Exercise 1

Determine the domain of definition of each of the following functions

$$f(x) = \frac{x^3 + 3}{1 - |x|}, \quad f(x) = \ln\left(\frac{2 + x}{2 - x}\right), \quad f(x) = \sqrt{x^2 + 3x - 10}, \quad f(x) = (x - 2)\sqrt{\frac{1 + x}{1 - x}}.$$

Exercise 2

❶ Study the parity of the following functions

$$f(x) = \frac{x^3}{1 + x^2}, \quad f(x) = \ln\left(\sqrt{1 + 4x^2} + 2x\right), \quad f(x) = \sin(x) + \cos(x).$$

❷ Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined by

$$f(x) = \frac{\cos(x)}{1 + x^2}, \qquad \forall x \in \mathbb{R}.$$

- Show that f is major and minor on $\mathbb{R}$.

❸ Study the monotonicity of the following function

$$f(x) = x^2, \qquad x \in \mathbb{R}_+ \text{ et } x \in \mathbb{R}_-.$$

Exercise 3

Calculate the following limits where they exist

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}, \quad \lim_{x \to +\infty} \sqrt{x + 8} - \sqrt{x - 4}, \quad \lim_{x \to 0} \frac{x^2 + 2|x|}{x}, \quad \lim_{x \to +\infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x}}}.$$

Exercise 4

❶ Study the continuity of the following functions

a) $f(x) = \begin{cases} \dfrac{\sin(x)}{|x|} & if \ x \neq 0, \\ 1 & if \ x = 0. \end{cases}$ b) $f(x) = \begin{cases} \dfrac{e^{4x} - 1}{x} & if \ x \neq 0, \\ 4 & if \ x = 0. \end{cases}$

Exercise 5

❶ Let f be a function defined by

$$f(x) = \begin{cases} (x-1)^2 & if \ x < -2, \\ a & if \ x = -2, \\ (2x+b)^2 & if \ x > -2. \end{cases}$$

Determine the real numbers a, b so that f is continuous on $\mathbb{R}$.

❷ Let f be a function defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} \frac{\ln(1+2x^2)}{x} - 1 & if \ x < 0, \\ b & if \ x = 0, \\ x^2 + x - a & if \ x > 0. \end{cases}$$

a) Determine the real numbers a, b so that f is continuous at the point $x_0 = 0$.

b) Show that the equation $f(x) = 0$ has at least one solution in the interval $[-1, 1]$.

Exercise 6

❶ Let g be a function defined on $\mathbb{R}_+$ by

$$g(x) = \begin{cases} \sqrt{x} & if \ 0 \le x \le 1, \\ ax^2 + bx + 1 & if \ x > 1. \end{cases}$$

Determine the real numbers a, b so that g is derivable on $\mathbb{R}_+^*$. Calculate $g'(x)$.

❷ Let h be a function defined on $\mathbb{R}$ by

$$h(x) = \begin{cases} x-1 & if \ x < e, \\ a\ln(x) + b & if \ x \ge e. \end{cases}$$

Determine the real numbers a, b so that h is derivable on $\mathbb{R}$. Calculate $h'(x)$.

Exercise 7

Calculate the derivatives of the following functions

$f(x) = \tan(x), \quad f(x) = \sin(2x+6) + \cos(3x+1), \quad f(x) = \ln(\ln(x)), \quad f(x) = \sqrt[3]{x^3+2},$
$f(x) = \sqrt{x + \sqrt{x}}.$