Chapter 2: Balances of matter, momentum and energy, Conservation equation of mass; Conservation equation of momentum; Energy conservation equation.

Goals

At the end of this chapter the student must be able to:

- To write the continuity equation,

- Calculate the mass and volume flow rates,

- To apply Bernoulli's theorem,

- To state Euler's theorem and evaluate the interaction efforts of a fluid with an obstacle.

Introduction

Fluid dynamics concerns the study of fluids in motion. This is a part important part of process engineering because the majority of operations require the transport of fluids or the production of fluid-fluid or fluid-solid mixtures. This chapter aims to address the fundamental equations that govern the dynamics of perfect incompressible fluids (we will not take into account the effects of viscosity $\mu = 0$ and $\rho = cte$), in particular:

- The continuity equation (conservation of mass),

- Bernoulli's theorem (conservation of energy)

- Euler's theorem (Conservation of momentum) from which, we

establishes the equations giving the dynamic force exerted by moving fluids (e.g. water jets).

4.1 General notions about flow

4.1.1 Permanent or stationary flow

A fluid flow is said to be permanent or stationary, if the parameters which characterize the fluid (pressure, speed, temperature, density) are independent of the time at each point of the flow. In other words, the partial derivatives with respect to at the time physical quantities are zero ($\partial/\partial t = 0$). Which means :

$$\vec{v}(\vec{r}, t) = \vec{v}(\vec{r})$$

$$u = u(x, y, z);\ v = v(x, y, z); w = w(x, y, z)$$

Otherwise, the flow is said to be non-permanent or unsteady.

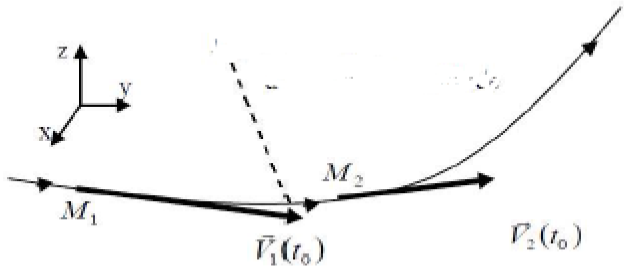
4.1.2 Trajectory and streamlines

4.1.2.1 Trajectory

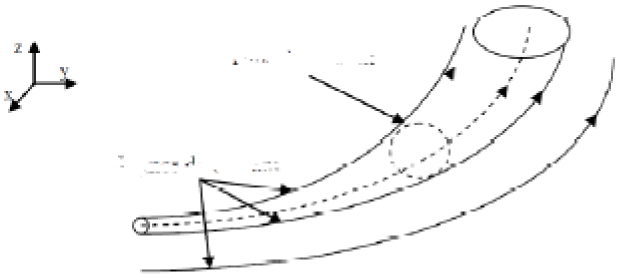
We call trajectory the oriented curve described by a particle during its movement, that is to say all of its positions occupied successively between two moments.

4.1.2.2 Current line

It is the tangent curve at any point in space to the velocity vector, at a given instant



A current tube (fluid vein) is the set of current lines relying on a closed contour



Noticed :

In steady flow, the streamline and the trajectory coincide.

4.2 Continuity equation or conservation of mass

Consider a perfect fluid (liquid), which flows in a steady state in the

pipe below.

We designate by:

S1 and S2 respectively the inlet section and the outlet section of the fluid at time t

· S1′ and S2 ′ respectively the inlet and outlet sections of the fluid at time t= t+dt ,

· V1and V2the flow speed vectors respectively through the sections S1and G S2

· dx1 and dx2 respectively the displacements of sections S1 and S2 during the interval of time ,

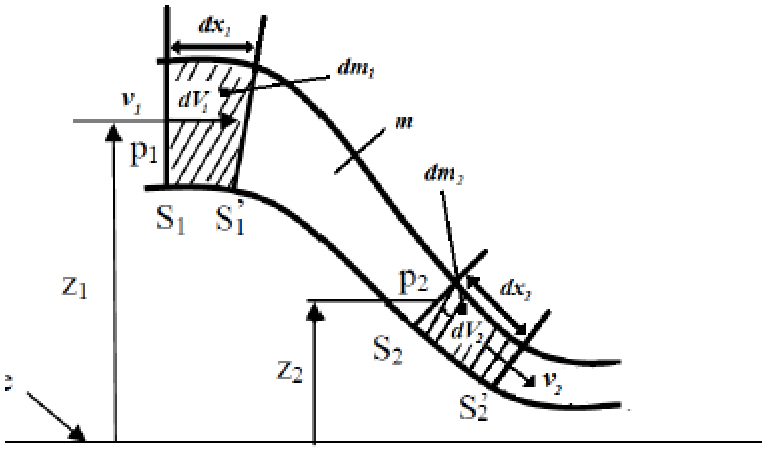
·dm1 incoming elementary mass between sections S1and S1′ ,

·dm2outgoing elementary mass between sections S2 and S2′,

· m: mass between S1and S2,

·dv1 incoming elementary volume included between sections S1 and S1 ′ ,

· dv2 elementary outgoing volume between sections S2and S2 ′,



At t time : the fluid between S1and S2 has a mass equal to dm+ m

At instant t+ dt the fluid between S1 ′ and S2′ to a mass equal to (m +dm2

By conservation of mass:

$$dm_1 + m = m + dm_2$$

Simplifying by m, we obtain

$$dm_1 = dm_2$$

So

$$\rho_1 . dV_1 = \rho_2 . dV_2$$

Or again:

$$\rho_1 . S_1 . dx_1 = \rho_2 . S_2 . dx_2$$

Dividing by gives us:

$$\rho_1 . S_1 . \frac{dx_1}{dt} = \rho_2 . S_2 . \frac{dx_2}{dt} \Leftrightarrow \rho_1 . S_1 . v_1 = \rho_2 . S_2 . v_2$$

Since the fluid is incompressible:

$$: \rho_1 = \rho_2 = \rho$$

We obtain the continuity equation as follows:

$$S_1.v_1 = S_2.v_2$$

This relationship represents the volume flow Q expressed in (m3/s). The continuity equation represents the law of conservation of mass.

4.3 Mass flow and volume flow

4.3.1 Mass flow

The mass flow rate q_m is the elementary mass of fluid passing through a straight surface elementary ds during a time interval dt

so

$$q_m = \frac{dm}{dt}$$

Taking into account the previous equations we obtain:

$$q_m = \frac{dm}{dt} = \rho.S_1.\frac{dx_1}{dt} = \rho.S_2.\frac{dx_2}{dt}$$

$$q_m = \rho.v_1.S_1 = \rho.v_2.S_2$$

Let in any straight section S of the pipe through which the fluid flows at average speed”:

$$q_m = \rho.v.S$$

Or :

q_m: Mass flow in [kg/s];

ρ: Density in [kg/m^3];

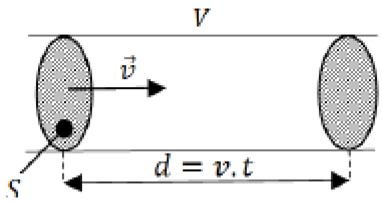
s: Section of the fluid vein in [m^2];

v: Average velocity of the fluid through G in [m/s].

4.3.2 Volume flow

The volume flow ím is the elementary volume of fluid which passes through a surface elementary line G during a time interval .

$$Q_v = \frac{dV}{dt}$$



Or :

dt: Time interval in seconds [s];

dv: Elementary volume, in [m3], having crossed a surface G during an interval of time dt

Q_V Volume of fluid per unit of time which passes through any straight section of the pipe, in [m3/s].

We can also write the volume flow as a function of the flow speed:

$$Q_v = \frac{dV}{dt} = S.\frac{dx}{dt} = S.v$$

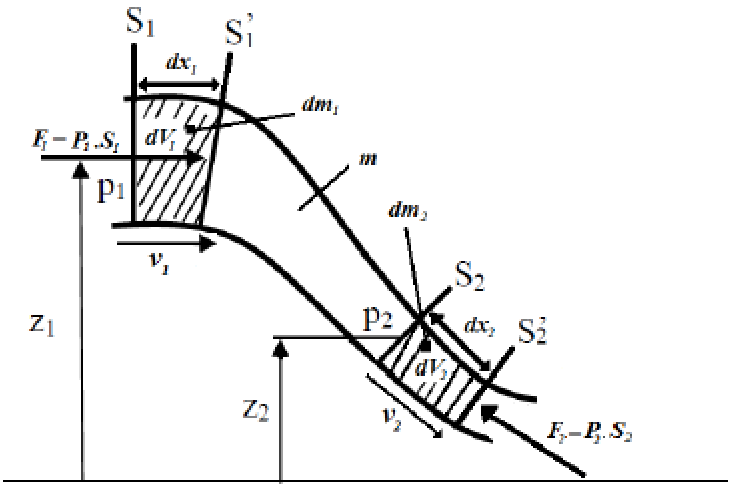
4.3.3 Relationship between mass flow and volume flow

The density is given by the relation:

$$\rho = \frac{dm}{dV} \qquad q_m = \rho.Q_V$$

4.4 Bernoulli's theorem

Or a permanent flow of a perfect incompressible fluid in a pipe. two sections S1 and S2 delimit at the moment a certain mass of fluid. We denote Z1 and Z2 respectively as the heights of the centers of gravity of the masses dm1 and dm2. We designate by Fp1 and Fp2 respectively the norms of the pressure forces of the fluid acting at the level of the sections S1 and S2 .At time t+ dt this mass moves and is between two sections S1′ and S2′. We have so dm= dm1=dm2



Let us apply the kinetic energy theorem to (The variation of kinetic energy is equal to the sum of the work of external forces

$$\Rightarrow \Delta E_C(dm) = \sum W_{1\rightarrow 2}$$

The variation of kinetic energy

$$\Delta E_C(dm) = \frac{1}{2}.dm.(v_2^2 - v_1^2)$$

Gravity force work

$$W_p = (z_1 - z_2).g.dm$$

Work of the internal forces is zero because the fluid is perfect (μ=0)

Work of pressure forces: On S1

$$W_{P1} = F_{P1}.x_1 = P_1.S_1.v_1.dt$$

On S2

$$S_2 : W_{P2} = -F_{P2}.x_2 = -P_2.S_2.v_2.dt$$

On side surface

$$W_{PL} = 0$$

$$\Rightarrow \frac{1}{2}.dm.(v_2^2 - v_1^2) = P_1.S_1.v_1.dt - P_2.S_2.v_2.dt + (z_1 - z_2).g.dm$$

$$\Rightarrow \frac{1}{2}.dm.\rho.(v_2^2 - v_1^2) = P_1.\rho.S_1.v_1.dt - P_2.\rho.S_2.v_2.dt + (z_1 - z_2).\rho.g.dm$$

Or, d'après l'équation de conservation du débit

$$dm = \rho.S_1.v_1.dt = \rho.S_2.v_2.dt$$

SO

$$\frac{1}{2}.dm.\rho.(v_2^2 - v_1^2) = P_1.dm - P_2.dm + (z_1 - z_2).\rho.g.dm$$

We arrive at Bernoulli's equation:

$$P_1 + \rho.g.z_1 + \frac{1}{2}\rho.v_1^2 = P_2 + \rho.g.z_2 + \frac{1}{2}\rho.v_2^2$$

$$\Rightarrow \; P + \rho.g.z + \frac{1}{2}\rho.v^2 = Cte \text{ en [Pa]}$$

The terms of this equation are energies per unit of volume [J/m^3], they are also

pressure terms [Pa].

Indeed, 1 J/m^3 = 1 Nm/m^3 = 1 N/m^2 =1 Pa

So Bernoulli's theorem translates the conservation of energy per unit volume.

P: Static pressure: This is the quantity that we measure, for example, by a pressure gauge or potential energy of pressure/unit of volume.

Pgz Position potential energy per unit volume

1/2ρv^2 Kinetic energy per unit volume or dynamic pressure

· Quantity ρ+ρgZ1/2ρv^2 is also called the fluid charge.

4.4.1 Other forms of Bernoulli's theorem

· By dividing the expression ρ+ρgZ1/2ρv^2 = cte   by ρg we obtain :

$$z + \frac{v^2}{2.g} + \frac{P}{\rho.g} = Cte \quad [m]$$

All terms are unit heights per meter [m] or energy per unit weight (J/N

= N.m / N = m).

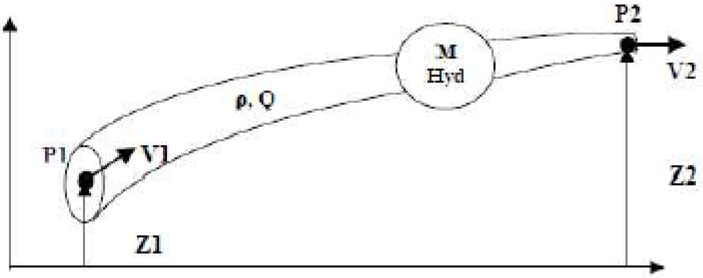
· By dividing the entire expression by ρ we obtain:

$$\frac{1}{2}v^2 + g.z + \frac{P}{\rho} = Cte \quad \text{en } [J/kg]$$

All terms are energies per unit mass

4.4.2 Bernoulli equation with work exchange

In a flow, the total mechanical energy per unit volume of fluid can be be modified from one section to another by introducing a machine into the circuit hydraulic. The machine can be a receiver (turbine) or a generator (pump).



When fluid passes through a hydraulic machine, it exchanges energy with this machine in the form of work ΔW for a duration Δt. The power P exchanged is:

$$P = \frac{\Delta W}{\Delta t} \quad [J/s] \equiv [W]$$

If P > 0: the machine supplies energy to the fluid (pump);

If P < 0: the machine extracts energy from the fluid (turbine).

The application of the Bernoulli equation between the points P_1 and P_2 allows you to write:

$$P_1 + \rho.g.z_1 + \frac{1}{2}\rho.v_1^2 = P_2 + \rho.g.z_2 + \frac{1}{2}\rho.v_2^2 \mp \frac{P}{Q_V}$$

4.6 Euler's theorem

Bernoulli's theorem is of too limited use. In fact, it does not allow to express the mechanical actions that can appear between fluids and solids by example. A direct application of Euler's theorem is the evaluation of the forces exerted by water jets. These are exploited in various fields: energy production electric from hydraulic energy thanks to turbines, cutting of materials, etc. From where the need to introduce a second theorem. Let us consider a portion of incompressible fluid circulating in a pipe animated by a permanent flow. This portion of fluid is delimited by a closed surface (a entrance surface S1 and an exit surface S2 the representation of this surface makes it possible to define the system.

We establish Euler's theorem from the fundamental relation of dynamics:

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

P is the momentum of the system:

$$d\vec{P} = d(m.d\vec{v}) = m_2.\vec{v}_2 - m_1.\vec{v}_1$$

Gold;

$$V_1 = S_1.dx_1 \text{ et } V_2 = S_2.dx_2$$

$$d\vec{P} = \rho.S_2.dx_2.\vec{v}_2 - \rho.S_1.dx_1.\vec{v}_1 = \rho.S_2.v_2.dt.\vec{v}_2 - \rho.S_1.v_1.dt.\vec{v}_1$$

From the continuity equation, we have

$$Q_V = S_2.v_2 = S_1.v_1$$

Which gives

$$: d\vec{P} = \rho.Q_V.dt.\vec{v}_2 - \rho.Q_V.dt.\vec{v}_1$$

Hence

$$: \frac{d\vec{P}}{dt} = \rho.Q_V.(\vec{v}_2 - \vec{v}_1)$$

Euler's theorem therefore becomes:

$$\sum \vec{F}_{ext} = q_m.(\vec{v}_2 - \vec{v}_1)$$

such that ΣF_{ext} the vector sum of the external forces (the resultant of the forces external) applied to an isolated section of fluid, which are:

- The weight of the fluid between S_1 and S_2 the tube;

- The pressure forces applied to section S_1 and S_2 the tube;

- The forces of the fluid on the pipe.

qm: The mass flow rate of the fluid;

v1: Velocity vector of the fluid entering S_1

V2: Velocity vector of the fluid leaving S_2