Exercise 5.1

Find the power series for the following functions:

$$f_1(x) = e^x \qquad f_2(x) = \cos x \qquad f_3(x) = \sin\ x$$

$$f_4(x) = \ln(x+1) \qquad f_5(x) = \frac{1}{x-1} \qquad f_6(x) = \frac{1}{x+1}$$

Exercise 5.2

Find Taylor series (power series) of order n=3 for the following functions about $x_0 = 0$.

$$f_1(x) = \sqrt{1+x} \qquad f_2(x) = \frac{e^x - 1 - x}{x^2}$$

$$f_3(x) = \ln(2+x) \qquad f_4(x) = \frac{e^x}{x+e^x}$$

Exercise 5.3

Consider the following function defined on $\mathbb{R}$ by:

$$f(x) = \frac{1}{1+e^x}$$

1. Find Taylor series (power series) of order $n = 3$ about $x_0 = 0$ for the function $f(x)$.
2. We denote by (C) the representative curve of f Write the equation of the tangent line to (C) at the abscissa point $x_0 = 0$.
3. Prove that the tangent crosses the curve at 0. Such a point is called the inflection point.

Power series of standard functions

e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{4!}+...+\frac{x^n}{n!}+x^n\varepsilon(x)$
$\mathrm{ch}(x)$	$1+\frac{x^2}{2!}+\frac{x^4}{4!}+...+\frac{x^{2n}}{(2n)!}+x^{2n+1}\varepsilon(x)$
$\mathrm{sh}(x)$	$x+\frac{x^3}{3!}+\frac{x^5}{5!}+...+\frac{x^{2n+1}}{(2n+1)!}+x^{2n+2}\varepsilon(x)$
$\cos(x)$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}+...+(-1)^n\frac{x^{2n}}{(2n)!}+x^{2n+1}\varepsilon(x)$
$\sin(x)$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}+...+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+x^{2n+2}\varepsilon(x)$
$\ln(x+1)$	$x-\frac{x^2}{2}+\frac{x^3}{3}+...+(-1)^{n-1}\frac{x^n}{n}+x^n\varepsilon(x)$
$(1+x)^\alpha$	$1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+...+\frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!}x^n+x^n\varepsilon(x)$
$\frac{1}{1+x}$	$1-x+x^2-x^3+...+(-1)^n x^n+x^n\varepsilon(x)$
$\frac{1}{1-x}$	$1+x+x^2+x^3+...+x^n+x^n\varepsilon(x)$
$\sqrt{1+x}$	$1+\frac{x}{2}-\frac{x^2}{8}+...+(-1)^{n-1}\frac{1\times3\times5\times...\times(2n-3)}{2\times4\times6\times...\times2n}x^n+x^n\varepsilon(x)$