Exercise 1.1

Which of the following are well formed statements (propositions)?

1. $\forall\, P\, Q$

2. $\left(\overline{(P \rightarrow (Q \wedge P))}\right)$

3. $\overline{P \rightarrow (Q = P)}$

4. $\left(\blacklozenge(q \vee p)\right)$

5. $(P \cap \bar{Q}) \cup (Q \rightarrow R)$

6. $P\, \bar{R}$

Exercise 1.2

Construct the truth table of $(F \cup G) \cap \overline{(F \cap G)}$.

Exercise 1.3

Use the truth tables method to determine whether $(P \rightarrow Q) \cup (P \rightarrow \bar{Q})$ is valid (true).

Exercise 1.4

Which of the following statements are true, which are false, and why?

$a.$ $(2 < 3)\ et\ (2\ divise\ 4)$

$b.$ $(2 < 3)\ et\ (2\ divise\ 5)$

$c.$ $(2 < 3)\ ou\ (2\ divise\ 5)$

Exercice 1.5

Complete the dotted lines with the appropriate logical operator: ⇔, ⇐, ⇒

1. $\forall x \in R : x^2 = 4 \quad \ldots \quad x = 2$
2. $\forall z \in C : z = z \ldots z \in R$
3. $\forall x \geq 0 : x^2 = 1 \ldots x = 1$

Exercise 1.6

Consider the following four assertions:

$a.$ $\exists x \in R, \forall y \in R : x + y > 0$

$b.$ $\forall x \in R, \exists y \in R : x + y > 0$

$c.\ \forall x \in R, \forall y \in R: x + y > 0$

$d.\ \exists x \in R, \forall y \in R: y2 > x$

1. Are statements a, b, c, d true or false?
2. Give their negation.

Exercice 1.7

Show that the product of two odd integers is odd.

Exercise 1.8

1. Show that if n^2 is even, then n is even, for n an integer.
2. show that $\sqrt{2}$ is irrational.

Hint: Suppose by contradiction that $\sqrt{2}$ is rational, that is $\sqrt{2} = \frac{m}{n}$, for m and n integers with no common factor. Show that m has to be even, that is m = 2k. Compute m^2 , and deduce that n has to be even too, with is a contradiction

Exercice 1.9

Let the sequence $(x_n)n \in N$ be defined by

$$x_0 = 4, \qquad and\ x_{n+1} = \frac{2x_n^2 - 3}{x_n + 2}$$

Show that: ∀n ∈ N: $x_n > 3$

Exercice 1.10

Prove that: $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)\ , \qquad \forall n \in N^*$