Abdelhafid Boussouf University Center- Mila
Institute of Sciences and Technology
Module: Analysis 1

Academic Year: 2023/2024
First Year Engineering
Semester 1

Tutorial exercises set 3: Analysis 1

Exercise 01:

Determine the domain of definition of the following functions:

عين ميدان (مجال) تعريف الدوال التالية:

1. $f(x) = \frac{x+1}{1-e^{\frac{1}{x}}}$
2. $f(x) = \frac{1}{\sqrt{\sin x}}$
3. $f(x) = e^{\frac{1}{1-x}}\sqrt{x^2-1}$
4. $f(x) = (1+\ln x)^{\frac{1}{x}}$
5. $f(x) = \frac{1}{\lfloor x \rfloor}$
6. $f(x) = \begin{cases} \sqrt{x-2}, & \text{if } x > 1 \\ \ln(x+2), & \text{if } x \leq 1 \end{cases}$

Exercise 02:

Calculate the limits of the following functions:

أحسب نهايات الدوال التالية:

1. $l_1 = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$
2. $l_2 = \lim_{x \to +\infty} x \sin\left(\frac{1}{x}\right)$
3. $l_3 = \lim_{x \to 0} \frac{x-\sin(2x)}{x+\sin(3x)}$
4. $l_4 = \lim_{x \to 0} \frac{\tan x}{x}$
5. $l_5 = \lim_{x \to \pi} \frac{\sin x - \cos x}{1-\tan x}$
6. $l_6 = \lim_{x \to 0} \frac{\sin(x \ln x)}{x^2}$
7. $l_7 = \lim_{x \to a^+} \frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$
8. $l_8 = \lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^x$
9. $l_9 = \lim_{x \to +\infty} \left(\sin\sqrt{x+1} - \sin\sqrt{x}\right)$
10. $l_{10} = \lim_{x \to 1} (1-x)\tan\left(\frac{\pi x}{2}\right)$

Exercise 03:

Using the definition of the limit of a function, show that

باستعمال تعريف نهاية دالة، بين أن

1. $\lim_{x \to 4} (2x - 1) = 7$
2. $\lim_{x \to +\infty} \frac{3x-1}{2x+1} = \frac{3}{2}$
3. $\lim_{x \to +\infty} \ln x = +\infty$
4. $\lim_{x \to -3^+} \frac{4}{x+3} = +\infty$

Exercise 04:

1. Demonstrate that the function:

   برهن أن الدالة:

   $$f(x) = \begin{cases} x\sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 3, & \text{if } x = 0 \end{cases}$$

   continuous at $x = 0$ (مستمرة عند).

2. What is the redefinition of $f(0)$ that makes $f(x)$ continuous at $x = 0$?

   2. ما هو إعادة تعريف $f(0)$ الذي يجعل $f(x)$ مستمرة عند $x = 0$ ؟

Exercise 05:

Demonstrate that the function $f(x) = x^2$ is:

برهن أن الدالة $f(x) = x^2$:

1. continuous at $x = 3$ (مستمرة عند)
2. uniformly continuous in $]0, 1[$. (مستمرة بإنتظام على المجال $]0, 1[$)

Exercise 06:

Demonstrate that the function $f(x) = \frac{1}{x}$ is:

برهن أن الدالة $f(x) = \frac{1}{x}$:

1. not uniformly continuous in $]0, 1[$. (ليست مستمرة بإنتظام على المجال $]0, 1[$).
2. uniformly continuous in $]2, +\infty[$. (مستمرة بإنتظام على المجال $]2, +\infty[$).

Exercise 07:

Prove that, if $f(x)$ has a derivative at $x = x_0$, then $f(x)$ must be continuous at x_0.

أثبت أنه إذا كانت $f(x)$ لها مشتقة عند $x = x_0$ ، فيجب أن تكون $f(x)$ مستمرة عند x_0 .

Exercise 08:

Considering the function: (نعتبر الدالة:)

$$f(x) = \begin{cases} x\sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

1. Study the continuity of $f(x)$ at $x = 0$. (أدرس إستمرارية $f(x)$ عند $x = 0$)
2. Is the function $f(x)$ differentiable at $x = 0$?(هل الدالة $f(x)$ قابلة للإشتقاق عند $x = 0$ ؟)

Exercise 09:

Considering the function: (نعتبر الدالة:)

$$f(x) = \begin{cases} x^2\sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

1. Is the function $f(x)$ differentiable at $x = 0$?(هل الدالة $f(x)$ قابلة للإشتقاق عند $x = 0$ ؟)
2. Study the continuity of $f'(x)$ at $x = 0$. (أدرس إستمرارية $f'(x)$ عند $x = 0$)

Exercise 10:

Differentiate the function f where $f(x)$ is:

إشتق الدالة f حيث $f(x)$:

1. $2x^{\frac{7}{2}}$
2. $x + \sqrt{x}$
3. $2ax^3 - \frac{x^2}{b} + c$
4. $\frac{x}{a} + \frac{b}{x} + \frac{x^2}{a^2} + \frac{a^2}{x^2}$
5. $\frac{nx^2}{x^{\frac{1}{3}}} + \frac{m}{x\sqrt{x}} + \frac{x^{\frac{1}{3}}}{\sqrt{x}}$
6. $\sin(\ln x)$
7. $\ln\left(\frac{1}{\cos x}\right)$
8. $\frac{\sinh^2 x}{e^x}$ where $\sinh = \frac{e^x - e^{-x}}{2}$
9. $\frac{\cosh^2 x}{e^x}$ where $\cosh = \frac{e^x + e^{-x}}{2}$
10. $\arctan x$
11. $\cos(\arcsin x)$
12. $\arctan\left(\frac{2x}{3+x}\right)$