(\cos(\theta)\vec{i} + \sin(\theta)\vec{j})$ 0.5

$\dot{\vec{u_\rho}} = \dot{\theta}\vec{u_\theta}$
$\dot{\vec{u_\theta}} = -\dot{\theta}\vec{u_\rho}$

we have, $\vec{r} = \rho \vec{u_\rho}$

$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\rho}\vec{u_\rho} + \rho\dot{\vec{u_\rho}}$

$\boxed{\vec{v} = \dot{\rho}\vec{u_\rho} + \rho\dot{\theta}\vec{u_\theta}}$ 0.5

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow$

$\vec{a} = \ddot{\rho}\vec{u_\rho} + \dot{\rho}\dot{\vec{u_\rho}} + \dot{\rho}\dot{\theta}\vec{u_\theta} + \rho\ddot{\theta}\vec{u_\theta} + \rho\dot{\theta}\dot{\vec{u_\theta}}$

$\vec{a} = (\ddot{\rho} - \rho\dot{\theta}^2)\vec{u_\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\vec{u_\theta}$ 0.5

Ex2

$dw = \vec{F} \cdot d\vec{r}$

$w = \int \vec{F} \cdot d\vec{r}$

$w = \|f\| \cos(\theta) \cdot \ell$

$\|\ell\|$ is a given distance.

$w_p = 0$ because $\cos(\theta) = 0$

$w_N = 0$ because $\cos(\theta) = 0$

$w_{f_1} = \|f_1\| \cdot \cos(\theta_1) \cdot \|\ell\|$

ℓ: is an algebric value

$w_{f_1} = 100 \cdot \cos(40) \cdot \|\ell\|$
$= 76{,}60 \cdot \|\ell\|$

$w_{f_2} = \|f_2\| \cdot \cos(\theta_2)(-\|\ell\|)$

$w_{f_2} = -80 \cdot \cos(10) \cdot \|\ell\|$

$w_{f_2} = -78{,}78 \cdot \|\ell\|$

the resultant force is

$\vec{F_r} = \vec{f_1} + \vec{f_2}$

$\vec{f_1} = \|\vec{f_1}\| \cdot \cos(\theta)\vec{i} + \|\vec{f_1}\| \cdot \sin(\theta)\vec{j}$

$\vec{f_2} = -\|\vec{f_2}\| \cdot \cos(\theta)\vec{i} - \|f_2\| \sin(\theta)\vec{j}$

the perpendicular component do not work

$\vec{F_r} = \|\vec{f_1}\| \cdot \cos(\theta_1)\vec{i} - \|\vec{f_2}\| \cdot \cos(\theta_2)\vec{i}$

$\vec{f_r} = 76{,}60\,\vec{i} - 78{,}78\,\vec{i}$

$\vec{F_r} = -2{,}18\,\vec{i}$

the body will move in negative direction.

$\text{Power} = \frac{\|F_r\| \cdot \|\ell\|}{2}$

$$\boxed{\text{Power} = \frac{1}{2}(2{,}18) \cdot \|\ell\|}$$

$\frac{2}{4}$

Ex 3

$x(t) = 2\cos(\frac{1}{2}t)$; $y(t) = 2\sin(\frac{1}{2}t)$

$$\begin{cases} x^2 = 4\cos^2(\frac{1}{2}t) \\ y^2 = 4\sin^2(\frac{1}{2}t) \end{cases}$$ 0,25

$\Rightarrow \boxed{x^2 + y^2 = 4}$ $x^2+y^2 = R^2$

$R = 2$ 0,25

the trajectory is a circle.

We have: $\vec{r} = x\vec{i} + y\vec{j}$

$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j}$ 0,25

$\vec{v} = -\sin(\frac{t}{2})\vec{i} + \cos(\frac{t}{2})\vec{j}$ 0,25

$\vec{a} = -\frac{1}{2}\cos(\frac{t}{2})\vec{i} - \frac{1}{2}\sin(\frac{t}{2})\vec{j}$ 0,25

tangential acceleration.

$a_t = \frac{d\|\vec{v}\|}{dt}$ 0,25

$\|\vec{v}\| = \sqrt{\sin^2(\frac{t}{2}) + \cos^2(\frac{t}{2})} = \sqrt{1} = 1$

$\Rightarrow a_t = \frac{d\|\vec{v}\|}{dt} = 0$ 0,25

Normal acceleration 0,25

$\vec{a} = \vec{a_t} + \vec{a_n}$ $\Rightarrow \vec{a_n} = \vec{a}$.

$\Rightarrow \|\vec{a_n}\| = \frac{1}{2}$ 0,25

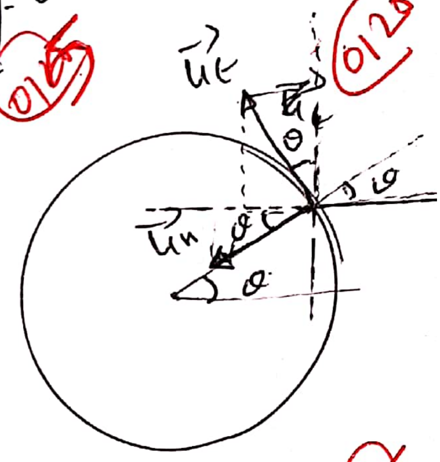
acceleration is constant; then the movement is uniform circular.

∘ Curvilinear abscissa.

$v = \frac{ds}{dt}$ 0,25 $\Rightarrow ds = v.dt$

$v = 1. \Rightarrow \int_0^s ds = \int_0^t dt$ 0,25

$\Rightarrow \boxed{s = t}$ 0,25

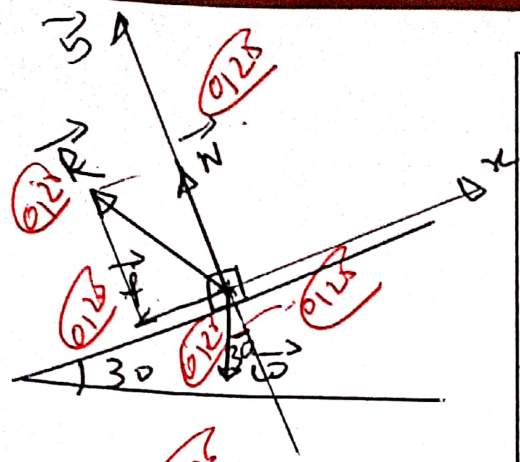


$\vec{u_t} = -\sin(\theta)\vec{i} + \cos(\theta)\vec{j}$ 0,25

$\vec{u_n} = -\cos(\theta)\vec{i} - \sin(\theta)\vec{j}$ 0,25

$\frac{3}{4}$

Ex4



$\sum \vec{F} = m\vec{a}$

$\vec{P}\begin{pmatrix}-mg\sin(\alpha)\\ -mg\cos(\alpha)\end{pmatrix}$

$\vec{N}\begin{pmatrix}0\\ N\end{pmatrix}$; $\vec{f_f}\begin{pmatrix}-f_f\\ 0\end{pmatrix}$

After projection:

$$\begin{cases} -mg\sin(\alpha) - f_f = m\frac{dv}{dt} & (1)\\ -mg\cos(\alpha) + N = 0 & (2)\\ f_f = \mu_k\cdot N = \mu_k mg\cos(\alpha) & (3)\end{cases}$$

from (1) $\Rightarrow$

$\frac{dv}{dt} = g\sin(\alpha) - \mu_k g\cos(\alpha)$

$\frac{dv}{dt} = \underbrace{g(\sin(\alpha) - \mu_k\cos(\alpha))}_{b}$

$\boxed{b = 0{,}41}$

$\frac{dv}{dt} = b$

$\frac{dv}{dt} = b \Rightarrow \frac{dv}{dt}\cdot dx = b\cdot dx$

$\frac{dx}{dt}\cdot dv = b\cdot dx$; $\frac{dx}{dt} = v$

$\Rightarrow v\cdot dv = b\cdot dx \Rightarrow \int_{v_0}^{v} v\cdot dv = \int_0^x dx$

$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = b\cdot x$

at the end $V = 0$, so

$\|x\| = \frac{v_0^2}{2b} = 0{,}13$ m.

We have $\sin(\alpha) = \frac{h}{x}$

$\Rightarrow h = x\cdot\sin(\alpha)$

for the first case (without friction)

$\mu_k = 0$ then $b = 0{,}5$

$\|x\| = 0{,}9$ m.

$h = x\cdot\sin(\alpha)$