Solution TD: 04

$$\begin{cases} U_1 = 2 \\ U_{n+1} = 3U_n - 2 \end{cases}$$

1) let define $v_n = u_n + \alpha$.

(v_n) geometric $\Leftrightarrow \exists q \in \mathbb{R}^* ;\ v_{n+1} = q \cdot v_n$.

$* \ v_{n+1} = u_{n+1} + \alpha = 3u_n - 2 + \alpha = 3(\overset{u_n}{\overset{\shortparallel}{v_n - \alpha}}) - 2 + \alpha$

$v_{n+1} = 3v_n - 3\alpha - 2 + \alpha = \boxed{\underset{q}{\overset{\shortparallel}{3}} v_n} \underbrace{- 2 - 2\alpha}_{0}$

so $-2 - 2\alpha = 0 \Rightarrow \boxed{\alpha = -1}$

so for $\boxed{\alpha = -1}$; (v_n) is a geometric with reason $\boxed{q = 3}$

2) $v_n = v_p \times q^{n-p} = v_1 \times 3^{n-1}$ $\quad (v_1 = u_1 + \alpha = 2 - 1 = 1$

$$\boxed{v_n = 1 \times 3^{n-1}}$$

$$u_n = v_n - \alpha = v_n + 1 = \boxed{3^{n-1} + 1 = u_n}$$

Ex no2

1) $v_n = (-1)^n + 1 \Rightarrow v_0 = 2 ; v_1 = 0 ; v_2 = 2 , v_3 = 0 \dots$

So we can't specify the variation (no monotone)

* $\lim v_n = \begin{cases} 2 & \text{if } n \text{ (زوجي)} \\ 0 & \text{if } n \text{ (فردي)} \end{cases}$; so there is no limite

2)

$v_n = -3 \times 2^n$.

$v_{n+1} - v_n = -3 \times 2^{n+1} - (-3 \times 2^n) = -3 \times 2^{n+1} + 3 \times 2^n$

$= 3 \times 2^n \underbrace{(-2+1)}_{=-1} = -3 \times 2^n < 0$

So (v_n) is increase.

* $\lim v_n = \lim -3 \times 2^n = -\infty \quad (q = 2 > 1)$

3) $V_n = 3\left(\frac{-2}{3}\right)^n + 5$

* $V_{n+1} - V_n = 3\left(\frac{-2}{3}\right)^{n+1} + \cancel{5} - \left(3\left(\frac{-2}{3}\right)^n + \cancel{5}\right)$

$= 3\left(\frac{-2}{3}\right)^n \left(\frac{-2}{3} - 1\right) = 3\left(\frac{-2}{3}\right)^n \left(\frac{-5}{2}\right)$

we can not define the sign of $V_{n+1} - V_n$; so we can not specify the variation of (V_n)

$\lim V_n = \lim \cancel{3\left(\frac{-2}{3}\right)^n} + 5$

$= 5.$

$\lim \left(\frac{-2}{3}\right)^n = 0 \quad (-1 < q < 1$

4) $v_n = \left(\frac{-5}{3}\right)^{n+1}$

$v_0 = -\frac{5}{3}$; $v_1 = +\frac{25}{9}$, $v_2 = -\frac{125}{27}$

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we can not specify the variation of (v_n)

$$\lim_{n\to+\infty} v_n = \lim_{n\to+\infty}\left(-\frac{5}{3}\right)^{n+1} = \begin{cases} +\infty & \text{if } n \text{ (زوجي)} \\ -\infty & \text{if } n \text{ (فردي)} \end{cases}$$

So there is no limite (المتتالية [illegible])

Exo 3 :

Prove $1+3+5+\dots+(2n-1) = n^2$ — $P(n)$

$n \geq 1$

verification for $\boxed{n=1}$, we have $1 = (1)^2 = 1$ ✓

, So we propose $P(n)$ true and we prove $P(n+1)$

So, we have $1+3+5+\dots+(2n-1) = n^2$.

$1+3+5+\dots+(2n-1)+(2(n+1)-1) =$

$= \underbrace{1+3+\dots+(2n-1)}_{n^2} + (2n+1) = n^2+2n+1$

$= (n+1)^2$ ✓ $P(n+1)$

So for all $n \geq 1$ the proposition $P(n)$ is true

Exo 4 : $3n^2+3+6$ multiple of 6

it mean $(3n^2+3n+6) = 6k$ (cero) $k \in \mathbb{N}$

* verification for $n=0$: $0+6 = 6\times 1$. $(k=1)$

* We porpose $P(n)$ true and we prove $P(n+1)$

$$3n^2+3n+6 = 6k$$

$$3(n+1)^2+3(n+1)+6 = 3n^2+6n+3+3n+3+6$$

$$= \underbrace{3n^2+3n+6}_{6k} + 6n+6$$

$$= 6k + 6(n+1)$$

$$= 6(k+n+1) = 6k' \quad ; k' \in \mathbb{N}$$

$P(n+1)$

So for all n, $P(n)$ is true.