

Exercice 1

For each of the following cases, find the values of m that make the root α of many Polynomes P:

$$\textcircled{1} \quad P(x) = mx^2 - 2mx - 4m - 2 \quad \leftarrow \alpha = 4$$

$$\textcircled{2} \quad P(x) = 2m^2x^2 + mx - m \quad \leftarrow \alpha = -1$$

$$\textcircled{3} \quad P(x) = -m^2x^3 + m(2m - 3)x^2 + 3x \quad \leftarrow \alpha = 2$$

Exercice 2

In each of the following cases, indicate whether α is a root of the polynomial Then analyze P(x), then deduce the other roots

$$\textcircled{1} \quad \alpha = -3 \quad \leftarrow P(x) = x^3 + x^2 - 7x - 3$$

$$\textcircled{2} \quad \alpha = -1 \quad \leftarrow P(x) = 4x^3 + 4x^2 - 9x - 9$$

$$\textcircled{3} \quad \alpha = 2 \quad \leftarrow P(x) = \frac{1}{2}x^3 - \frac{1}{4}x^2 + x - 5$$

$$\textcircled{4} \quad \alpha = 1 \quad \leftarrow P(x) = x^4 - 3x^2 + 2x$$

Exercice 3

Express the following as a sum of partial fractions

$$\text{a)} \quad \frac{5x^2 + 17x + 15}{(x+2)^2(x+1)} \quad \text{b)} \quad \frac{x}{(x-3)^2(2x+1)}$$

$$\text{c)} \quad \frac{3x+1}{(x-1)^2(x+2)} \quad \text{d)} \quad \frac{x^2+1}{(x-1)^2(x+1)}$$

Exercice 4

Express the following as a sum of partial fractions

$$\text{a)} \quad \frac{x^2 - 3x - 7}{(x^2 + x + 2)(2x - 1)} \quad \text{b)} \quad \frac{13}{(2x + 3)(x^2 + 1)}$$

$$\text{c)} \quad \frac{x}{(x^2 - x + 1)(3x - 2)}$$

Exercice 5

Express the following as a sum of powers of x and partial fractions

$$\text{a)} \quad \frac{x^3 + 1}{x^2 + 1} \quad \text{b)} \quad \frac{2x^4 + 3x^2 + 1}{x^2 + 3x + 2} \quad \text{c)} \quad \frac{7x^2 - 1}{x + 3}$$