Abd El HafidBoussouf University Center – Mila
Institute of Science and Technology
Process Engineering

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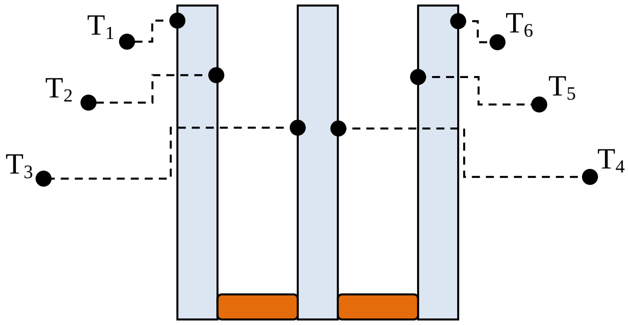
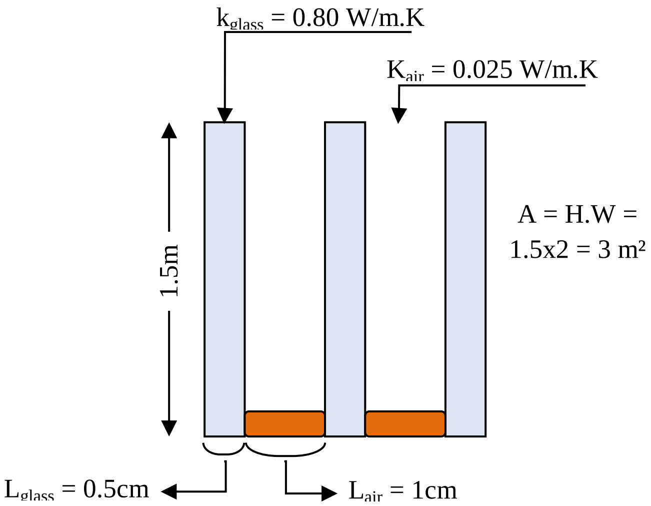
Heat Transfer Final Exam

(Standard Correction)

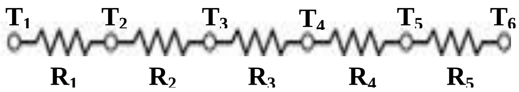
<u>Problem 1</u> (3 marks):





<u>To find $\dot{q}$</u> :

a) **<u>Draw thermal circuit</u>:**



b) **<u>Calculate resistances</u>:**

$$R_1 = R_3 = R_5 = \frac{L_{glass}}{k_{glass}.A} = \frac{0,005m}{\left(0,80\frac{W}{m.K}\right)(3\ m^2)} = 0,002083\ (\frac{K}{W})$$

$$R_2 = R_4 = \frac{L_{air}}{k_{air}.A} = \frac{0,01m}{\left(0,025\frac{W}{m.K}\right)(3\ m^2)} = 0,1333\ (\frac{K}{W})$$

$$R_{total} = 3.R_1 + 2.R_2 = (3 \text{ x } 0,002083) + (2 \text{ x } 0,1333) = 0,2729\ (\frac{K}{W})$$

c) **<u>Calculate the rate of heat loss through the window ($\dot{q}$)</u> :**

$$\dot{q} = \frac{T_1 - T_6}{R_{total}} = \frac{(10\ °C - 0\ °C)}{0,2729\ (\frac{K}{W})} = 36,64\ W$$

Problem 2 (3 marks):

D = 2,5 mm = 0,0025 m

L = 20 mm = 0,02 m

$T_\infty = 20\ °C$

ε = 0,80

$h = 25\ (\frac{W}{m^2.K})$

$T_s = 400\ °C$

Find $\dot{q}$:

$\dot{q} = \dot{q}(conv) + \dot{q}(rad)$

$\dot{q} = h.A.(T_s - T_\infty) + ε.σ.A.(T_s^4 - T_{surr}^4)$

A = π.D.L = π.(0,0025 m).(0,02 m) = 1,571 x 10^{-4} m²

$$\dot{q}(conv) = \left(25\left(\frac{W}{m^2.K}\right)\right).(1{,}571.10^{-4} m^2).(400\ °C - 20\ °C) = 1{,}492\ W$$

$$\dot{q}(rad) = (0{,}80).\left(5{,}67.10^{-8}\left(\frac{W}{m^2.K^4}\right)\right).(1{,}571.10^{-4} m^2).\left((400+273)K - (20+273)\ K\right) = 1{,}49\ W$$

$\dot{q} = 1{,}492\ W + 1{,}409\ W = 2{,}901\ W$

Problem 3 (4 marks):

1) Determine if the flow is entirely laminar, entirely turbulent, or whether it transitions somewhere along the plate:

$$Re_L = \frac{\rho\,.U_\infty.x_{cr}}{\mu} = \frac{867\left(\frac{kg}{m^3}\right).\ 2{,}5\left(\frac{m}{s}\right).\ x_{cr}}{0{,}06108\left(\frac{kg}{m.s}\right)} = 5 \text{ x } 10^5 \rightarrow x_{cr} = 14{,}1\ m\ > 10\ m$$

So, LAMINAR over the entire plate.

2) Find the total rate of heat transfer per unit width from the *ThermoKool* to the plate:

$q = \bar{h}.A.(T_\infty - T_s)$, so we need to calculate: $\bar{h}$

$$Re_L = \frac{\rho\,.U_\infty.x_{cr}}{\mu} = \frac{867\left(\frac{kg}{m^3}\right).\ 2{,}5\left(\frac{m}{s}\right).\ 10\ (m)}{0{,}06108\left(\frac{kg}{m.s}\right)} = 354\ 900$$ (Laminar, flat plate, constant T_s B.C)

Then, $\overline{Nu} = 0{,}664.{Re_L}^{0{,}5}.Pr^{1/3}$

$$\overline{Nu} = 0{,}664.(354\ 900)^{0{,}5}.(1551)^{1/3} = 4\ 579$$

$$\overline{Nu} = \frac{\bar{h}\,.L}{k} \rightarrow \bar{h} = \frac{\overline{Nu}\,.L}{k} = \frac{4\ 579\,.(0{,}1414)}{10} \rightarrow \bar{h} = 647\left(\frac{W}{m^2.°C}\right)$$

So,

$$q = 647\left(\frac{W}{m^2.°C}\right).(10\ m\ .\ 1m).(80\ °C - 30\ °C)$$

$$q = 32{,}37\ W/m$$

Problem 4 (10 marks):

L = 17 m ; r_1 = 15 cm ; r_2 = 20 cm ; k = 14 W/m.K ; e_{gen} = 4000 kW ; T_1 = 60 ºC ; T_2 = 200 ºC

Determine the temperature of the pipe wall at a radius of r = 17.5 cm:

Assumptions:

1) One-dimensional heat conduction ;
2) Steady-state ;
3) Constant K.

ENERGY EQUATION:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\underbrace{\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)}_{0}+\underbrace{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{0}+\dot{e}_{\text{gen}}=\underbrace{\rho C\frac{\partial T}{\partial t}}_{0}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\dot{e}_{\text{gen}}=0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{gen}}{k}$$

$$\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right)=-\left(\frac{\dot{e}_{gen}}{k}\right).r=\beta.r$$

$$\left(r\frac{dT}{dr}\right)=\beta.\frac{r^2}{2}+C_1$$

$$\left(\frac{dT}{dr}\right)=\beta.\frac{r}{2}+\frac{C_1}{r}$$

$$\boldsymbol{T(r)=\beta.\frac{r^2}{4}+C_1.\ln(r)+C_2} \qquad (\textit{Temperature Distribution})$$

Evaluate constants using Boundary Conditions:

$T(r_1)=T_1 \quad \rightarrow \quad T_1=\beta.\frac{r_1^2}{4}+C_1.\ln(r_1)+C_2 \qquad (eqn.1)$

$T(r_2)=T_2 \quad \rightarrow \quad T_2=\beta.\frac{r_2^2}{4}+C_1.\ln(r_2)+C_2 \qquad (eqn.2)$

Subtract (eqn.1) from (eqn.2):

$$T_2-T_1=\frac{\beta}{4}(r_2^2-r_1^2)+C_1.(\ln(r_2)-\ln(r_1))$$

$$(T_2-T_1)-\frac{\beta}{4}(r_2^2-r_1^2)=C_1.\ln\left(\frac{r_2}{r_1}\right)$$

$$\boldsymbol{C_1=\frac{(T_2-T_1)-\frac{\beta}{4}(r_2^2-r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}}$$

Solve for $\boldsymbol{C_2}$ in (eqn.1):

$$C_2 = T_1 - \beta.\frac{r_1^2}{4} - C_1.\ln(r_1)$$

$$\boldsymbol{C_2 = T_1 - \beta.\frac{r_1^2}{4} - (\frac{(T_2 - T_1) - \frac{\beta}{4}(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}).\ln(r_1)}$$

So,

$$T(r) = \beta.\frac{r^2}{4} + (\frac{(T_2 - T_1) - \frac{\beta}{4}(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}).\ln(r) + T_1 - \beta.\frac{r_1^2}{4} - (\frac{(T_2 - T_1) - \frac{\beta}{4}(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}).\ln(r_1)$$

$$T(r) = T_1 + \frac{\beta}{4}(r^2 - r_1^2) + (\frac{(T_2 - T_1) - \frac{\beta}{4}(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}).\ln\left(\frac{r}{r_1}\right)$$

$$\boldsymbol{T(r) = T_1 + \frac{\beta}{4}(r^2 - r_1^2) + \left((T_2 - T_1) - \frac{\beta}{4}(r_2^2 - r_1^2)\right).\left(\frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)}\right)}$$

Check: let $\boldsymbol{\dot{e}_{gen} = 0}$

$$T(r) = T_1 + (T_2 - T_1).\left(\frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)}\right)$$

Find ($\boldsymbol{r = 17,5}$ cm) :

$$-\frac{\dot{e}_{gen}}{4.k}(r^2 - r_1^2) = \frac{4{,}280\,.\,10^6\,(\frac{\text{W}}{\text{m}^3})}{4\,.\,14\,(\frac{\text{W}}{\text{m.K}})}\left[(0.175\text{ m})^2 - (0{,}150\text{ m})^2\right] = -\,620{,}9\text{ K}$$

$$-\frac{\dot{e}_{gen}}{4.k}(r_2^2 - r_1^2) = \frac{4{,}280\,.\,10^6\,(\frac{\text{W}}{\text{m}^3})}{4\,.\,14\,(\frac{\text{W}}{\text{m.K}})}\left[(0.200\text{ m})^2 - (0{,}150\text{ m})^2\right] = -1\,337{,}5\text{ K}$$

$$\frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{\ln\left(\frac{0{,}175}{0{,}150}\right)}{\ln\left(\frac{0{,}200}{0{,}150}\right)} = 0{,}5358$$

$$T(r = 17{,}5\text{ cm}) = (333\,K) + (-620{,}9\,K) + \left[(200\,°C - 60°C) - (-1\,337{,}5\,K)\right].(0{,}5358)$$

$$\boldsymbol{T(r = 17{,}5\text{ cm}) = 503{,}74\,K}$$